A Theory of Organizational Dynamics: Internal Politics and Efficiency*

Hongbin Cai[†] Hong Feng[‡] Xi Weng [§]

September 2015

Abstract

In this paper, we develop an infinite horizon dynamic model to study how internal politics affects an organization's admission of new members, and investigate the implications of the dynamic interactions between internal politics and admission of new members on the organization's long-term outcomes and welfare. We consider a three-member organization in which one member retires in each period and the incumbent members vote to admit a candidate to fill the vacancy. Agents differ in quality, which is valued equally by members of the organization, and each agent belongs to one of two types, where members of the majority type in any period control the organization's rent distribution and share the total rent of that period among themselves. We characterize the symmetric Markov equilibria with undominated strategies of the model and develop a method to compare the long-term welfare outcomes among them. It is found that the organization should require consensus in admitting new members: unanimity voting does a better job than majority voting in terms of long-term welfare. In addition, internal politics can be a useful incentive instrument in organizational design: organizations with a certain degree of incongruity perform better in the long run than either harmonious or very divided organizations.

JEL Classification: D72, D71, C73

Keywords: Internal Politics, Organizational Efficiency, Organizational Dynamics

*We wish to thank Botond Kőszegi (the editor) and three anonymous referees for comments and suggestions that greatly improved the article. We thank seminar participants at Hong Kong University, UCLA, USC, University of Texas at Austin and Texas A&M University for helpful comments and suggestions. Remaining errors are our own. Cai and Weng acknowledge support from Key Laboratory of Mathematical Economics and Quantitative Finance (Peking University), Ministry of Education. Weng also acknowledges financial support from the National Natural Science Foundation of China (Grant No. 71303014) and Guanghua Leadership Institute (Grant No. 12-02).

[†]Guanghua School of Management and IEPR, Peking University, Beijing, China 100871. Contact: 86-10-62765132, hbcai@gsm.pku.edu.cn

[‡]RIEM, SWUFE, Chengdu, China 610074.

[§]Guanghua School of Management and IEPR, Peking University, Beijing, China 100871.

1 Introduction

The long-term health and survival of an organization depend crucially on its ability to consistently attract high-caliber new members, because existing members inevitably have to exit the organization for retirement or other reasons. However, admission of new members is often interfered with internal politics, whereby different groups of incumbent members vie for control over the decision-making power of the organization.[1] In the process of admitting a candidate into an organization, incumbent members will look not only at his qualification, but also how his admission affects the future power structure of the organization. In this paper, we develop an infinite horizon dynamic model to study how internal politics affects an organization's admission of new members, and investigate the implications of the dynamic interactions between internal politics and admission of new members on the organization's long-term outcomes and welfare.

We consider a three-member organization (club) in which one of the incumbent members is chosen randomly to exit in each period and, before knowing who will exit, the incumbent members vote to admit a candidate to fill the vacancy. Each player has two characteristics: *quality* and *type*. The uniformly-drawn *quality* represents a player's skills, prestige or resources, which are valuable to every member of the organization. However, internal politics often is anchored on things other than quality, such as race, gender, ideology, specialization (e.g., theorists versus empiricists), or personality. While the political structure of many organizations is often quite complicated, for simplicity we suppose that every player belongs to one of two *types*: left or right. Type matters because we consider *distributive politics* in the sense that there is a fixed amount of rent (e.g., research funds, perks, prestigious positions) in each period and the majority type controls the rent allocation and distributes it among members of its type.[2]

In our model, the club's welfare is independent of its political structure because the amount of rent is fixed. Thus, its per-period welfare is simply the sum of the quality of the club members minus the total search costs. In the first best solution, the social planner of the club optimally trades off the search costs and the benefits of setting a high standard (same for both types of candidates) to get high quality candidates. In another benchmark, we suppose that there is no internal politics in the club (when there is no rent to grab), so all incumbent members have identical preferences and will choose the same admission standard for both types of candidates. In this case, the equilibrium (called the "*harmonious equilibrium*") admission policy is inefficient because there is an "intertemporal free-riding" problem in that incumbent members do not take into account the effects of their admission decisions on future generations of club members.

[1]See March (1962) and Pfeffer (1981) for discussions of internal politics. Other related discussions can be found in Milgrom and Roberts (1988, 1990) and Meyer, Milgrom, and Roberts (1992).

[2]The model can be readily extended to other specifications of rent distributions. See Section 8 for more discussions.

Consequently, all incumbent members search less relative to the efficient level by setting admission standards inefficiently low.

In the presence of internal politics, incumbent members treat different types of candidates differently, and the club's admission policy in each period depends on its power structure in that period. In choosing their strategies, incumbent members not only need to calculate the benefits and costs of admitting a candidate of a given quality, but also need to take into account how the type of the admitted candidate affects the power structure (type profile) of the club in the future. To simplify matters, we focus on symmetric Markov equilibria with undominated strategies in which incumbent members' strategies depend only on the current period type profile of the club, and develop a method to compare their long-term welfare.

We then solve the most efficient equilibrium under both majority and unanimity voting rules in selecting new members. Under either voting rule, the solution crucially depends on the value of a variable which is a function of the model's primitive parameters. This variable can be interpreted as the *degree of incongruity* of the club. It is smaller, or the club is more congruous, when the rent to fight for in each period is smaller (so the gain from internal politics is smaller), or when the uncertainty over candidate quality is greater (so searching for good candidates is more important), or when the delay cost is higher (so the cost of internal politics is larger).

It turns out that the most efficient equilibria can be divided into two categories: "*power-switching*" equilibria, in which both types of candidates are admitted with positive probabilities in every state so power switches back and forth between the two types over time; and "*glass-ceiling*" equilibria, in which candidates of the minority type are never admitted in contentious states (when both types of incumbents are present), and hence the club will never experience power switches. Not surprisingly, under either voting rule, "power-switching" equilibria arise when the club is relatively congruous while "glass-ceiling" equilibria arise when the club is relatively incongruous. The most efficient equilibria under the two voting rules are different in the following two aspects. First, "glass-ceiling" equilibria exist for a much larger range of parameter values under majority rule than under unanimity rule. This is because under majority rule, the majority-type incumbents can easily exclude candidates of the minority type but doing so is very costly under unanimity rule. Second, under majority rule, in a power-switching equilibrium, the majority-type incumbents always discriminate against candidates of the minority type in that candidates of the minority type face higher admission standards than those of the majority type. Such an equilibrium is labeled "pro-majority power-switching." However, under unanimity rule, when the club is sufficiently congruous, the opposite happens in a power-switching equilibrium:: candidates of the majority type are discriminated against in contentious states. This is because the majority-type incumbents have weaker incentives to fight with the minority-type incumbent so the minority-type incumbent can insist on admitting his favored candidate with higher probability. Such an equilibrium is labeled "pro-minority power-switching." As the club becomes more incongruous, the most efficient equilibrium under unanimity rule converts back to pro-majority power-switching, before turning into the glass-ceiling equilibrium.

There are two main findings by comparing the long-term equilibrium outcomes. First, the long-term welfare under unanimity voting is always greater than or equal to that under majority voting. Unanimity voting outperforms majority voting in two scenarios. In one scenario, when the degree of incongruity is intermediate, unanimity rule still allows a power-switching equilibrium, but under majority rule only the less efficient glass-ceiling equilibrium exists. Intuitively, in a dynamic setting with internal politics, majority voting is more exclusive than unanimity voting, as under majority voting the majority incumbent members are more likely to set a glass-ceiling for the opposite type and keep control of the organization forever. In the second scenario, when the degree of incongruity is low, the pro-minority power-switching equilibrium under unanimity rule achieves greater long-term welfare than the pro-majority power-switching equilibrium under majority rule. In the pro-majority power-switching equilibrium, the admission standard of majority-type candidates is set sufficiently low by the majority-type incumbents to keep control of the club, exacerbating the intertemporal free-riding problem in searching for high quality candidates. On the other hand, in the pro-minority power-switching equilibrium under unanimity voting, both types of candidates face quite stringent admission standards, which helps overcome the intertemporal free-riding problem and improves long-term welfare.

Our second main finding is that when the club is relatively congruous, the pro-minority power-switching equilibrium under unanimity voting can yield greater long-term welfare than the harmonious equilibrium. The intuition is that, in the presence of mild internal politics, unanimity voting allows both types of incumbent members in a contentious state to raise the admission standards for candidates of the opposite type. In equilibrium, they will agree on admission standards that are reasonably high but not too high to cause stalemates. Thus, compared with the inefficiently low admission standards in the harmonious equilibrium, politicking by incumbent members can result in more efficient admission standards and thus greater long-term welfare.

Real-world organizations that fit our stylized model include academic departments, social clubs, professional societies, condominium associations and partnership firms, etc.[3] Our paper has two interesting implications about organizational design for such organizations. First, our first main finding provides a new rationale for the optimality of unanimity voting rule and for consensus-based decision-making requirements. Compared with majority voting, unanimity voting allows all incumbent members, regardless of their type, to have voices and decision-making power in an organization. This results in a relatively balanced power structure, which is good

[3]The insight of the model may also apply to organizations that fit some but not all features of the model. For example, in the boards of directors of many public firms, non-profit organizations and local governments (e.g., a city's education board), even though incumbent members do not directly select new members, quite often they do have substantial influence in the selection process.

for the long-term welfare of the organization. Furthermore, by giving real power to the minority-type incumbent members, unanimity voting can avoid favoritism by the majority-type incumbent members toward candidates of their own type, hence maintaining reasonably high admission standards for candidates of both types. Therefore, although unanimity voting and consensus building may involve long decision processes, they may well be what the organization needs in important decisions such as admitting new members. Secondly, our second main finding suggests that there is an optimal degree of organizational incongruity. In a homogeneous organization, it is easy to make decisions but people tend to shirk in searching for high quality candidates. On the other hand, in a highly divided organization, internal politicking is so intense that decision-making processes are long and costly, and the organization eventually becomes perpetually dominated by one type. A good organizational design should avoid these two extremes by trying to achieve the right degree of incongruity. In other words, if properly designed (that is, it has the right amount of rents for discretionary use and unanimity requirements in admitting new members), internal politics can be a useful incentive instrument. In such a case, the organization will remain balanced over time and all members, regardless of their type, are engaged in the admission of new members, resulting in better decisions and better long-term outcomes for the organization.

The rest of the paper is organized as follows. The next section reviews the literature. Section 3 presents the model, and Section 4 solves for two benchmarks: the first best solution of the model and the "harmonious equilibrium" in a politics-free world. We then provide a method to solve for the long-term stationary outcome and welfare in Section 5. In Section 6, we solve for the symmetric Markov equilibria under both majority and unanimity voting rules. Then Section 7 derives the optimal voting rule and other implications for organizational design, and Section 8 contains discussions and concluding remarks.

2 Related Literature

To study dynamic interactions between internal politics and the admission of new members, our paper combines collective search and dynamic club formation. In so doing, our paper is related to both the dynamic club formation literature and the collective search literature.

The dynamic club formation literature stems from the seminal work of Roberts (1999), who studies a dynamic model of club formation in which current members of the club vote by majority rule on whether to admit new members from a fixed population of potential members. He defines Markov Voting Equilibrium (MVE) in this setting, and develops techniques to show the uniqueness of MVE and analyze the steady state of MVE, with a focus on the club size in equilibrium. Instead of considering majority voting rule, Barbera, Maschler, and Shalev (2001) and Granot, Maschler, and Shalev (2002) study how a group of heterogeneous members admit new members in a finite horizon game under other voting rules (quota-1 rule in Barbera, Maschler, and Shalev (2001) and unanimity rule in Granot, Maschler, and Shalev (2002)). Both papers show that a variety of equilibrium outcomes with different club formations can arise due to strategic voting by incumbent members, and offer further equilibrium refinements. The theoretical analysis has been widely applied to investigate how distribution of political power evolves over time in contexts such as immigration laws, suffrage, and constitutional rules (see, e.g., Jehiel and Scotchmer (2001), Lizzeri and Persico (2004), Jack and Lagunoff (2006), Acemoglu, Egorov, and Sonin (2012), and Acemoglu, Egorov, and Sonin (2014)). In particular, building on Roberts (1999), Acemoglu, Egorov, and Sonin (2014) study important issues in political economy such as enfranchisement and political transitions. In a general dynamic setting, they consider how a winning coalition among "ordered" players (e.g., along an ideological line) decides on political changes (i.e., transition to another state) over time in a stochastic environment. They prove the existence and uniqueness of a pure-strategy Markov Voting Equilibrium, and derive interesting comparative statics results regarding repression and other features of political changes.[4]

Our paper differs from the dynamic club formation literature in several aspects. First, the existing literature focuses on how the club size is determined and who will be included in the club or excluded from it, while we study a dynamic club formation problem with a fixed club size. In terms of club composition, our model, with its random exits by incumbent members, is an over-generation model of dynamic club formation; we are interested in the long-term power structure of the club, not the identities of the club members. Second, we embed a collective search problem in the dynamic club formation process to study how internal politics affects the search incentives of club members and the long-term welfare of the club, while the existing literature is mostly abstract regarding what the club actually does besides voting on membership changes. Third, while the existing literature emphasizes positive analysis of the evolution of club formation, we develop a model with more structures on payoffs (values from membership and rent allocation) to allow for comparison of the club's long-term welfare in different situations, so as to facilitate normative analysis of optimal voting rule and other issues of organizational design. We find that unanimity voting dominates majority voting, and internal politics can be welfare-improving under certain conditions.

In the collective search literature (see, e.g., Albrecht, Anderson, and Vroman (2010), Compte and Jehiel (2010), and Moldovanu and Shi (2013)), researchers consider a search problem where a *once-and-for-all* decision to stop is made by a voting committee who examines sequentially each available option ("candidate" in our model). Committee members have different fixed preferences over the options, and collective decisions are made according to a pre-specified voting rule. A main

[4]One of the key assumptions in Acemoglu, Egorov, and Sonin (2014) is a "single crossing" condition, which in our context would imply that right-type members would have higher stage payoffs in the more "right" states. This does not hold in our model, as right-type incumbents prefer the state with two right-type incumbents to the state with all three right-type incumbents because of rent dilution concerns.

focus of this literature is to compare collective search with single-agent search, and to examine how committee composition and decision rules affect search outcomes. In particular, Compte and Jehiel (2010) allow different types of committee members and multi-dimensional candidate attributes, which is similar to our setting. Differing from the collective search literature, our paper studies an infinitely *repeated* problem of collective search in which the formation of the decision-making body endogenously changes over time. In our model, the value of admitting a candidate to an incumbent member is endogenous, in the sense that the change of power structure brought by admitting him affects how members make future admission decisions and hence affects future payoffs. By embedding collective search in the dynamic club formation process, we are interested in how the club's power structure evolves over time, and how internal politics affects the club's admission policies. Our analysis show that unanimity voting is optimal in providing search incentives to incumbent members, and internal politics can promote an organization's long-term welfare under certain conditions.

In the literature, Schmeiser (2012) is the most closely related paper to ours. He considers a model of the dynamics of board composition in which corporate insiders are more able than outsiders but are also more self-interested. As in our model, in each period, existing board members vote to admit a new member to replace a randomly retired member. But unlike our model, in each period, two candidates are simultaneously observed: one insider and one outsider, and the organization must hire one of them. Therefore, there is no collective search, which is essential to our analysis and drives the intertemporal free-riding problem. Another difference between Schmeiser (2012) and our model is that while Schmeiser (2012) uses his model to show that regulations such as a minimal ratio of outsiders on corporate boards can enhance firm value, we focus on the interaction between internal politics and the admission of new members, and derive the optimal voting rule in such a setting.

Our paper is also related to the literature studying how admission standards for new members evolve over time in organizations (see, e.g., Athey, Avery, and Zemsky (2000) and Sobel (2000, 2001)). Our model differs from these existing works in several aspects. First, we focus on the dynamic effects of internal politics on the admission of new members, which is not considered in these papers. Secondly, we build a model that is suitable for welfare analysis, so as to analyze organizational design questions such as the optimal voting rule. For their different purposes, the above-mentioned models by construction do not allow welfare analysis. Thirdly, while Athey, Avery, and Zemsky (2000) and Sobel (2000, 2001) all have a fixed body of decision-makers controlling the admission of new members, in our model the composition of the club changes over time and dynamically interacts with admission policies.

3 The Game

3.1 Model Setup

We consider an infinite-horizon game in discrete time indexed by $t = 0, 1, 2, \cdots$. There is a club of fixed size $N = 3$.[5] In each period t, one of the incumbent members is chosen randomly to exit the club, and before this exit occurs, the three members must select one new member from a large pool of outside candidates who want to join the club. All of the players are risk neutral and maximize their expected utility. For simplicity, we assume there is no discounting.[6] We also normalize the outside option for each player to zero such that it is always desirable to join the club.

A player, either an incumbent member or a candidate, is characterized by his quality and his type. A player's quality, denoted by v, represents the skills, prestige or resources that he can bring to the club and are valuable to the whole club. We suppose that a player of quality v brings a common value of v per period to *every member* of the club including himself, so his total contribution to the club value per period is $3v$. Players in the population differ in quality. For the population, suppose v is distributed according to a uniform distribution function $F(v)$ on $[\underline{v}, \bar{v}]$, where $0 \leq \underline{v} < \bar{v}$. When, in a given period, the club's members have qualities v_k, $k \in \{1, 2, 3\}$, each member's benefit from club membership in that period is $V = \sum_{k=1}^{3} v_k$.

Aside from quality heterogeneity, players belong to one of two types, "left" type and "right" type, and each type is equally represented in the population. Players' types are exogenously given and cannot be changed afterwards.[7] Type is important because club politics is centered on such characteristics. We consider the situation of distributive politics in the following sense. In each period, there is a fixed amount of total rent B in the club to be distributed to its members. We suppose that distribution of rent B is determined by majority voting of the three members in each period, and for simplicity, members of the majority type share the rent equally

[5]The collective search model with heterogeneous committee member types and multi-dimensional candidates is in general very complicated to solve analytically. For example, Compte and Jehiel (2010) focus on the limiting case where the discount rate goes to one. In the Online Appendix, we solve numerically the model with five members, and show that the qualitative results of the baseline model still hold.

[6]In our model random exits from the club serve the role of discounting, thus no discounting over time is needed. In the Online Appendix, we extend the model to allow for more general discounting.

[7]Depending on the applications, type can be interpreted as race, gender, ideology (or party affiliation), or specialization. The exogenous type assumption is plausible for the following reasons. (1) There are cases in which types are either fixed (e.g., race and gender) or very difficult to change (e.g., specialization). (2) In cases where types are choices made by the members (e.g., ideology), there are still conditions under which no one switches type in equilibrium (for example, in the static sense, the current majority members would reject party-switching by the minority member because this would dilute their share of rent).

among themselves.[8] The total benefit to a club member A in a period is the sum of his benefit from club membership V and the rent he receives in that period. Formally, let a single variable $I \in \{0, 1, 2, 3\}$ indicate the number of right types among the club's incumbent members. We will call I the "state" of the club. The states can be further divided into two groups. Contentious states 1 and 2 are respectively called left-majority and right-majority states, and states 0 and 3 are respectively called left-homogeneous and right-homogeneous states. In a club with qualities v_k, $k \in \{1, 2, 3\}$, a right-type incumbent member's current period total benefit is $\sum_{k=1}^{3} v_k + \frac{B}{3}$ in state 3, is $\sum_{k=1}^{3} v_k + \frac{B}{2}$ in state 2, and is $\sum_{k=1}^{3} v_k$ otherwise.

Each period t is divided into three stages. The first (selection) stage may consist of an infinite number of rounds. In each round, a candidate is randomly drawn from the population. His quality and type are then revealed to the incumbent members, who then vote whether to accept him as a new member. Under majority (unanimity) rule, if a candidate gets two (three) or more yes votes, then he is admitted to the club and the selection stage of the current period is over. If a candidate does not get the required yes votes, then the club draws another candidate from the population and uses the same selection procedure to decide whether to admit him. This selection process continues until a candidate is admitted. We suppose that each selection round imposes a cost of $\tau > 0$ to every incumbent member.[9] Since member selection takes at least one round, we count selection costs only if it takes more than one round.

After the admission of a new member, in the second (exiting) stage, one of the incumbent members is chosen randomly to exit the club permanently for exogenous reasons (e.g., natural death, family reasons, retirement). In the third (political decision) stage, the two remaining incumbent members and the new member together decide on club politics (e.g., the distribution of rent B). The same process repeats in each period infinitely.

We want to make several remarks about our model setting.

First, the sequence of move within each period as specified above is convenient for our analysis, because it ensures that there are an odd number of voting members in both the candidate selection stage and the club's political decision stage. One can think of alternative sequences of move, but our results are quite robust in this aspect. For example, suppose at the beginning of each period one of the three incumbent members randomly exits the club (e.g., one faculty member retires in June), and a new member who was admitted in the previous period joins the club (e.g., a new faculty member hired in March arrives in September). The three members then decide on club politics and admission of a new member for the following period (e.g., recruiting season is

[8] For example, one can imagine that the club elects a chairman or president by majority voting, who then decides on distributing some monetary or non-monetary resources (e.g., research funds, office spaces, other perks). The elected official is loyal to his "party", and distributes the rent to members of his type only.

[9] Such a cost can take many forms, e.g., reviewing files, interviewing, meetings, and the opportunity costs of leaving the position vacant.

in January). Our analysis will be completely unchanged with this sequence of move.[10]

Second, in the above formulation of club politics, we make two assumptions. One is of the nature of "incomplete contracts," namely, there are certain rents of the club that cannot be specified in contracts clear enough among club members and hence are subject to ex post negotiations/politicking by the members. This should be true for most organizations: otherwise there is little point in setting up an organization if all of its resources and rents are completely pre-determined in contracts. Moreover, as our results will show, it is actually not always in the best interest of the club to pre-determine rent distribution even if all rents are contractible.

Another assumption in our formulation of club politics is that, by distributive politics, the total amount of rent is constant in each period independent of power structure. This assumption is likely to be satisfied in applications where the discretionary resources of the organization are more or less fixed, e.g., research funds, office spaces, or prestigious positions. By this assumption, the total value of the club depends only on the qualities of its members and is independent of its power structure, which greatly facilitates welfare comparison. One implication of fixed total rent is that each member of the majority type gets a smaller share of the total rent as the majority increases. Thus majority-type incumbents would favor candidates of the opposite type if they are assured of keeping control over the internal politics of the club. In the Online Appendix, we show that the main results of our model can be extended to the case in which the per-capita rent, instead of the total rent, is fixed each period. See Section 8 for more discussions of this assumption.

Finally, we suppose that the club's voting rule for admitting new members is fixed at the beginning of the game and cannot be modified later. This is of course for analytical simplicity, but it is also consistent with the observation that many organizations have very strict requirements for changing their chapter rules or constitutions. Our central interest is in finding the best voting rule for admitting new members in terms of the long-term welfare of the club.

3.2 Strategies and Solution Concepts

In any period, each incumbent member's strategic decision is to vote on whether to accept a candidate or not in the selection stage. There are no actions to be taken in the exiting stage or the political decision stage. In general, an incumbent member's voting decision in a selection stage can depend on the quality and type of the candidate, the quality and type profiles of the

[10] For another alternative sequence of move, suppose at the beginning of each period the club has four members, and one of the incumbent members exits. The three remaining members vote to admit a new member. Then in the political decision stage, the four members vote under majority rule with a pre-specified tie-breaking rule. With some minor modifications in solving the model, our qualitative results should still hold with this alternative sequence of move.

three incumbent members, and all previous histories up to the current round in the current period.

Throughout the paper, we focus on equilibria in which players use Markov strategies. Without putting restrictions on strategies, the game admits trivial equilibria in the following sense. In any given period, if every incumbent member votes "no" on *any* candidate, then it is indeed an equilibrium that no candidate will be admitted forever. But in this equilibrium every incumbent member gets a payoff of negative infinity (as long as τ is positive)! Using this equilibrium as a punishment, then any outcome can be supported in equilibrium. It does not seem reasonable that players can credibly commit to such punishments. By focusing on Markov strategies, we rule out such trivial equilibria by ruling out history-dependent award and punishment schemes.

By definition, in each period t, a Markov strategy of an incumbent $k = 1, 2, 3$ can be written as

$$\sigma_k : [\underline{v}, \bar{v}]^3 \times [\underline{v}, \bar{v}] \times \{L, R\}^3 \times \{l, r\} \to \{yes, no\},$$

where the four determinants of the mapping are the quality profile of the incumbents, the quality of the candidate, the type profile of the incumbents and the type of the candidate, respectively. Note that the specification is time-invariant and history-independent. Denote $\sigma = \{\sigma_k\}_{k=1}^3$ to be the combination of these three incumbent members' strategies.

Let $b \in \{L, R\}$ denote the type of an incumbent member, and $b' \in \{l, r\}$ denote the type of a candidate. Each strategy σ_k determines an incumbent k's acceptance region $\mathcal{A}_k \subset [\underline{v}, \bar{v}] \times \{l, r\}$. Given the strategy profile $\sigma = \{\sigma_k\}_{k=1}^3$ and the club's voting rule, we can hence uniquely determine the club's acceptance region $\mathcal{A} \subset [\underline{v}, \bar{v}] \times \{l, r\}$. This in turn determines the expected quality of the newly admitted candidate $E[v_{\text{new}}|\sigma]$, each member's expected rent conditional on his survival $E[\mu|\{b_k\}, \sigma]$, and the expected search length (or expected delay) $E[d|\sigma]$. Notice that the expected rent also depends on the incumbent members' type profile $\{b_k\}$.

At any time t_0, suppose the quality profile of the incumbent members is $\{v_k\}_{k=1}^3$ and their type profile is $\{b_k\}_{k=1}^3$. For a given admission strategy profile σ, we can calculate the total expected payoff of an incumbent member k. Denote this value as $u_k(\{v_k\}_{k=1}^3, \{b_k\}_{k=1}^3, \sigma)$, and u_k can be determined recursively as

$$\begin{aligned} u_k(\{v_k\}_{k=1}^3, \{b_k\}_{k=1}^3, \sigma) &= \frac{2}{3}\Bigg\{ v_k + E[\mu|\{b_k\}, \sigma] + E[v_{\text{new}}|\sigma] \\ &+ \frac{1}{2}\sum_{j \neq k} \Big[v_j + E[u_k(\{v_k, v_j, v_{\text{new}}\}, \{b_k, b_j, b_{\text{new}}\}, \sigma)|\sigma] \Big] \Bigg\} - \tau E[d|\sigma]. \end{aligned} \quad (1)$$

The interpretation is as follows. The term $\frac{2}{3}$ is the probability that member k survives one period, otherwise member k exits the club and gets the normalized outside option of zero. In the bracket, v_k is member k's own quality, $E[\mu|\{b_k\}, \sigma]$ represents member k's expected rent in this period, and $E[v_{\text{new}}|\sigma]$ is the expected quality of the newly admitted member. Conditional on member k's survival, $\frac{1}{2}$ is the probability that any other member $j \neq k$ survives one period. If member j survives, member k receives v_j, member j's quality, and the expected value in the next period, which is $E[u_k(\{v_k, v_j, v_{\text{new}}\}, \{b_k, b_j, b_{\text{new}}\}, \sigma)|\sigma]$. The last term $\tau E[d|\sigma]$ is the expected search cost to member k in this period.

For simplicity, we focus on symmetric Markov equilibria with weakly stage-undominated strategies of the game, which henceforth we refer to as an "equilibrium." Formally, we say that incumbent member k votes sincerely if for any profile $\{v_k, b_k\}_{k=1}^3$, a candidate with characteristics $(\tilde{v}, b')$ belongs to $\mathcal{A}_k$, if and only if

$$u_k(\{v_k\}_{k=1}^3, \{b_k\}_{k=1}^3, \sigma) - \tau \leq \frac{2}{3}\left[v_k + \tilde{v} + E[\mu|\{b_k\}, b'] + \frac{1}{2}\sum_{j\neq k}\left[v_j + u_k(\{v_k, v_j, \tilde{v}\}, \{b_k, b_j, b'\}, \sigma)\right]\right]. \tag{2}$$

The right-hand side expression of Condition (2) is the total expected payoff to member k from admitting the candidate with characteristics $(\tilde{v}, b')$ right away, and the left-hand side expression is his total expected payoff from rejecting the candidate and searching for another candidate in the next round. So the condition requires sincere voting in the selection stage.[11] This is needed to rule out trivial voting equilibria.

Definition 1 *A symmetric Markov equilibrium with weakly stage-undominated strategies consists of a strategy profile* $(\sigma_1, \sigma_2, \sigma_3)$ *and value functions* (u_1, u_2, u_3) *which satisfy the following conditions:*

(i) For each k, u_k satisfies Equation (1).

(ii) For any $j, k \in \{1, 2, 3\}$, if $v_k = v_j$ and $b_k = b_j$, then $\sigma_k = \sigma_j$.

(iii) Denote $\tilde{b}$ ($\tilde{b}'$) to be the opposite type of type b (b'), and then $\sigma_j(\{v_k\}, v, \{b_k\}, b') = \sigma_j(\{v_k\}, v, \{\tilde{b}_k\}, \tilde{b}')$.

(iv) Each incumbent member k votes sincerely.

The symmetry requirement consists of two parts, (ii) and (iii). First, it requires that incumbent members with the same type and quality choose the same strategies. Moreover, since

[11] This is a common assumption in the literature to rule out equilibria of coordination failure in voting, i.e., voting "no" on a preferred outcome is a weakly dominated best response if everyone else does so (see, e.g., Chan and Suen (2013)). A "trembling hand" argument ensures that voters do not use weakly dominated strategies because there is always a positive probability that he is pivotal. Alternatively, if incumbent members vote sequentially in each selection round, then they will vote their true preferences as well.

the model is symmetric with respect to the two types, right-type incumbents in state i are in the same strategic position as left-type incumbents in state $3-i$. Symmetry requires that in equilibrium, when facing a type b' candidate, right-type incumbents in state i choose the same strategies as left-type members in state $3-i$, facing the opposite type candidate.

4 Two Benchmarks

4.1 The First Best Solution

In this section, we solve for the first best solution for the club as a benchmark case. Since the two types are symmetric, the social planner of the club should have the same admission policy for both types. It is easy to see that the social planner's optimal admission policy should take the following cutoff form: admit a candidate if and only if his quality is at least v^*. Since every member of the club is admitted by such a policy, each member's *expected benefit from club membership per period* is $3E[v|v \geq v^*]$. To calculate the expected search cost in each period, note that the probability that a candidate is admitted is $x^* = 1 - F(v^*)$. Given v is uniformly distributed on $[\underline{v}, \bar{v}]$, denote $a \equiv \bar{v} - \underline{v}$ to be the spread of the quality distribution and $F(v^*) = (v^* - \underline{v})/a$. Hence the expected delay in each period is

$$E[d^*] = \sum_{d=1}^{\infty} x^*(1-x^*)^d d = (1-x^*)/x^* = F(v^*)/(1-F(v^*)).$$

Each member's *expected net value per period* is therefore $3E[v|v \geq v^*] - \tau F(v^*)/(1-F(v^*))$. Maximizing this function, we obtain the following proposition (proof omitted).

Proposition 1 *In the first best solution,*

(i) when $\tau \geq 3a/2$, the club admits any candidate (i.e., $v^* = \underline{v}$).

(ii) when $\tau < 3a/2$, the club admits candidates whose quality is above v^*, where $v^* = \bar{v} - \sqrt{2a\tau/3}$.

In the optimal policy, the social planner trades off the cost of delay and the benefits of setting a high standard to get high quality candidates. When search is very costly, the club admits any candidate to avoid paying the search cost. When the unit search cost τ is not too large, the social planner has an optimal interior searching rule: she will search until a candidate's quality is above a pre-fixed level v^*.

In the interior solution ($v^* = \bar{v} - \sqrt{2a\tau/3}$), the probability that a candidate is admitted in the first best solution can be expressed as $x^* = (\bar{v} - v^*)/a = \sqrt{2\tau/(3a)}$. This has a very simple interpretation. The smaller a is, the smaller is the benefit of searching for one more round.[12] Thus, the admission probability will be higher (or the admission standard will be lower) if the unit search cost τ is higher or the quality distribution has a smaller spread. The club's expected net value in the first best solution can be calculated as $U^* = 3Ev + 1.5a - \sqrt{6a\tau} + \tau$, where Ev denotes the expectation of v.

4.2 Equilibrium without Internal Politics

We now consider another benchmark case in which internal politics is of no importance. This happens when $B = 0$ or equivalently when the club's rent is pre-determined and not subject to the internal politicking of its members. In such a case, all incumbent members have identical preferences over admission policies since they care only about the candidate's quality. In an equilibrium with weakly stage-undominated strategies, they only need to solve for the optimal admission policy that maximizes their payoffs. We call the equilibrium in this case the "harmonious equilibrium." In the harmonious equilibrium, the incumbent members need to solve an optimal stopping problem: admit a candidate if and only if his quality is at least $\hat{v}$.

The expected value to an incumbent member if a candidate with quality $\hat{v}$ is admitted is:[13]

$$\frac{2}{3}\left(1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + ...\right)\hat{v} = \hat{v}.$$

Let w be the *expected net value* an incumbent member can get from selecting a new member using the optimal rule. Clearly, $w \in [\underline{v}, \bar{v}]$.[14] By the definition of $\hat{v}$, it must be that

$$\hat{v} = \max\{w - \tau, \underline{v}\}. \tag{3}$$

When $w - \tau \geq \underline{v}$, this means that if the candidate's quality happens to be $\hat{v}$, the incumbent members must be indifferent between admitting him now (i.e., receiving value $\hat{v}$) and rejecting and waiting to see another candidate in the next round. In the latter case, an incumbent will receive a value of w (from the same optimal admission policy next round) but will incur the waiting cost of τ. When $w - \tau < \underline{v}$, waiting never makes sense so the club should admit any candidate, that is, set $\hat{v} = \underline{v}$.

By the definition of w, we have

$$w = \int_{\hat{v}}^{\bar{v}} v dF(v) + F(\hat{v})(w - \tau) \tag{4}$$

[12]This is analogous to option value increasing in the variance of the return of the underlying asset.

[13]This is because a new member of quality v contributes a value of v in each period he remains in the club, and he is in the club for sure in the period he is admitted and has a survival chance of 2/3 in each of the future periods.

[14]The reason $w \geq \underline{v}$ is that the club can always admit everybody (i.e., $\hat{v} = \underline{v}$).

where the first term is the expected value in the event that the candidate's quality is above $\hat{v}$ (so he is admitted), and the second term is the expected net value in the event that the candidate's quality is below $\hat{v}$ (so the club has to search further).

Equations (3) and (4) define the optimal $\hat{v}$ and the resulting expected net value w. We have the following result (proof omitted).

Proposition 2 *The club's optimal admission policy in the harmonious equilibrium can be characterized as follows:*

(i) when $\tau \geq a/2$, the club admits any candidate (i.e., $\hat{v} = \underline{v}$).

(ii) when $\tau < a/2$, the club admits candidates whose quality is above $\hat{v}$, where $\hat{v} = \bar{v} - \sqrt{2a\tau}$.

(iii) When $\tau < 3a/2$, the admission standard in the harmonious equilibrium is strictly lower than the first best level.

The characterization of the harmonious equilibrium in Proposition 2 is easy to understand. What is interesting is that even in a politics-free world, the club's admission policy is inefficient. In the harmonious equilibrium, an incumbent member only gets a marginal benefit of v from admitting a candidate with quality v, while the social planner's marginal benefit from admitting this candidate is $3v$. Facing the same marginal search cost, an incumbent member in the harmonious equilibrium thus sets a lower standard than the social planner. This is similar to the under-provision of public goods in the standard static model of clubs. However, in our model, inefficiency does not come from free riding among incumbent members in a given period. The joint surplus of all incumbent members in any given period is maximized in the harmonious equilibrium. The source of inefficiency in the harmonious equilibrium is "*intertemporal free-riding*", because incumbent members in the current period do not take into account the benefits of having high quality new members to future generations of club members. Thus, they search less relative to the efficient level by having lower admission standards.[15]

In the interior solution ($\hat{v} = \bar{v} - \sqrt{2a\tau}$), the probability that a candidate is admitted in the harmonious equilibrium is $\hat{x} = (\bar{v} - \hat{v})/a = \sqrt{2\tau/a}$. The club's expected net value in the harmonious equilibrium is $U^h = 3Ev + 1.5a - 2\sqrt{2a\tau} + \tau$, which is strictly lower than that in the first best solution.

[15]As shown by Cai and Feng (2007), an early version of this paper, the result holds for an arbitrary number of members and any distribution of quality.

5 Model Analysis

Suppose $B > 0$ so there is internal politics. In this section, we will introduce the framework for equilibrium analysis and welfare comparison.

5.1 Equilibrium Characterization

Equation (1) provides a recursive definition of value function u_k. To solve the model, we compute the value functions by deduction. At any time t_0, suppose the quality profile of the incumbent members is $\{v_k\}_{k=1}^3$ and the state is i. For a given admission strategy profile σ, we can calculate the expected payoff of an incumbent member, say $k = 1$, who is of type $b \in \{L, R\}$, at time $t = t_0$ as

$$Eu_1(t = t_0) = \frac{2}{3}\left[v_1 + \frac{1}{2}\sum_{k=2}^{3} v_k + E[v_{\text{new}}^{t_0}|\sigma] + E[\mu^{t_0}|\{b_k^{t_0}\}, \sigma]\right] - \tau E[d^{t_0}|\sigma].$$

The interpretation is as follows. The term $\frac{2}{3}$ is the probability that member 1 survives one period, otherwise member 1 exits the club and gets zero payoff. In the square bracket, v_1 is member 1's own quality, $\frac{1}{2}$ is the probability that any other incumbent member $k > 1$ survives one period conditional on member 1's survival, so the second term is the expected total value to member 1 from the other surviving incumbent member. The term $E[v_{\text{new}}^t|i, \sigma]$ is the expected quality of the newly admitted member in the period, the next term $E[\mu^t|i, \sigma]$ represents member 1's expected rent in this period, and the last term $\tau E[d^t|i, \sigma]$ is the expected search cost to member 1 in this period. All three of these terms depend only on the current state of the club and the admission policy. Note that the above equation differs from Equation (1) in that it is the current period payoff; thus it does not have the terms of continuation payoffs found in Equation (1).

Similarly, member 1's expected payoff in the next period $t = t_0 + 1$, $Eu_1(t = t_0 + 1)$, is given by

$$\left(\frac{2}{3}\right)^2\left[v_1 + \frac{1}{2}\left[\frac{1}{2}\sum_{k=2}^{3} v_k + E[v_{\text{new}}^{t_0}|\sigma]\right] + E[v_{\text{new}}^{t_0+1}|\sigma] + E[\mu^{t_0+1}|\{b_k^{t_0+1}\}, \sigma]\right] - \tau E[d^{t_0+1}|\sigma].$$

By deduction, member 1's value function u_1 under strategy profile σ can be written as

$$\begin{aligned} u_1(\{v_k\}_{k=1}^3, \{b_k\}_{k=1}^3, \sigma) &= \sum_{t=t_0}^{\infty} Eu_1(t) \\ &= \sum_{n=1}^{\infty}(\frac{2}{3})^n v_1 + \sum_{n=1}^{\infty}(\frac{1}{3})^n (v_2 + v_3) + \pi_1(\{b_k\}, \sigma) \\ &= 2v_1 + \frac{1}{2}(v_2 + v_3) + \pi_1(\{b_k\}, \sigma), \end{aligned} \tag{5}$$

where the *searching payoff* $\pi_1(\{b_k\}, \sigma)$ contains all the terms about the expected qualities of newly admitted members in each period, the expected rent member 1 gets in each period, and the expected search cost in each period. In other words, $\pi_1(\{b_k\}, \sigma)$ is the expected value incumbent member 1 can get through the club's admissions of new members in current and future periods. It is also worth noting that the coefficient for v_1 and those for v_2 and v_3 are different. This is because member 1 has to stay in the club to enjoy positive benefits. But conditional on member 1's survival, the survival probability for the other two has to be lower.

A key observation is that the quality profile $\{v_k\}$ enters each incumbent member's value function as a constant, and does not directly affect $\pi_k(\{b_k\}, \sigma)$. By Markov properties, member k's admission strategies can be further simplified to

$$\sigma_k : [\underline{v}, \bar{v}] \times \{L, R\}^3 \times \{l, r\} \to \{yes, no\}.$$

where now the strategy σ_k is independent of the quality profile of the incumbent members.

Since strategies do not depend on the quality profile, the symmetry condition (ii) in Definition 1 implies that members of the same type will choose the same admission strategy. Moreover, the distribution of rent is purely determined by the state variable I, defined as the number of right-type incumbent members. Therefore, the same type b incumbent member will receive the same searching payoff in state i, and we denote this searching payoff $\pi_i^b(\sigma)$. The strategies of member k who is type b in state i can be rewritten as

$$\sigma_i^b : [\underline{v}, \bar{v}] \times \{l, r\} \to \{yes, no\}.$$

In our equilibrium analysis, $\pi_i^b(\sigma)$ will play a central role because it is essentially the value function of type b member in state i.

Given Markov strategy σ_i^b and searching payoff π_i^b, Conditions (i)-(iv) in Definition 1 are equivalent to:

(i) For each state i and type b, π_i^b satisfies the following equation:

$$\pi_i^b = \frac{2}{3}\Big[E[v_{\text{new}}|\sigma] + E[\mu|b, i, \sigma] + E[\pi_{i'}^b|i, \sigma]\Big] - \tau E[d|\sigma], \tag{6}$$

where $E[\mu|b, i, \sigma]$ represents type b member's expected rent in state i and i' is the state in the next period;

(ii) Denote $\tilde{b}$ to be the opposite type of type b, and then $\sigma_i^b(v, b') = \sigma_{3-i}^{\tilde{b}}(v, \tilde{b}')$;

(iii) Voting is sincere for each state i and type b.

The sincere voting condition implies that an equilibrium admission strategy profile σ_i^b should take the following cutoff form (minimal admission standards): an incumbent in state i votes "yes" on a candidate of types $b' \in \{l, r\}$ if the candidate's quality is higher than a quality standard $v_i^{b'}$. By symmetry condition (ii) of Definition 2, we only need to specify a right-type incumbent's equilibrium cutoffs (v_i^r, v_i^l), where r, l is the candidate's type, because a left-type incumbent's equilibrium cutoffs in state i are those of the right-type incumbent in state $3-i$. Together with the voting rule, an equilibrium admission strategy profile, now consisting of the quality standards for the two types of candidates set by the two types of incumbents, determines the club's equilibrium admission policy. Under majority voting, the equilibrium admission policy in a contentious state is the same as the majority-type incumbent's equilibrium cutoffs $v_i^{b'}$; while under unanimity voting, the equilibrium admission policy in a contentious state is the larger of the equilibrium cutoffs set by the two types of incumbents.

Given the complexity of the model, even with so many restrictions on equilibrium strategies, there may still be multiple equilibria. We solve this problem by selecting the equilibrium with the highest long-term welfare. One imagines that the founders of the club would want to ensure that the club selects the most efficient equilibrium and commits to the optimal rule which achieves this equilibrium.[16]

5.2 Long-Term Welfare Analysis

Since both the qualities and the types of candidates are uncertain before they arrive, the club's value and type composition are stochastic over time. Our welfare analysis will focus on the long-term (stationary) behavior of these stochastic processes.

With an equilibrium admission policy (v_i^l, v_i^r) in state i, the probability of the newly admitted member being the right type, p_i^r, must satisfy

$$p_i^r = 0.5[1 - F(v_i^r)] + 0.5F(v_i^r)p_i^r + 0.5F(v_i^l)p_i^r.$$

That is, the new member can be of the right type in one of the three events whose probabilities correspond to the three terms above, respectively: (1) the first candidate is of the right type with quality above v_i^r and so is admitted; (2) the first candidate is of the right type with quality below v_i^r and so is rejected but the club admits a right-type candidate eventually; and (3) the first candidate is of the left type with quality below v_i^l and so is rejected, but the club admits a right-type candidate eventually. Solving for p_i^r, we have

$$p_i^r = \frac{1 - F(v_i^r)}{2 - F(v_i^r) - F(v_i^l)} = \frac{\bar{v} - v_i^r}{2\bar{v} - v_i^r - v_i^l}. \tag{7}$$

Similarly, the probability of the newly admitted member in state $i \in \{0, 1, 2, 3\}$ being of the left type is given by

$$p_i^l = \frac{\bar{v} - v_i^l}{2\bar{v} - v_i^r - v_i^l}.$$

[16] For example, Barzel and Sass (1990) provide evidence that developers of condominiums choose voting rules for condominium homeowner's associations to maximize the value of condominium purchase to potential homeowners.

The evolution of the state variable, the number of right-type incumbents in the club I, constitutes a Markov chain. Its transition probability matrix can be written as

$$\mathbf{P} = (p_{ij})_{i,j\in\{0,1,2,3\}} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & \quad 1 & \quad\quad 2 & \quad\quad 3 \end{array} \\ \begin{pmatrix} p_0^l & p_0^r & 0 & 0 \\ \frac{1}{3}p_1^l & \frac{1}{3}p_1^r + \frac{2}{3}p_1^l & \frac{2}{3}p_1^r & 0 \\ 0 & \frac{2}{3}p_2^l & \frac{2}{3}p_2^r + \frac{1}{3}p_2^l & \frac{1}{3}p_2^r \\ 0 & 0 & p_3^l & p_3^r \end{pmatrix} \end{array}.$$

Given $\mathbf{P}$, the stationary probability distribution $\mathbf{Q}$ is given by

$$\mathbf{Q} = \mathbf{P}'\mathbf{Q}, \tag{8}$$

where $\mathbf{Q} = \{q_i\}$ and $q_i \in [0, 1]$ is the stationary probability of the state i for $i \in \{0, 1, 2, 3\}$ such that $\sum_i q_i = 1$, and $\mathbf{P}'$ is the transpose of $\mathbf{P}$.

Given $\mathbf{Q}$ and the club's equilibrium admission policy in each state, we can evaluate the club's long-term welfare. First, the long-term *expected quality* of a representative club member in the club, denoted by s, can be calculated as follows:

$$s = \sum_{i=0}^{i=3} q_i \left(p_i^r E[v|v \geq v_i^r] + p_i^l E[v|v \geq v_i^l] \right); \tag{9}$$

where for each state i, q_i is the probability that state i happens, p_i^r (resp., p_i^l) is the probability that a newly admitted member will be of the right type (resp. left type), and $E[v|v \geq v_i^r]$ (resp., $E[v|v \geq v_i^l]$) is the expected quality of a newly admitted right-type (resp. left-type) member. Notice that the expression $p_i^r E[v|v \geq v_i^r] + p_i^l E[v|v \geq v_i^l]$ is the expected quality of a new member in a given state i under the equilibrium admission policy (v_i^l, v_i^r). Taking expectation over the states using the stationary probability distribution thus gives the expected quality of a representative member in the club.

To calculate the expected search cost in the long-term stationary world, note that for any given state i and equilibrium admission policy (v_i^l, v_i^r), a candidate is admitted with probability of

$$x_i = 0.5(1 - F(v_i^r)) + 0.5(1 - F(v_i^l)) = 1 - 0.5F(v_i^r) - 0.5F(v_i^l).$$

The expected delay in state i is then given by

$$E[d_i] = \sum_{d=1}^{\infty} x_i(1 - x_i)^d d = (1 - x_i)/x_i.$$

Thus, the expected delay in the long-term stationary world is

$$D = \sum_{i=0}^{i=3} q_i E[d_i]. \quad (10)$$

The long-term welfare of the club can be measured as follows:

$$U = 3s - \tau D = 3 \sum_{i=0}^{i=3} q_i \left(p_i^r E[v|v \geq v_i^r] + p_i^l E[v|v \geq v_i^l] \right) - \tau \sum_{i=0}^{i=3} q_i E[d_i]. \quad (11)$$

Once we have solved the equilibrium admission policy, the above formula can be used to compare long-term welfare under different voting rules.

6 Equilibria with Internal Politics

In this section we characterize the most efficient equilibrium of the model with internal politics under majority voting and unanimity voting. To avoid trivial corner solutions, we suppose that the unit search cost τ is less than $a/4$. This assumption appears to be reasonable in most applications, because the selection costs involved in recruiting one candidate, such as the time costs of reading files and going to meetings, should be small relative to the importance of admitting high quality new members.

6.1 Equilibrium under Majority Voting

Under majority voting, the majority type in the current period determines the equilibrium admission policy in that period. Specifically, in each of the four states (left homogeneous, right homogeneous, left majority, right majority), the majority type decides the minimal quality standards to admit left- and right-type candidates. In a symmetric equilibrium, four admission standards need to be determined: admission standards for left- and right-type candidates in the right-homogeneous state and the right-majority state, and then admission standards in the left-homogeneous state and the left-majority state can be found identically for the opposite types of candidates. Fixing admission policies, the value functions of type b incumbent members, π^b, can be calculated in the way described in the previous section. With these value functions, we can analyze the optimal admission policy for the incumbent members in each state. The detailed steps for characterizing the most efficient equilibrium and the proofs of our results in the rest of the paper are relegated to the Appendix.

Intuitively, when deciding whether to admit a candidate, the majority-type incumbents take into account four factors: (i) his qualifications, (ii) the search cost (if he is to be rejected and another candidate sought)), (iii) the effect on rent allocation in the current period, and (iv) the effect on future power structures of the club. Factor (iv) depends on how power structures evolve in the future, which depends on admission policies in different states. For example, in a right-majority state, the right-type incumbents may lose control of the rent distribution in the current period if admitting a left-type candidate, as well as future control of the club (if the left-type incumbents, when in power, do not admit right-type candidates).

In equilibrium, it turns out that the admission policies and the pattern of power changes hinge on one simple variable, which is defined as $c \equiv B/(12\sqrt{a\tau})$. The variable c can be interpreted as the club's *degree of incongruity*. It is small (or, the club is congruous) when the rent B (the gain from politicking) is small, or when admitting high quality candidates is important (the uncertainty of candidate quality a is relatively large), or when delay is costly (τ is relatively large).

The next proposition characterizes the most efficient equilibrium under majority voting.

Proposition 3 *Under majority voting rule,*

(i) when the club is relatively congruous ($0 < c < 0.43$*), the most efficient equilibrium is the "power-switching equilibrium" in which both types of candidates are admitted with positive probabilities so that power switches back and forth between the two types;*

(ii) when the club is relatively incongruous ($c > 0.43$*), the most efficient equilibrium is the "glass-ceiling equilibrium" in which, in contentious states, the majority-type incumbents will never admit candidates of the opposite type so that the club will never experience a change of power.*

Proposition 3 says that the pattern of the most efficient symmetric equilibrium under majority voting crucially depends on the degree of incongruity c. Intuitively, when the club is relatively congruous (c is small), searching for better candidates is more important than grabbing rent through internal politics, so the majority-type incumbents will admit candidates of the opposite type who are of high quality. As a result, the control over rent allocation will change hands between the two types in this power-switching equilibrium. However, if the club is relatively incongruous (c is large), controlling rent allocation becomes the dominant concern for the majority-type incumbents, so they do not admit candidates of the opposite type no matter how qualified they are. Consequently, in this case, the type controlling rent allocation at the very beginning will always hold power in the club, and the minority type will never have a real say in the internal politics, thus the name "glass-ceiling equilibrium."[17]

[17] The existence of the glass-ceiling equilibrium crucially depends on the assumption that v has bounded support. In the Online Appendix, we study the case where v follows an exponential distribution with parameter λ. It turns out that our main results (Proposition 5) do not change in that case.

With internal politics, admission policies will be distorted. To be more explicit about the distortions, we compare admission probabilities instead of admission standards. Recall that the admission probability for any type of candidate is $x^* = \sqrt{2\tau/(3a)}$ in the first best and is $\hat{x} = \sqrt{2\tau/a}$ in the harmonious equilibrium in the absence of internal politics. Define $x_i^{b'} \equiv (\overline{v} - v_i^{b'})/a$ as the probability that a type b' candidate will be admitted in state i in an equilibrium under internal politics, where v_i' is the admission standard. The proof of Proposition 3 also implies the following corollary.

Corollary 1 *In both the power-switching and glass-ceiling equilibria, the majority-type incumbents in contentious states favor candidates of their own type and discriminate against candidates of the opposite type: $x_2^r > \hat{x} = \sqrt{2\tau/a} > x_2^l$; but in homogeneous states they have lower standards for the opposite type than for their own type: $x_3^l > \hat{x} = \sqrt{2\tau/a} > x_3^r$. Moreover, the distortions are greater in contentious states than in homogeneous states.*

Corollary 1 says that under majority rule, the candidates of the majority type always have a higher probability of being admitted in contentious states but a lower probability in homogeneous states. In contentious states, the majority-type incumbents fear that admitting a candidate of the opposite type may shift the balance of power against them and hence set much higher standards for candidates of the opposite type than for those of their own type. When the club is relatively incongruous, the majority-type incumbents' discrimination goes to the extreme and candidates of the opposite type are completely excluded. In contrast, when all three members are of the right (or left) type, they are safely in control of the power over rent distribution. Since they prefer sharing rent with fewer members of their own type, they will favor candidates of the opposite type and discriminate against those of their own type. The distortion of admission standards is smaller in homogeneous states than in contentious states because majority-type incumbents do not need to worry about losing control over rent allocation in the current period.

6.2 Equilibrium under Unanimity Voting

Under unanimity voting rule, all incumbent members need to reach a consensus about admitting a candidate. This is easily achieved in homogeneous states. But in contentious states, the majority- and minority-type incumbents will have different standards for each type of candidate, and the admission criterion is given by the higher standard between the two types of incumbent members.

Using an approach similar to that used in solving for equilibria under majority rule, we can characterize the most efficient equilibrium under unanimity voting rule.

Proposition 4 *Under unanimity voting rule,*

(i) *when the club is congruous* ($0 < c < 0.47$)*, the most efficient equilibrium is the "pro-minority power-switching" equilibrium in which candidates of both types are admitted in each state with positive probabilities, but candidates of the majority type in contentious states have a lower probability of being admitted than those of the minority type;*

(ii) *when the degree of incongruity is intermediate* ($0.48 < c < 1.97$)*, the most efficient equilibrium is the "pro-majority power-switching" equilibrium in which candidates of the majority type in contentious states have a higher probability of being admitted than those of the minority type;*

(iii) *when the club is very incongruous* ($c > 1.97$)*, the most efficient equilibrium is the glass-ceiling equilibrium.*

Similar to Proposition 3, the most efficient equilibrium under unanimity rule also involves power-switching when the club is relatively congruous and glass-ceiling when the club is relatively incongruous. However there are important differences between the two cases.

Figure 1 below depicts the normalized equilibrium admission probabilities in the right-majority state when the degree of incongruity c is small ($c < 0.43$).[18]

Insert Figure 1 here.

As shown in Figure 1, under majority rule, the "power-switching" equilibrium favors the majority-type candidates, in the sense that candidates of the majority type are admitted with higher probability (lower standard) than those of the minority type in contentious states. However, under unanimity rule, candidates of the left type are more likely to be admitted than those of the right type in the right-majority state. We call this "pro-minority power-switching" equilibrium to emphasize the contrast to the "pro-majority power-switching" equilibrium. In the pro-minority power-switching equilibrium, the minority incumbent member has more power than his majority peers in selecting new members. This is because under unanimity rule, the minority-type incumbent has strong incentives to block the majority-type candidates so that he may gain the control over the rent allocation. In contrast, the majority-type incumbents can choose whether or not to fight with the minority-type incumbent. In contentious states, each majority member still has a 50% chance of being in power after the admission of a minority-type candidate (conditional on his remaining in the club), thus he has weaker incentives to block the minority-type candidates.

[18] To simplify comparison, we normalize admission probabilities by multiplying them by $\sqrt{a/\tau}/2 > 1$. The normalized admission probabilities take values between zero and one.

Thus, when the club is congruous (admitting high-quality candidates is more important than controlling rent allocation), the majority-type incumbents will avoid fighting with the minority-type incumbent and hence the minority member can take advantage of this to admit his favored candidate with higher probability. It can be shown that the “pro-minority power-switching” equilibrium is the unique (hence trivially the most efficient) equilibrium when c is small, as stated in Proposition 4(i).

Proposition 4(ii) says that as the degree of incongruity c falls into an intermediate range, the most efficient equilibrium is the pro-majority power-switching equilibrium. In this range, there always exist multiple equilibira, but the pro-majority power-switching equilibrium leads to greater long-term welfare than other equilibria. The coexistence of the pro-majority and pro-minority power-switching equilibria is like a Game of Chicken. In contentious states, the minority-type incumbent will be tough if he expects the majority-type incumbents to be soft, which leads to the pro-minority power-switching equilibrium. On the other hand, the minority-type incumbent will be soft if he expects the majority-type incumbents to be tough, which leads to the pro-majority power-switching equilibrium. When both exist, the efficiency comparison of the two equilibria depends on the degree of incongruity. For a small range of c ($0.43 < c < 0.47$), both equilibria exist, but the pro-minority power-switching equilibrium dominates in efficiency. This case is included in Proposition 4(i). As c increases, in the pro-minority power-switching equilibrium, the minority-type incumbent keeps raising the admission standard for the majority-type candidates in contentious states (see Figure 1), resulting in more and more welfare loss. On the contrary, in the pro-majority power-switching equilibrium, when c increases, the distortion of admission standards by the majority-type incumbents does not increase as fast as in the pro-minority power-switching equilibrium, because the majority-type incumbents fear the loss of power less than the minority-type incumbent does in contentious states. Therefore, for $c > 0.47$, when both exist, the pro-majority power-switching equilibrium dominates the pro-minority power-switching equilibrium in efficiency .

As in the case of majority voting (Proposition 3), the welfare comparison between the pro-majority power-switching equilibrium and the glass-ceiling equilibrium favors the former for relatively small c, and becomes reversed for sufficiently large c. For relatively small c, in the range when both equilibria exist, the majority-type incumbents in contentious states are still willing to admit high quality minority-type candidates, because of their high quality and also because of the need to reduce unworthy search costs. Since internal politics is not very important, the majority-type incumbents are not very afraid of losing control in the future by admitting candidates of the opposite type. Thus, for relatively small c, the pro-majority power-switching equilibrium leads to less distortion in admission policies and greater long-term werlfare than the glass-ceiling equilibrium.

Proposition 4(ii) gives the range of c in which the pro-majority power-switching equilibrium is the most efficient, by combining the ranges of c in which it dominates either the pro-minority power-switching equilibrium or the glass-ceiling equilibrium, or both.[19]

Proposition 4(iii) says that for sufficiently large c, the most efficient equilibrium is the glass-ceiling equilibrium. Intuitively, when internal politics is very important, in contentious states, the majority-type incumbents are reluctant to admit candidates of the opposite type because they expect that power will be difficult to regain once they lose it, and the minority-type incumbent wants to insist on high standards for candidates of the majority type because control over rent allocation is too important to give up. Thus, there will be large political costs in contentious states. And this will make incumbents in homogeneous states hesitant to admit candidates of the opposite type. Therefore, as the club becomes very incongruous, candidates will face very stringent admission standards (except those of the same type as the incumbents in homogeneous states) and hence the club experiences inefficiently long delays in selecting new members. As a result, for sufficiently large c, the most efficient equilibrium switches to the glass-ceiling equilibrium, which mitigates internal politics by eliminating the chance of the minority's having a say.

7 Optimal Voting Rule and Organizational Design

Using the equilibrium characterization results of the preceding section, in this section we investigate the optimal voting rule and other organizational design issues. Naturally, we suppose that the founder or social planner of the club adopts the voting rule that yields the greatest long-term welfare for the club.

Equation (11) of Section 5 gives the definition of long-term welfare for the club. It can be shown that the welfare function in the cases we are interested in can be expressed as

$$U = 3Ev + \frac{3}{2}a + \tau - 2\sqrt{a\tau}\gamma,$$

where γ summarizes the total long-term expected welfare of the club in each case. Aside from the model's parameters, the welfare function of the club depends only on γ: the smaller γ is, the more efficient it is for the club. This greatly simplifies our welfare caparison.

It can be calculated that in the first best solution $\gamma^* = \sqrt{6}/2$, and in the harmonious equilibrium $\hat{\gamma} = \sqrt{2}$. In the case of majority voting,

$$\gamma^m \equiv 4q_3/\left(y_3^r + y_3^l\right) + 4q_2/\left(y_2^r + y_2^l\right),$$

[19]For some range of c, there exist other types of equilibria, which are easily dominated. The Online Appendix provides a complete equilibrium characterization under unanimity voting.

where q_3 (q_2) is the long-term stationary probability of the club being in the right-homogeneous (majority) state, and $y_i^{b'} = x_i^{b'}\sqrt{a/\tau}/2$ is the normalized admission probability of a candidate of type $b' = l, r$ in state i.

In the unanimity voting case, similarly we have

$$\gamma^u \equiv 4q_3/(y_3^r + y_3^l) + q_2(1 + 3(y_1^l)^2 + 3(y_2^l)^2)/(y_1^l + y_2^l).$$

From the above expressions, we can calculate the expected welfare loss in each case and obtain the following result.

Proposition 5 *(i) For every c, the club can achieve greater or equal long-term welfare under unanimity voting than under majority voting.*

(ii) For $c < 0.42$, under unanimity voting the club can achieve higher long-term welfare than in the harmonious equilibrium. At $c = 0.24$, the club achieves the highest long-term welfare under unanimity voting.

(iii) Under majority voting, the club cannot achieve greater long-term welfare than it can in the harmonious equilibrium.

Insert Figure 2 here.

Figure 2 illustrates the comparison of welfare losses γ in different cases. Evidently, welfare loss is never lower under majority voting than under unanimity voting, thus the club achieves greater or equal long-term welfare under unanimity voting than under majority voting. Thus, Proposition 5(i) says that unanimity is a better voting rule than majority when the club's admission of new members is influenced by internal politics. As can be seen from Figure 2, unanimity voting outperforms majority voting in two scenarios. First, when c is low, the pro-minority power-switching equilibrium under unanimity rule achieves greater long-term welfare than the pro-majority power-switching equilibrium under majority rule. As shown in Figure 1, candidates of both types face stringent admission standards in the pro-minority power-switching equilibrium under unanimity voting, while candidates of the majority type are admitted with a much lower standard in the pro-majority power-switching equilibrium under majority voting. As a result, by giving both types of incumbent members more balanced power in admitting new members, unanimity voting can avoid straightforward favoritism by the majority-type incumbents and motivate all members to search for high quality candidates. Secondly, for $0.43 < c < 1.97$, unanimity voting still allows power-switching equilibria, but majority rule only allows the glass-ceiling equilibrium which is much less efficient. Intuitively, majority voting gives the majority-type incumbent members unlimited power to exclude the opposite type candidates. Thus, as the stake of internal politics becomes large, they will completely exclude the opposite type and keep control of the club firmly in their own hands.

Also evident from Figure 2, Proposition 5(ii) says that for $c < 0.42$, the pro-minority power-switching equilibrium under unanimity voting yields greater long-term welfare than the harmonious equilibrium. The reason can be clearly seen from Figure 1, as the admission standards for both types of candidates are higher in the pro-minority power-switching equilibrium than in the harmonious equilibrium. Intuitively, when internal politics is mild, under unanimity voting both types of incumbent members will set stringent standards to admit candidates of the opposite type, which helps offset the intertemporal free riding in the harmonious equilibrium.

Proposition 5 (iii) says that majority voting always yields lower long-term welfare than the harmonious equilibrium. In either the pro-majority power-switching or the glass-ceiling equilibrium, admission standards are biased relative to those in the harmonious equilibrium in that candidates of one type face a much lower standard than that applied to candidates of the other type. Thus, as the welfare loss function is convex in admission standards, the divergence of admission standards for the two types of candidates (relative to that in the harmonious equilibrium) leads to lower long-term welfare under majority voting than in the harmonious equilibrium. Moreover, from Figure 2, we see that the long-term welfare under majority voting is decreasing in the degree of incongruity: organizations with a higher degree of incongruity always perform worse in the long run under majority voting.

We should point out that the superiority of unanimity voting in our model crucially depends on our focus on the most efficient equilibria. Compared with majority voting, unanimity voting is more likely to have multiple equilibria because coordination between different members in the same generation and across generations is more important. Thus, whether unanimity voting rule is good for the club depends on whether the club members can manage to select the best equilibrium. If they fail to do so, unanimity voting can lead to worse outcomes than majority voting.[20]

By showing that unanimity rule dominates majority rule, Proposition 5 provides a new rationale for unanimity voting. In the common-value voting literature, unanimity voting is found to be an inferior collective decision mechanism (see, e.g., Feddersen and Pesendorfer (1998)). These different results should not be viewed as contradictory, because the contexts are quite different.

[20] In the Online Appendix, we characterize all different kinds of equilibria under unanimity voting. For $c > 1.97$ there still exists a power-switching equilibrium under unanimity voting, in which incumbents of the two types engage in intensive politicking and take very long delays in admitting a new member. It is shown that such an equilibrium is worse for the club than the glass-ceiling equilibrium under majority voting.

In our model, voting is used to aggregate preferences in a collective search situation, while the common-value voting literature considers information aggregation in collective decisions.[21] Another common criticism of committee decision making is that it causes inefficiently long delays in reaching agreements,[22] and clearly delays will be the worst under unanimity voting. Our analysis shows that in the presence of internal politics, unanimity voting is effective in motivating members to engage in costly search and thus raises the club's long-term welfare. Indeed it takes longer to reach a decision under unanimity voting than under majority voting in our model, but this is actually good for the organization (although not necessarily for individual members who have to incur personal delay costs).[23]

Our analysis suggests that organizations may benefit from requiring important decisions to be made by consensus. For example, quite often university administrations approve senior hiring proposals by academic departments only if those proposals have had super-majority or even unanimous support within the departments, a mere majority support is usually perceived as a weak signal by university administrations. Similarly, in many partnership firms, new partners may only be admitted by unanimous vote of the existing partners. A casual argument for such requirements is that even though decision-making processes may take a long time in organizations that emphasize consensus-building, they tend to make better decisions as all members are involved in decision-making and tend to be more balanced as no group of members can dominate by forming a majority coalition. Our analysis provides a clear mechanism and conditions under which such requirements are indeed optimal.[24]

Our results also suggest that there is an optimal degree of organizational incongruity. In a homogeneous organization, it is easy to reach agreements but members tend to shirk in their efforts at making important decisions for the organization due to the intertemporal free-riding

[21]It would be an interesting empirical question to distinguish preference aggregation from information aggregation in collective decisions, and to test the different predictions from the two theoretical perspectives.

[22]Such as "A committee is a thing which takes a week to do what one good man can do in an hour " by Elbert Hubbard.

[23]Albrecht, Anderson, and Vroman (2010)) also show that unanimity voting is optimal in a collective search model, but in a limiting sense when search cost per round goes to zero (in their model the discounting factor between search rounds goes to one).

[24]For another example, due to cultural influences, firms in Japan, Korea and other East Asian countries tend to emphasize consensus-building as a distinct trait of corporate culture. In some Japanese companies, agreement is normally obtained by circulating a document which must first be signed by the lowest level manager, and then upwards, and may need to be revised and the process may have to start over again (Verma (2009)). As argued by Ouchi (1981), the consensual decision-making style in Japan improves firm performance, which is supported by several empirical studies (see, e.g., Dess (1987)). On the other hand, other factors not considered here can be important in optimal corporate decision-making structures. For instance, consensus decision-making tends to be more conservative and less risk-taking, which may or may not be an advantage depending on environments.

problem. On the other hand, in a highly divided organization, internal politicking is so intense that decision-making processes are exceedingly long and costly, and the organization eventually becomes perpetually dominated by one type. A good organizational design should avoid these two extremes by trying to achieve the right degree of incongruity.[25] In other words, internal politics, whereby members of an organization compete for discretionary rents, if designed properly, can be a useful incentive instrument. In such cases, the organization will remain balanced over time and members of different types are all engaged in the important decisions of the organization, resulting in better decisions and better long-term outcomes for the organization.

As Proposition 5(ii) shows, the optimal level of incongruity c is about 0.24. To be at this optimal level, it requires that $B = 2.88\sqrt{a\tau}$. Thus, B should be larger if a or τ is greater. Specifically, in organizations where admitting high quality candidates is very important (large a), then rents available for discretionary use (e.g., prestigious positions, discretionary resources) should be relatively large. Similarly, if it is difficult to motivate members to be engaged in the admission of new members (τ is large), then organizations should make available more discretionary rents.

By conducting a welfare comparison, we derive Proposition 5 as the main result of our normative analysis. Our model also allows us to conduct positive analysis of the equilibrium behavior of the organization, and derive implications that are potentially testable in empirical contexts. There are at least three aspects of organizational behavior to investigate. First of all, Figure 3 depicts the expected search length, measured by the weighted average of the expected length in each state, where the weight is the long-term probability of each state. Figure 3 shows that under both majority and unanimity voting, the expected search length is increasing in c. That is, more incongruous organizations have longer decision time. Moreover, unanimity voting always yields a higher expected search length than majority voting. This is because under unanimity voting, both types of incumbent members set stringent standards for the admission of candidates of the opposite type, while under majority voting the majority incumbent members favor their own type.

Secondly, Figure 4 depicts the fraction of time spent in contentious vs. homogeneous states, measured by the relative frequency in the long run: q_3/q_2. It can be seen that this relative frequency is discontinuous in c under both majority and unanimity voting. For majority voting, the discontinuity reflects the switch from the power-switching equilibrium to the glass-ceiling equilibrium; for unanimity voting, the discontinuity reflects the switch from the pro-minority equilibrium to the pro-majority equilibrium. Moreover, it is interesting to notice that this relative frequency is non-monotonic in c under both voting rules. This reflects two opposite effects caused

[25]In an interesting study, Milliken and Martins (1996) find that diversity has negative effects on group outcomes early in a group's life, but after this stage, once a certain level of behavioral integration has been achieved, groups may be able to obtain benefits from diversity.

by an increase of c. For example, when c becomes larger, the majority incumbent members on the one hand have higher incentives to stay in the homogeneous states to avoid the risk of losing control. But on the other hand they also have higher incentives to switch to the contentious states to avoid "dilution" of rent.

Finally, Figure 5 depicts the expected length of time between switches of control. Under majority voting, this expected length is increasing in c when $c < 0.43$: as c becomes larger, the majority incumbent members are less reluctant to lose control. For $c > 0.43$, the expected length is infinity as there is no switch in the glass-ceiling equilibrium. Under unanimity voting, the expected length is decreasing in c in the pro-minority power-switching equilibrium. However, in the pro-majority power-switching equilibrium, the expected length is again non-monotonic in c as shown in Figure 4.

Insert Figure 3 here.

Insert Figure 4 here.

Insert Figure 5 here.

8 Discussions and Concluding Remarks

In this paper we build an infinite-horizon dynamic model to study the interactions of an organization's internal politics and its decisions to admit new members. Among other things, we find that it is beneficial for organizations to build consensus in the presence of internal politics: unanimity voting does a better job than majority voting in terms of long-term welfare. In addition, internal politics can be a useful incentive instrument in organizational design: organizations with a certain degree of incongruity perform better in the long run than either harmonious or very divided organizations.

Our model can be extended in different directions. In the Online Appendix, we present several extensions, including (i) different quality distributions, (ii) different discounting factors, and (iii) a larger club size. It turns out that the main results of the baseline model are mostly robust in these extensions. It would also be interesting to consider other extensions in future research, such as a situation where candidates can endogenously choose qualities by human capital investments as in Athey, Avery, and Zemsky (2000), Sobel (2000) and Sobel (2001). Given the admission biases in each of the equilibria of the model (pro-majority power-switching, pro-minority power-switching, glass-ceiling), candidates of the two types will have different incentives to make human capital investments, which in turn will affect how the club admits different types of candidates.

An important extension for future research would consider different kinds of internal politics. To make welfare comparison simple, we have considered distributive politics where the total rent in each period is constant and is shared by the majority-type incumbents, so that the type profile of the club does not affect welfare directly. In other situations, the total rent available in each period may not be constant and may depend on the type profile of the club. For example, there can be situations in which each of the majority-type members can get a fixed amount of rent regardless of the size of the majority. Or, besides his quality providing a common value to every member, a candidate may bring an additional common value *only* to incumbent members of his type (e.g., a new theorist benefits incumbent theorists in a department). In these situations, rent dilution is of less or no concern to the majority-type incumbents, which makes them more likely favor candidates of the same type than in the current model. In the Online Appendix, we study the extension where the per-capita rent is fixed, and show that most findings on the long-term welfare are similar to the current model. In addition, welfare is higher in the model with fixed total rent than in the model with fixed per-capita rent when the degree of incongruity is small, but the opposite is true when the degree of incongruity is relatively large. In future research it would be interesting to investigate the organizational design that would yield the most desirable rent distribution.

Appendix

A.1 Equilibrium Analysis under Majority Voting

Under majority voting, consider a right-type incumbent member A. From Section 5.1, we only need to solve A's searching payoff where the subscript i denotes the current state, the superscript R denotes A's type, and the admission policy σ is suppressed to simplify notation.

In state $i = 2$, if the club admits a right-type candidate with quality v^r in the first selection round, A's expected searching payoff is

$$\pi_2^R(v^r, \text{yes}) = \frac{2}{3}\left[v^r + \frac{1}{2}\left(\frac{B}{2} + \frac{1}{2}v^r + \pi_2^R\right) + \frac{1}{2}\left(\frac{B}{3} + \frac{1}{2}v^r + \pi_3^R\right)\right]. \tag{12}$$

With probability $\frac{2}{3}$, A survives one period. In that event, A receives v^r, the quality of the admitted candidate. Moreover, conditional on A's survival, the other left- and right-type incumbent each exit with probability $\frac{1}{2}$. If the right-type exits, A receives rent $\frac{B}{2}$ and the continuation searching payoff is π_2^R. Moreover, from Equation (5), the future expected value brought by the quality of the newly admitted candidate (taking into account the survival rate) is $\frac{1}{2}v^r$.[26] If the left-type exits, A receives rent $\frac{B}{3}$ and continuation value $\frac{1}{2}v^r + \pi_3^R$. Notice that the qualities of the incumbent members do not appear in Equation (12) because by definition, π only contains information about the expected qualities of newly admitted members in each period, the expected rent member A gets in each period, and the expected search cost in each period.

Equation (12) can be further simplified as

$$\pi_2^R(v^r, \text{yes}) = v^r + \frac{5B}{18} + \frac{1}{3}\pi_2^R + \frac{1}{3}\pi_3^R,$$

which again implies that the expected value to incumbent member A if a candidate with quality v^r is admitted is v^r.

Similarly, in state $i = 2$, if a left-type candidate with quality v^l is admitted, a right-type incumbent member A's expected π_2^R is

$$\pi_2^R(v^l, \text{yes}) = v^l + \frac{1}{3}\pi_1^R + \frac{1}{3}\left(\frac{B}{2} + \pi_2^R\right). \tag{13}$$

[26] To understand why this term is necessary, maybe it is helpful to compare with another scenario in which instead of the right-type candidate John with quality v^r, another right-type candidate Joe with quality v' is admitted. Suppose the other right-type incumbent member exits in the current period. Then A, the left-type incumbent, and Joe (or John) will be the incumbents in the next period. Either Joe or John is admitted, the right-type is still the majority in the next period, and A will get the same expected searching payoff π. But the quality difference between John and Joe will make a difference for A in the next period, whether he goes to the next period with John or Joe. And this difference is precisely reflected in the term of $1/2v^r$.

In state $i = 3$, member A's expected searching payoff from admitting a left-type candidate with quality v^l can be calculated as follows:

$$\pi_3^R(v^l, \text{yes}) = v^l + \frac{B}{3} + \frac{2}{3}\pi_2^R. \tag{14}$$

If the club admits a right-type candidate with quality v^r, then

$$\pi_3^R(v^r, \text{yes}) = v^r + \frac{2B}{9} + \frac{2}{3}\pi_3^R. \tag{15}$$

Similarly, in state $i = 2$, the expected expressions of searching payoff for a left-type incumbent member from admitting a left-type candidate with quality v^l and from admitting a right-type candidate with quality v^r are given by

$$\pi_2^L(v^l, \text{yes}) = v^l + \frac{B}{3} + \frac{2}{3}\pi_1^L, \tag{16}$$

and

$$\pi_2^L(v^r, \text{yes}) = v^r + \frac{2}{3}\pi_2^L. \tag{17}$$

If a candidate is rejected by the club, no matter what the type or quality of the candidate is, a type $b \in \{L, R\}$ incumbent member's searching payoff simply becomes $\pi_i^b - \tau$.

Notice that under majority voting, the block of right-type incumbents decide the admission policy (v_i^l, v_i^r) in state $i = 2, 3$. As a result, Equation (6) implies:

$$\pi_i^R = E\left[\frac{1}{2}\max\left\{\pi_i^R(v^r, \text{yes}), \pi_i^R - \tau\right\} + \frac{1}{2}\max\left\{\pi_i^R(v^l, \text{yes}), \pi_i^R - \tau\right\}\right]. \tag{18}$$

For an admission policy (v_i^l, v_i^r) to be optimal for a right-type incumbent in a state i, it must be that, for candidate of types $b' \in \{l, r\}$,

$$v_i^{b'} \begin{cases} = \underline{v} & , \quad \text{if } \pi_i^R(v_i^{b'} = \underline{v}, \text{yes}) \geq \pi_i^R - \tau; \\ \in (\underline{v}, \bar{v}) & , \quad \text{if } \pi_i^R(v_i^{b'}, \text{yes}) = \pi_i^R - \tau; \\ = \bar{v} & , \quad \text{if } \pi_i^R(v_i^{b'} = \bar{v}, \text{yes}) \leq \pi_i^R - \tau; \end{cases} \tag{19}$$

where $\pi_i^R(\cdot, \text{yes})$ is defined by Equations (12) to (15).

Given the equilibrium admission policy (v_i^l, v_i^r), we can now calculate the expected searching payoff of a type $b \in \{R, L\}$ incumbent member in state i as follows:

$$\pi_i^b = 0.5\left[\int_{v_i^r}^{\bar{v}} \pi_i^b(v_i^r, \text{yes})dF(v_i^r) + F(v_i^r)(\pi_i^b - \tau) + \int_{v_i^l}^{\bar{v}} \pi_i^b(v_i^l, \text{yes})dF(v_i^l) + F(v_i^l)(\pi_i^b - \tau)\right]. \tag{20}$$

Proof of Proposition 3 Step i) Characterizing the power-switching equilibrium: Under majority voting rule, the first possibility is that both types of candidates are admitted at state 2. Then Condition (19) is satisfied with equality for $i = 2, 3$ and $b' = l, r$. After some algebra calculation, we have

$$
\begin{aligned}
\frac{2}{3}\left[\frac{3}{2}v_3^r + \frac{B}{3} + \pi_3^R\right] &= \pi_3^R - \tau; && (21)\\
\frac{2}{3}\left[\frac{3}{2}v_3^l + \frac{B}{2} + \pi_2^R\right] &= \pi_3^R - \tau; && (22)\\
\frac{2}{3}\left[\frac{3}{2}v_2^r + \frac{5B}{12} + \frac{1}{2}\pi_2^R + \frac{1}{2}\pi_3^R\right] &= \pi_2^R - \tau; && (23)\\
\frac{2}{3}\left[\frac{3}{2}v_2^l + \frac{B}{4} + \frac{1}{2}\pi_1^R + \frac{1}{2}\pi_2^R\right] &= \pi_2^R - \tau. && (24)
\end{aligned}
$$

By Equation (20), we can obtain, for $i = 2, 3$ and $b = r$,

$$
\begin{aligned}
\pi_3^R &= \frac{\overline{v} - v_3^r}{3a}\left[\frac{B}{3} + \pi_3^R\right] + \frac{\left[\overline{v}^2 - (v_3^r)^2\right]}{4a} + \frac{v_3^r - \underline{v}}{2a}[\pi_3^R - \tau]\\
&+ \frac{\overline{v} - v_3^l}{3a}[\frac{B}{2} + \pi_2^R] + \frac{\left[\overline{v}^2 - \left(v_3^l\right)^2\right]}{4a} + \frac{v_3^l - \underline{v}}{2a}[\pi_3^R - \tau]; \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
\pi_2^R &= \frac{\overline{v} - v_2^r}{3a}\left[\frac{5B}{12} + \frac{1}{2}\pi_2^R + \frac{1}{2}\pi_3^R\right] + \frac{\left[\overline{v}^2 - (v_2^r)^2\right]}{4a} + \frac{v_2^r - \underline{v}}{2a}[\pi_2^R - \tau]\\
&+ \frac{\overline{v} - v_2^l}{3a}\left[\frac{B}{4} + \frac{1}{2}\pi_1^R + \frac{1}{2}\pi_2^R\right] + \frac{\left[\overline{v}^2 - \left(v_2^l\right)^2\right]}{4a} + \frac{v_2^l - \underline{v}}{2a}[\pi_2^R - \tau]. \qquad (26)
\end{aligned}
$$

Also by Equation (20), and using the fact that $\pi_2^L = \pi_1^R$, we have

$$
\begin{aligned}
\pi_1^R &= \frac{\overline{v} - v_2^r}{3a}\pi_1^R + \frac{\left[\overline{v}^2 - (v_2^r)^2\right]}{4a} + \frac{v_2^r - \underline{v}}{2a}[\pi_1^R - \tau]\\
&+ \frac{\overline{v} - v_2^l}{3a}[\frac{B}{2} + \pi_2^R] + \frac{\left[\overline{v}^2 - \left(v_2^l\right)^2\right]}{4a} + \frac{v_2^l - \underline{v}}{2a}[\pi_1^R - \tau]. \qquad (27)
\end{aligned}
$$

Thus, we have a system of seven equations (21)-(27) with seven unknowns: $v_2^r, v_2^l, v_3^r, v_3^l, \pi_1^R, \pi_2^R, \pi_3^R$.

Substituting (21) and (22) into (25) and simplifying, we can get $4a\tau = (\overline{v} - v_3^r)^2 + (\overline{v} - v_3^l)^2$. Using our variable transformation $x_i^{b'} \equiv (\overline{v} - v_i^{b'})/a$, we have

$$(x_3^r)^2 + (x_3^l)^2 = 4\tau/a. \tag{28}$$

Similarly, substituting Equations (23) and (24) into (26) and simplifying, we can get $4a\tau = (\overline{v} - v_2^r)^2 + (\overline{v} - v_2^l)^2$, or,

$$(x_2^r)^2 + (x_2^l)^2 = 4\tau/a. \tag{29}$$

From Equations (21), (22) and (24), we can get

$$\begin{aligned}
\pi_3^R &= \frac{2B}{3} + 3\tau + 3v_3^r; \\
\pi_2^R &= \frac{B}{2} + 3\tau + \frac{9}{2}v_3^r - \frac{3}{2}v_3^l; \\
\pi_1^R &= \frac{B}{2} + 3\tau + 9v_3^r - 3v_3^l - 3v_2^l.
\end{aligned}$$

Substituting π_3^R and π_2^R into (23) gives $\frac{B}{6} = 2v_3^r - v_3^l - v_2^r = (\overline{v} - v_2^r) + (\overline{v} - v_3^l) - 2(\overline{v} - v_3^r)$. Thus,

$$x_2^r + x_3^l - 2x_3^r = \frac{B}{6a}. \tag{30}$$

Substituting π_1^R, π_2^R into (27) and manipulating terms, we can obtain

$$(x_2^r + x_2^l)B/a = 3x_2^r x_3^r + 3x_2^l x_3^l + 6x_2^l x_2^r - 12(x_2^l)^2. \tag{31}$$

Thus, we have four equations (28)-(31) and four unknowns: x_3^r, x_3^l, x_2^r and x_2^l. To further simplify things, let $y_i^{b'} = x_i^{b'}\sqrt{a/\tau}/2$, for $i = 1, 2, 3, 4$ and $b' = l, r$. Define $c = B/(12\sqrt{a\tau})$. Then (28)-(31) become

$$\begin{aligned}
(y_3^r)^2 + (y_3^l)^2 &= 1 \\
(y_2^r)^2 + (y_2^l)^2 &= 1 \\
y_2^r + y_3^l - 2y_3^r &= c \\
y_2^r y_3^r + y_2^l y_3^l + 2y_2^l y_2^r - 4(y_2^l)^2 &= 2c(y_2^r + y_2^l).
\end{aligned} \tag{32}$$

By the first two equations of (32), all $y_i^{b'}$ must be in $(0, 1)$. A solution to (32) must also have the following properties:

Claim 1: If $c = 0$, then $y_i^{b'} = \sqrt{2}/2$ is a solution, which coincides with the harmonious equilibrium.

Proof: It is easy to check that $y_i^{b'} = \sqrt{2}/2$ is a solution to (32) when $c = 0$. Then $x_i^{b'} = 2y_i^{b'}\sqrt{\tau/a} = \sqrt{2\tau/a}$. By our calculation in Section 5, in the harmonious equilibrium, $\hat{x} = (\bar{v} - \hat{v})/a = \sqrt{2\tau/a}$. *Q.E.D.*

Claim 2: y_2^l cannot be the largest among the four unknowns. Otherwise, the RHS of the last equation of (32) is negative. Contradiction.

Claim 3: $y_3^r \leq y_3^l$.
Proof: Otherwise, if $y_3^r > y_3^l$, the third equation of (32) implies that

$$y_2^r = 2y_3^r + c - y_3^l > y_3^l.$$

Then it must be that $y_2^r > y_3^r > y_3^l > y_2^l$, where the last inequality follows from $(y_3^r)^2 + (y_3^l)^2 = (y_2^r)^2 + (y_2^l)^2$. However, substituting the third equation (as the expression of c) into the last equation of (32) gives

$$2(y_2^r)^2 + 2y_2^r y_3^l + y_3^l y_2^l - 5y_2^r y_3^r + 4(y_2^l)^2 - 4y_3^r y_2^l = 0.$$

This is inconsistent with the fact that y_2^r and y_3^r are the largest. Contradiction. *Q.E.D.*

Claim 4: $y_3^r \leq y_2^r$. Otherwise, it must be that $y_2^r < y_3^r \leq y_3^l < y_2^l$, since $(y_3^r)^2+(y_3^l)^2 = (y_2^r)^2+(y_2^l)^2$. But this violates Claim 2. Contradiction.

Claim 5: $y_2^r \geq y_3^l \geq \frac{\sqrt{2}}{2} \geq y_3^r \geq y_2^l$.
Proof: Suppose $y_3^l > y_2^r$. Then it must be that $y_3^l > \{y_2^r, y_2^l\} \geq y_3^r$. From the third equation of (32), $y_2^r = 2y_3^r + c - y_3^l$. Substituting this into the third term of the LHS of the last equation of (32), we have

$$y_2^r y_3^r - y_2^l y_3^l + 4y_2^l y_3^r - 4(y_2^l)^2 = 2cy_2^r.$$

The LHS is negative when $y_3^l > \{y_2^r, y_2^l\} \geq y_3^r$, because $4y_2^l y_3^r \leq 4\left(y_2^l\right)^2$ and $y_2^r y_3^r < y_2^l y_3^l$. Therefore, it must be that $y_2^r \geq y_3^l$. By Claims 4 and 5 and the fact that $(y_3^r)^2 + (y_3^l)^2 = (y_2^r)^2 + (y_2^l)^2 = 1$, it must be that $y_2^r \geq y_3^l \geq \frac{\sqrt{2}}{2} \geq y_3^r \geq y_2^l$. *Q.E.D.*

Substituting the first two equations of (32) into the last two gives

$$\begin{aligned} y_2^r + \sqrt{1-(y_3^r)^2} - 2y_3^r &= c \\ y_2^r y_3^r + \sqrt{1-(y_2^r)^2}\sqrt{1-(y_3^r)^2} + 2y_2^r\sqrt{1-(y_2^r)^2} - 4\left(1-(y_2^r)^2\right) &= 2c\left(y_2^r + \sqrt{1-(y_2^r)^2}\right). \end{aligned}$$

Substituting the first equation above into the second gives one equation in terms of y_3^r only. Based on this, define function:

$$
\begin{aligned}
\Omega(y;c) &= \left(c+2y-\sqrt{1-y^2}\right)y+\sqrt{1-\left(c+2y-\sqrt{1-y^2}\right)^2}\sqrt{1-y^2} \\
&+ 2\left(c+2y-\sqrt{1-y^2}\right)^2\sqrt{1-\left(c+2y-\sqrt{1-y^2}\right)^2}-4\left(1-\left(c+2y-\sqrt{1-y^2}\right)^2\right) \\
&- 2c\left[\left(c+2y-\sqrt{1-y^2}\right)+\sqrt{1-\left(c+2y-\sqrt{1-y^2}\right)^2}\right]
\end{aligned}
$$

It can be shown numerically that $\max_y \Omega(y;c) > 0$ for $c < 0.43$ and vice versa. Therefore, the power-switching equilibrium exists for $c < 0.43$. *Q.E.D.*

Proof of Proposition 3 Step ii) Characterizing the glass-ceiling equilibrium: Another possibility is that $v_2^l = \bar{v}$, and hence by Condition (19), the following condition must hold:

$$
\frac{2}{3}\left[\frac{3}{2}\overline{v}+\frac{1}{2}\pi_1^R+\frac{1}{2}\pi_2^R+\frac{B}{4}\right] \leq \pi_2^R-\tau. \qquad (33)
$$

With $v_2^l = \bar{v}$, Equations (21), (22), (23) and (25) should still hold and Equations (26) and (27) are changed to

$$
\pi_2^R = \frac{\pi_2^R-\tau}{2}+\frac{\overline{v}-v_2^r}{3a}\left[\frac{5B}{12}+\frac{1}{2}\pi_2^R+\frac{1}{2}\pi_3^R\right]+\frac{\left[\overline{v}^2-(v_2^r)^2\right]}{4a}+\frac{v_2^r-\underline{v}}{2a}[\pi_2^R-\tau]; \qquad (34)
$$

$$
\pi_1^R = \frac{\pi_1^R-\tau}{2}+\frac{\overline{v}-v_2^r}{3a}\pi_1^R+\frac{\left[\overline{v}^2-(v_2^r)^2\right]}{4a}+\frac{v_2^r-\underline{v}}{2a}[\pi_1^R-\tau]. \qquad (35)
$$

Thus we have six equations (21), (22), (23), (25), (34) and (35) with six unknowns: v_2^r, v_3^r, v_3^l, π_1^R, π_2^R, π_3^R. The solution to this equation system must also satisfy (33) for it to constitute an equilibrium.

Similarly define $x_i^{b'} \equiv (\overline{v}-v_i^{b'})/a$ and $y_i^{b'} = x_i^{b'}\sqrt{a/\tau}/2$. From the first and third equations of (32), $(y_3^r)^2+\left(y_3^l\right)^2 = 1$ and $y_3^l-2y_3^r = c-1$. We can obtain the following solution:

$$
\begin{aligned}
y_3^r &= \frac{1}{5}\left(\sqrt{4+2c-c^2}-2c+2\right) \\
y_3^l &= \frac{1}{5}\left(2\sqrt{4+2c-c^2}+c-1\right).
\end{aligned}
$$

Then from Equations (34) and (35), we can get

$$\pi_3^R = \frac{2B}{3} + 3\tau + 3v_3^r$$
$$\pi_2^R = 3\overline{v} + 3\tau + \frac{3}{4}B - 3\sqrt{a\tau}\,(1 + y_3^r)$$
$$\pi_1^R = 3\overline{v} + 3\tau - 6\sqrt{a\tau}.$$

Substituting π_2^R and π_1^R into (33), we get $y_3^r < 2c$. This is satisfied if and only if $c > \frac{10}{29}$. It is straightforward to check that when $c > \frac{10}{29}$, $y_3^l > y_3^r$ since y_3^l is increasing in c and y_3^r is decreasing in c. Also notice that when $c > 2$, $\frac{1}{5}\left(\sqrt{4 + 2c - c^2} - 2c + 2\right) < 0$. Actually in this case it's not difficult to verify that in the glass-ceiling equilibrium, $y_3^r = 0, y_3^l = 1$. *Q.E.D.*

Proof of Proposition 3 Step iii) Comparing the long-term welfare: Under majority voting, using Equation (11), we can show that the long-term welfare of the club is given by

$$U^m = 3Ev + \frac{3}{2}a + \tau - 2\sqrt{a\tau}\gamma^m$$

where $\gamma^m \equiv 4q_3/\left(y_3^r + y_3^l\right) + 4q_2/\left(y_2^r + y_2^l\right)$. For $c \in (10/29, 0.43)$, both the power-switching and glass-ceiling equilibria exist. But the power-switching equilibrium dominates the glass-ceiling equilibrium in welfare. This completes the proof of the proposition. *Q.E.D.*

A.2 Equilibrium Analysis under Unanimity Voting

Abusing notation slightly, let v_2^r (v_1^r) and v_2^l (v_1^l) be the right-type incumbents' preferred quality standards in state 2 (1) for right- and left-type candidates, respectively. By symmetry, v_1^r (resp., v_1^l) is the left-type incumbent's preferred standard for a left-type (resp. right-type) candidate in state 2. Then, the admission criterion in state 2 is $\tilde{v}_2^r = \max\{v_2^r, v_1^l\}$ for right-type candidates, and $\tilde{v}_2^l = \max\{v_2^l, v_1^r\}$ for left-type candidates. Lemma 1 below says that under unanimity voting rule, the admission criterion for a candidate is determined by the preferred standard of the incumbent members of his opposite type.

Lemma 1 *Under unanimity voting rule, in any equilibrium $v_2^r \leq v_1^l$ and $v_2^l \geq v_1^r$. Thus, $\tilde{v}_2^r = v_1^l$ and $\tilde{v}_2^l = v_2^l$.*

Proof of Lemma 1: First we can show the following result:

Lemma 2

$$(x_2^r)^2 + (x_2^l)^2 = \frac{4\tau}{a} + \left[\max\{x_2^r, x_1^l\} - x_1^l\right]^2 + \left[\max\{x_2^l, x_1^r\} - x_1^r\right]^2$$
$$(x_1^r)^2 + (x_1^l)^2 = \frac{4\tau}{a} + \left[\max\{x_2^r, x_1^l\} - x_2^r\right]^2 + \left[\max\{x_2^l, x_1^r\} - x_2^l\right]^2.$$

Proof: Since the admission criterion is now given by $\tilde{v}_2^r = \max\{v_2^r, v_1^l\}$ and $\tilde{v}_2^l = \max\{v_2^l, v_1^r\}$, Equations (26) should be modified as follows:

$$\begin{aligned} \pi_2^R \quad = \quad & \frac{\overline{v} - \tilde{v}_2^r}{3a}\left[\frac{5B}{12} + \frac{1}{2}\pi_2^R + \frac{1}{2}\pi_3^R\right] + \frac{\left[\overline{v}^2 - (\tilde{v}_2^r)^2\right]}{4a} + \frac{\tilde{v}_2^r - \underline{v}}{2a}[\pi_2^R - \tau] \\ & + \frac{\overline{v} - \tilde{v}_2^l}{3a}\left[\frac{B}{4} + \frac{1}{2}\pi_1^R + \frac{1}{2}\pi_2^R\right] + \frac{\left[\overline{v}^2 - \left(\tilde{v}_2^l\right)^2\right]}{4a} + \frac{\tilde{v}_2^l - \underline{v}}{2a}[\pi_2^R - \tau]. \end{aligned}$$

Moreover, Equations (23) and (24) should also be satisfied by the requirement of sincere voting. Using Equations (23) and (24), we can simplify the above equation as

$$(\overline{v} - v_2^r)^2 + (\overline{v} - v_2^l)^2 = 4a\tau + (\tilde{v}_2^r - v_2^r)^2 + (\tilde{v}_2^l - v_2^l)^2.$$

Since

$$\frac{\tilde{v}_2^r - v_2^r}{a} = \frac{\overline{v} - v_2^r}{a} - \frac{\overline{v} - \tilde{v}_2^r}{a} = x_2^r - \min\{x_2^r, x_1^l\} = \max\{x_2^r, x_1^l\} - x_1^l$$

and similarly

$$\frac{v_1^l - v_2^l}{a} = \max\{x_2^l, x_1^r\} - x_1^r,$$

we get the first statement of the lemma. Using the same method on Equation (27), and with the fact that

$$\begin{aligned} \frac{2}{3}\left[\frac{3}{2}v_1^r + \frac{B}{2} + \pi_2^R\right] \quad &= \quad \pi_1^R - \tau; && (36) \\ \frac{2}{3}\left[\frac{3}{2}v_1^l + \pi_1^R\right] \quad &= \quad \pi_1^R - \tau, && (37) \end{aligned}$$

we can prove the second statement of the lemma. *Q.E.D.*

To prove the proposition, let's first consider an interior equilibrium where all the quality standards are less than $\overline{v}$. From Equations (21)-(24) and (36)-(37), we can eliminate all the πs to get

$$\begin{aligned} x_2^r + x_3^l - 2x_3^r \quad &= \quad \frac{B}{6a}; && (38) \\ x_3^r - x_2^l + x_2^r - x_1^l \quad &= \quad \frac{B}{3a}; && (39) \\ x_2^l + x_1^r - 2x_1^l \quad &= \quad \frac{B}{2a}. && (40) \end{aligned}$$

We now eliminate all the other possibilities to prove the proposition.

(a) Suppose $v_1^l \geq v_2^r, v_1^r > v_2^l$, then $x_1^l \leq x_2^r, x_1^r < x_2^l$. By the above lemma, we have

$$\begin{aligned}(x_2^r)^2 + (x_2^l)^2 &= \frac{4\tau}{a} + (x_2^r - x_1^l)^2 + (x_2^l - x_1^r)^2 \\ (x_1^r)^2 + (x_1^l)^2 &= \frac{4\tau}{a}.\end{aligned}$$

Substituting the second equation into the first equation, we can get $x_2^l x_1^r + x_2^r x_1^l = (x_1^r)^2 + (x_1^l)^2$. But this cannot hold, because by $x_1^l \leq x_2^r$ and $x_1^r < x_2^l$, the RHS is less than the LHS.

(b) Suppose $v_1^l < v_2^r, v_1^r \leq v_2^l$, then $x_1^l > x_2^r, x_1^r \geq x_2^l$. Following the same method as in part (a), we can get $x_2^r x_1^l + x_2^l x_1^r = (x_2^r)^2 + (x_2^l)^2$, which is impossible since $x_1^l > x_2^r, x_1^r \geq x_2^l$.

(c) Suppose $v_1^l < v_2^r, v_1^r > v_2^l$, then $x_1^l > x_2^r, x_1^r < x_2^l$. Equation (40) and $x_1^r < x_2^l$ imply that $x_1^l < x_2^l$. Equation (39) and $x_1^l > x_2^r$ imply that $x_2^l < x_3^r$. Thus, we have $x_2^r < x_1^l < x_2^l < x_3^r$. By Equation (38), we must have $x_3^l > x_3^r$. From Lemma 2 we have

$$\begin{aligned}(x_2^r)^2 + (x_2^l)^2 &= \frac{4\tau}{a} + (x_2^l - x_1^r)^2 \\ (x_1^r)^2 + (x_1^l)^2 &= \frac{4\tau}{a} + (x_1^l - x_2^r)^2.\end{aligned}$$

Summing them up and substituting $\frac{4\tau}{a}$ by $(x_3^r)^2 + (x_3^l)^2$ (since Equation (28) is still valid), we can get

$$(x_3^r)^2 + (x_3^l)^2 = x_1^r x_2^l + x_2^r x_1^l. \tag{41}$$

But this contradicts the fact that x_3^l and x_3^r are greater than all the four variables on the RHS.

In summary, in an interior equilibrium, it must be that $v_1^l \geq v_2^r$ and $v_1^r \leq v_2^l$.

Now consider that some of the standards are greater than $\overline{v}$. Part (a) and (b) of the above proof are still valid. For part (c), assuming $\widehat{v}_3^r$ satisfies Equation (21), which means

$$\frac{2}{3}\left[\frac{3}{2}\widehat{v}_3^r + \frac{B}{3} + \pi_3^R\right] = \pi_3^R - \tau$$

and assuming the same thing for Equations (22)-(24), (36)-(37), we can get $\widehat{v}_3^l, \widehat{v}_2^r, \widehat{v}_2^l, \widehat{v}_1^r, \widehat{v}_1^l$ respectively. It's obvious that $v_i^{b'} = \min\left\{\widehat{v}_i^{b'}, \overline{v}\right\}$.

Define $\widehat{x}_i^{b'} \equiv \frac{\overline{v} - \widehat{v}_i^{b'}}{a}$, then $x_i^{b'} = \max\left\{\widehat{x}_i^{b'}, 0\right\}$ and (38)-(40) become

$$\widehat{x}_2^r + \widehat{x}_3^l - 2\widehat{x}_3^r = \frac{B}{6a} \tag{42}$$

$$\widehat{x}_3^r - \widehat{x}_2^l + \widehat{x}_2^r - \widehat{x}_1^l = \frac{B}{3a} \tag{43}$$

$$\widehat{x}_2^l + \widehat{x}_1^r - 2\widehat{x}_1^l = \frac{B}{2a}. \tag{44}$$

Since $x_1^l > x_2^r, x_1^r < x_2^l$, it's straightforward that $\widehat{x}_1^l > \widehat{x}_2^r, \widehat{x}_1^r < \widehat{x}_2^l$. So we can follow the same analysis as in part (c) above to get that $\widehat{x}_3^r$ and $\widehat{x}_3^l$ are greater than the other four $\widehat{x}_i^{b'}$. Noting that at least one $\widehat{x}_2^{b'}$ should be positive (otherwise in state 2 the club will not hire any candidate and will receive an expected utility of negative infinity), so $\widehat{x}_3^r$ and $\widehat{x}_3^l$ must be positive. Then we have $x_3^r = \widehat{x}_3^r, x_3^l = \widehat{x}_3^l$ and $\max\left\{x_1^r, x_2^l, x_2^r, x_1^l\right\} = \max\left\{\widehat{x}_1^r, \widehat{x}_2^l, \widehat{x}_2^r, \widehat{x}_1^l\right\} < \min\left\{\widehat{x}_3^r, \widehat{x}_3^l\right\} = \min\left\{x_3^r, x_3^l\right\}$. Also notice that Equation (41) is always valid whether the standard is greater than $\overline{v}$ or not. So we can get the same contradiction as in part (c) above. *Q.E.D.*

Proof of Proposition 4: Using Lemma 2 and Proposition 1 we can easily get the following results:

$$(x_2^r)^2 + (x_2^l)^2 = \frac{4\tau}{a} + (x_2^r - x_1^l)^2 \tag{45}$$

$$(x_1^r)^2 + (x_1^l)^2 = \frac{4\tau}{a} + (x_1^r - x_2^l)^2. \tag{46}$$

So for solutions with quality standards lower than $\overline{v}$, we have six equations (28), (38)-(40), (45)-(46), and six unknowns $x_3^r, x_3^l, x_2^r, x_2^l, x_1^r, x_1^l$. Let

$$y_i^j = \sqrt{\frac{a}{4\tau}} x_i^j, \quad c = \frac{B}{12\sqrt{a\tau}}.$$

Then we can get a system of equations about $y_3^r, y_3^l, y_2^r, y_2^l, y_1^r, y_1^l$:

$$\begin{aligned}
(y_3^r)^2 + (y_3^l)^2 &= 1 \\
y_2^r + y_3^l - 2y_3^r &= c \\
y_3^r - y_2^l + y_2^r - y_1^l &= 2c \\
y_2^l + y_1^r - 2y_1^l &= 3c \\
(y_2^r)^2 + (y_2^l)^2 &= 1 + (y_2^r - y_1^l)^2 \\
(y_1^r)^2 + (y_1^l)^2 &= 1 + (y_1^r - y_2^l)^2.
\end{aligned}$$

Unlike the majority voting case, some of the y_i^b can be higher than one. There may be multiple solutions to the system of equations. The first possibility is the pro-minority power-switching equilibrium such that $y_1^l < y_2^l$. In particular, as y_1^l goes to zero, both y_2^l and y_1^r go to one from the last two equations of the above system of equations. Then, $y_2^l + y_1^r - 2y_1^l = 3c$ implies that c has to be smaller than $\frac{2}{3}$ to guarantee that such an equilibrium exists. For $c \leq \frac{2}{3}$, we can solve the system of equations numerically. The second possibility is the pro-majority power-switching equilibrium such that $y_1^l > y_2^l$. In particular, as y_2^l goes to zero, both y_1^l and y_2^r go to one from the last two equations of the above system of equations. c has to be larger than $\frac{10}{29}$ to guarantee such an equilibrium exists. For $c \geq \frac{10}{29}$, we can also solve the system of equations numerically.

There can also be equilibrium such that the equilibrium quality standards are $\bar{v}$. In particular, consider the glass-ceiling equilibrium where $\tilde{v}_2^l = v_2^l = \bar{v}$. Then the following inequalities must be satisfied

$$\frac{2}{3}\left[\frac{3}{2}\overline{v} + \frac{B}{4} + \frac{1}{2}\pi_1^R + \frac{1}{2}\pi_2^R\right] \leq \pi_2^R - \tau. \tag{47}$$

By the fact that $x_2^l = 0$ and Equations (45)-(46), we can easily derive

$$x_2^r = x_1^l = \sqrt{\frac{4\tau}{a}}.$$

Similar to the glass-ceiling equilibrium in the majority voting case, we have

$$\begin{aligned}
\pi_3^R &= 3\overline{v} + 3\tau + \frac{2B}{3} - 6\sqrt{a\tau}y_3^r; \\
\pi_2^R &= 3\overline{v} + 3\tau + \frac{3}{4}B - 3\sqrt{a\tau} - 3\sqrt{a\tau}y_3^r; \\
\pi_1^R &= 3\overline{v} + 3\tau - 6\sqrt{a\tau};
\end{aligned}$$

in which

$$y_3^r = \frac{1}{5}\left(\sqrt{4 + 2c - c^2} - 2c + 2\right).$$

Substituting π_1^R and π_2^R into (47), we can get $c > \frac{10}{29}$ to guarantee Inequality (47). Other types of equilibria may also exist under unanimity voting. For example, there may be an equilibrium such that in contentious states, only candidates of the minority type are admitted. But since the welfare of these equilibria cannot exceed that in the glass-ceiling equilibrium, we omit the discussion of these equilibria (see the Online Appendix for a complete equilibrium characterization).

Finally, using Equation (11), the long-term welfare under unanimity voting rule is given by

$$U^u = 3Ev + \frac{3}{2}a + \tau - 2\sqrt{a\tau}\gamma^u$$

where $\gamma^u \equiv \frac{4q_3}{y_3^r + y_3^l} + \frac{q_2}{y_1^l + y_2^l}(1 + 3(y_1^l)^2 + 3(y_2^l)^2)$. Comparing the welfare of different equilibria gives us the proposition. *Q.E.D.*

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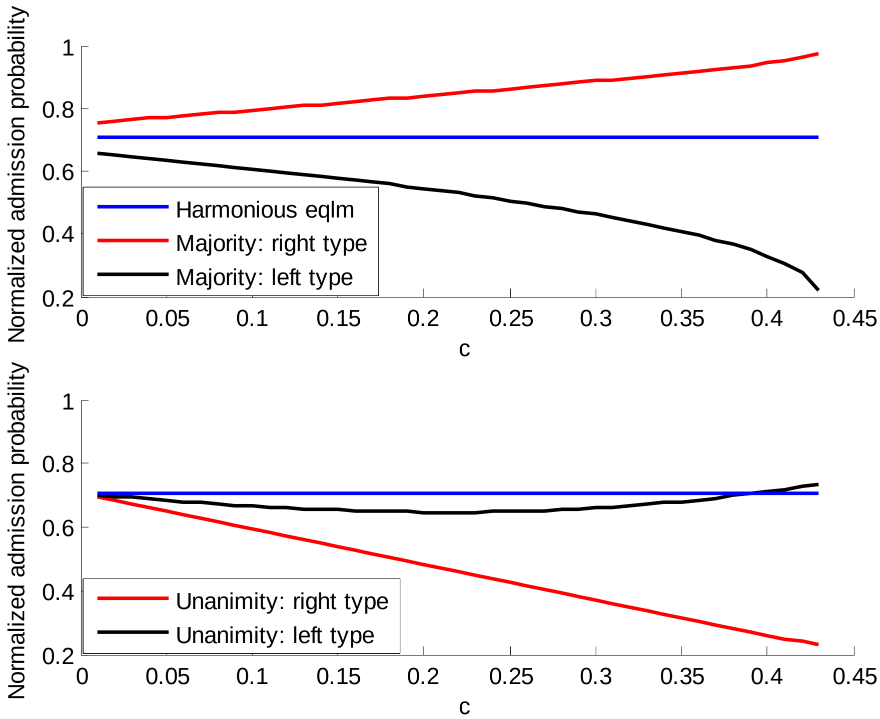

Figure 1: Upper Panel: Equilibrium Normalized Admission Probabilities ($\times\sqrt{\frac{a}{4\tau}}$) in State 2 (Contentious State with Majority Dominance) under Majority Voting, Candidates of the Majority Type are More Likely to be Admitted; Lower Panel: Equilibrium Normalized Admission Probabilities in State 2 under Unanimity Voting, Candidates of the Minority Type are More Likely to be Admitted

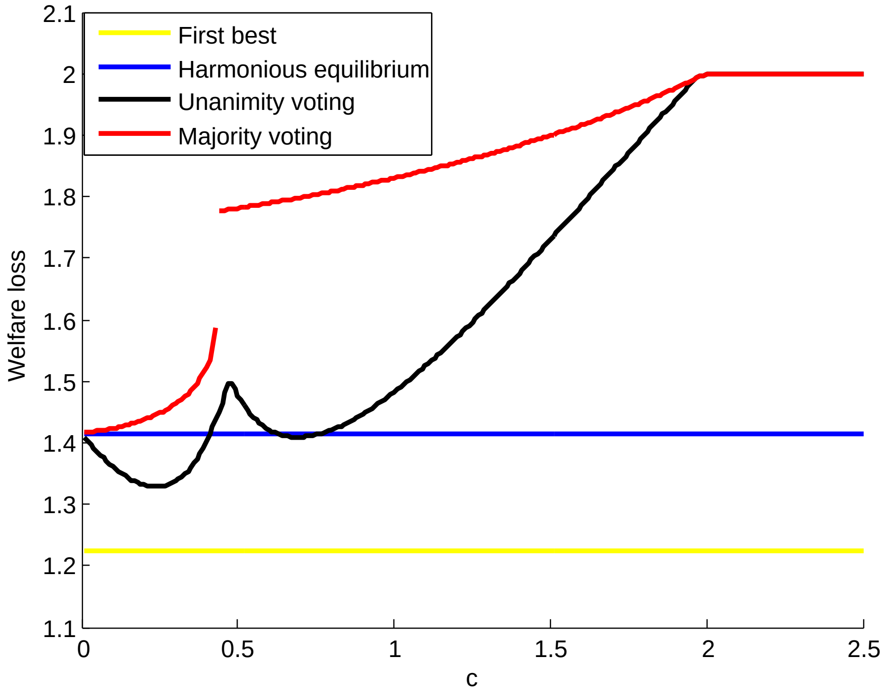

Figure 2: Long-Term Welfare Loss in the Most Efficient Equilibrium, Majority Voting is Always Worse than Harmonious Equilibrium and Unanimity Voting, and Unanimity Voting can be Better than Majority Voting when $c < 0.42$

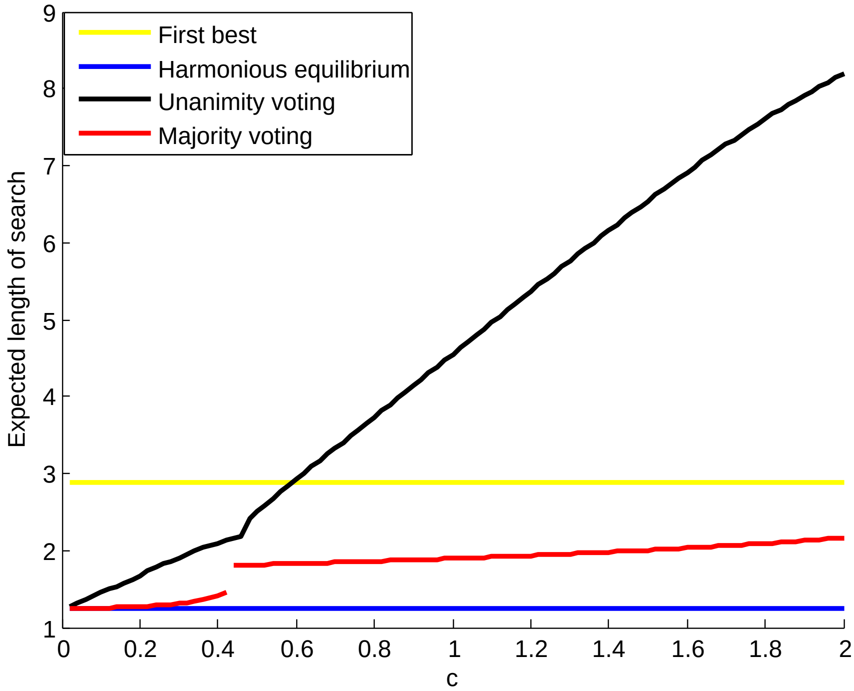

Figure 3: Long-Term Expected Delay in the Most Efficient Equilibrium, Unanimity Voting Always Causes a Longer Expected Delay than Majority Voting and Can Even Cause a Longer Expected Delay than First Best when c is Sufficiently Large

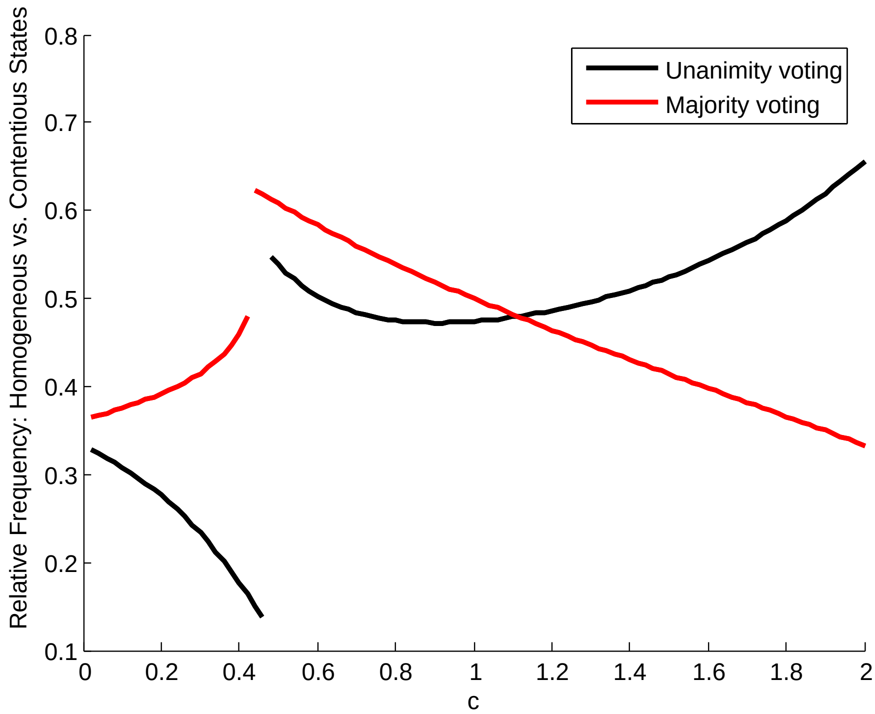

Figure 4: Relative State Frequency in the Most Efficient Equilibrium, Non-Monotonic Relationship in c under Both Majority Voting and Unanimity Voting

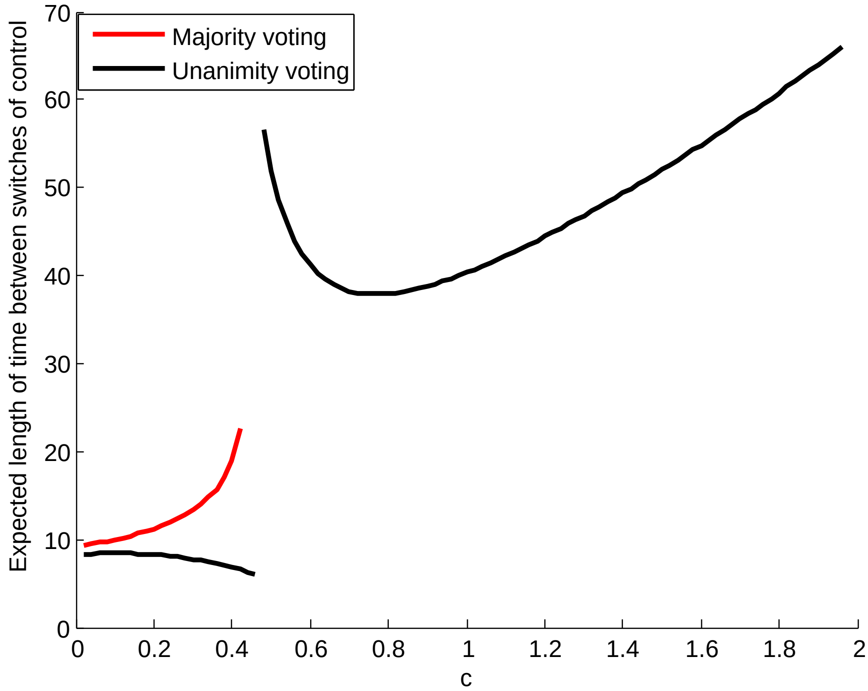

Figure 5: Expected Length of Time between Switches of Control in the Most Efficient Equilibrium, Non-Monotonic Relationship in c under Unanimity Voting