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Can Prospect Theory Explain the Disposition Effect? A New Perspective on Reference Points

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Abstract. There has been recent debate about whether prospect theory can explain the disposition effect. Using both theory and simulation, this paper shows that prospect theory often predicts the disposition effect when lagged expected final wealth is the reference point under the principle of preferred personal equilibrium, regardless of whether the reference point is updated or not. When initial wealth is the reference point, however, there is often no disposition effect. Models that use a reference point with no lag under the principle of preferred personal equilibrium or that determine the reference point using the principle of disappointment aversion cannot explain why the investor bought a stock in the first place. Reference point adjustment weakens the disposition effect, leads to more aggressive initial stock purchase strategies, and predicts history dependence in stock holding.

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Keywords: disposition effect • prospect theory • loss aversion • reference point • expectations

1. Introduction

The disposition effect refers to the propensity of certain investors to sell stocks that have risen in value rather than stocks that have fallen in value since purchase (Shefrin and Statman 1985, Odean 1998, Weber and Camerer 1998). The disposition effect poses a challenge to standard models that are based on maximization of investor utility.[1] Prospect theory, as developed by Kahneman and Tversky (1979), has been the most popular theory used to explain the disposition effect. This theory assumes that people derive utility from gains and losses relative to a reference point. The popular explanation, however, often relies on one of prospect theory's assumptions called diminishing sensitivity, which assumes that there is a concave utility function above the reference point and a convex utility function below the reference point (thus creating an S-shaped value function). With the implicit assumption of purchase price or initial wealth as the reference point, risk aversion in the gains domain (as implied by diminishing sensitivity) explains more sales of stocks that earn positive profits.

However, the core element of prospect theory is loss aversion, which assumes that losses relative to the reference point hurt an individual more than equal-sized gains bring satisfaction.[2] Surprisingly, Barberis and Xiong (henceforth, BX) (2009) suggest that loss aversion often leads to more sales of stocks that make negative profits. This effect dominates that of diminishing sensitivity in such a way that, taken together, prospect theory tends to predict a pattern opposite the disposition effect. While BX (2009) carefully examined the robustness of their conclusion in many directions, they did not focus on the role of the reference point. This paper is the first to formally analyze the implications of the reference point on the disposition effect.

The empirical measure of the disposition effect naturally sets average purchase price as the benchmark to define winning and losing stocks. Accordingly, most studies also assume the reference point in utility to be the initial wealth.[3] While this is consistent with the traditional status quo assumption of reference points in behavioral economics, the reference point to judge psychological gains and losses may deviate from average purchase price. For instance, an investor expecting to earn 5% returns from an investment portfolio may experience a 3% return as a loss of 2% rather than a gain of 3%. Inspired by Köszegi and Rabin (henceforth, KR)'s (2006, 2007, 2009) new reference-dependent models that endogenize reference points as rational expectations, many recent behavioral economics studies provide supporting empirical evidence for an expectations-based reference point. This paper, however, explores for the first time the implications of expectations-based reference points on investors' trading behaviors.

We build a dynamic model of individual trading behavior, assuming loss aversion and diminishing sensitivity, that formally considers rational expectations as an alternative reference point. Our model follows BX's (2009) setup as closely as possible for comparison purposes while following two major approaches to endogenize expectations as reference point: the preferred personal equilibrium proposed by KR (2006, 2007, 2009) and the disappointment aversion models (Bell 1985, Loomes and Sugden 1986, Gul 1991).[4] We find two main results. First, prospect theory often predicts the disposition effect when lagged expected final wealth is the reference point under the principle of preferred personal equilibrium, regardless of whether the reference point is updated or not. Second, models that do not include lag in reference point formation under preferred personal equilibrium or that determine the reference using the principle of disappointment aversion cannot explain why the investor bought a stock in the first place. The main message of this paper is that some expectations-based reference point models can explain the disposition effect, but there are limitations in applying the general idea of an expectations-based reference point to specific settings.

In the spirit of KR's (2006, 2007, 2009) preferred personal equilibrium, we analyze three specifications of the expectations-based reference point: initial expected final wealth (EC) as the constant reference point as well as both one-period-lagged (L1) and two-period-lagged expected final wealth (L2) as variable reference points. It is important to use the lagged wealth as the reference point because Corollary 2 shows that assuming current expected final wealth (no lag) as the reference point predicts no trading in the first place. All three of the aforementioned specifications provide strong intuition (confirmed by our simulation results) that loss aversion successfully predicts the disposition effect when lagged expected final wealth defines the reference point. Our robustness check also suggests that other simplifications relative to KR's (2006) assumptions in their new reference-dependent model (e.g., the stochastic reference point, the consideration of consumption utility in addition to gain–loss utility) and changing the length of the evaluation period, however, do not substantially affect the model's major predictions on the disposition effect. Since the effect of loss aversion is dominant in our model, prospect theory as a whole can thus often predict the disposition effect using an expectations-based reference point under the principle of preferred personal equilibrium. In reaching this conclusion, the disposition effect is measured, as before, by defining gains and losses relative to the average purchase price, only the reference point in the utility function is varied.

The intuition is simple. The kink generated by loss aversion implies a discontinuous change in the marginal utility around the reference point. This sharp change leads to behavior demonstrating excessive risk aversion (Rabin 2000), which in a stock-trading setting predicts the bunching of sales around the reference point. When stock trading profits are too low or too high relative to the reference point (wherein the probability of crossing the reference point in the future is low), the effect of the kink vanishes: Investors become less risk averse and are thus more likely to hold a stock. Given this relationship, the location of the reference point changes investors' risk attitudes. BX (2009) made an important observation that the stocks in question must have positive expected returns for loss-averse investors to purchase them in the first place. Such a returns distribution tends to generate large trading gains and small trading losses relative to average purchase price. When initial wealth is the reference point, loss aversion predicts a relatively smaller stock position in the domain of trading losses since generated trading gains are, on average, farther away from the reference point than trading losses. This paper, in contrast, shows that when the reference point is expected final wealth (which is typically higher than initial wealth), most trading gains are on average closer to the reference point than trading losses. Loss aversion therefore implies a strong bunching of sales around winning stocks rather than losing stocks, thus creating the disposition effect.

The role of diminishing sensitivity is more ambiguous in our setting. When the reference point is the status quo wealth, diminishing sensitivity strengthens the disposition effect as suggested by the traditional wisdom; however, when the reference point is lagged expectations, the effect of diminishing sensitivity is ambiguous, and hence we have to rely on simulation to determine the overall effect of loss aversion and diminishing sensitivity. It turns out that the role of diminishing sensitivity is second order compared to that of loss aversion in determining the disposition effect.

Our model, based on preferred personal equilibrium, generates novel predictions in addition to the disposition effect. We show that the adjustment in reference point leads to a weaker disposition effect, more aggressive shareholding strategy during the initial period, and history dependence on optimal stock holding. Since market experience can allow investors to admit gains or losses more easily and adjust their reference point more quickly, these results provide a reasonable explanation for how market experience reduces behavioral bias (List 2003, Feng and Seasholes 2005, Da Costa et al. 2013). When the reference point adjustment is slow, investors in our models also tend to sell stocks with small trading gains/losses more often than those with large trading gains/losses, a fact demonstrated in Table III of Odean (1998) yet not well explained by alternative theories. When the reference point is adjusted quickly, however, a reverse pattern appears that tends to predict the findings of Ben-David and Hirshleifer (2012) that investors tend to sell big winners and losers rather than small ones. Given that our model can generate either Odean's (1998) or Ben-David and Hirshleifer's (2012) empirical finding depending on the adjustment speed of the reference point, it may be possible to reconcile these two contradicting empirical findings using our model.

When the reference point is determined under the principle of disappointment aversion, however, the investor will not buy the stock in the first place. While the preferred personal equilibrium assumes that the reference point is fixed when comparing different choices, the disappointment aversion models assume that the reference point varies with choice in such a comparison. When the investor can determine reference point and stock holding at the same time, purchasing stock is always suboptimal. This is because when the reference point is the expected final wealth, purchasing stock always implies a negative expected gain–loss utility caused by loss aversion, because the distribution of gains and losses is roughly symmetric relative to expected value. Not purchasing stock thus is a dominant decision since it generates neither gains nor losses. This result suggests that the general idea of an expectations-based reference point may not work well in some specific forms and specific application settings.

In addition to BX (2009), several theoretical studies on prospect theory and the disposition effect assume initial wealth as the reference point. Kaustia (2010) as well as Hens and Vlcek (2011) use the same partial equilibrium approach as BX (2009) and reach similar conclusions. Li and Yang (2013) characterize the predictions of prospect theory in a general equilibrium framework, finding that diminishing sensitivity predicts the disposition effect, price momentum, reduced return volatility, and a positive return–volume correlation; loss aversion generally predicts the opposite. When stock returns are negatively skewed, however, loss aversion may actually predict the disposition effect.

BX (2009, 2012) develop an alternative explanation of the disposition effect based on realization utility, posing a distinction between paper and realized gains by assuming that additional gain–loss utility occurs at the moment of sale. The optimal solution is characterized by a threshold strategy whereby investors sell stocks once their gains reach a certain liquidation point. Combined with positive time discounting, realization utility in their model predicts the disposition effect among a wide range of other predictions. Ingersoll and Jin (2012) also assume realization utility, but add diminishing sensitivity to their model, which predicts the disposition effect as a dynamic result. McQueen and Vorkink (2004) investigate the effects of loss aversion and changing risk aversion on asset prices, with risk aversion and attention to news depending on past investment performance.[5]

This paper contributes to the broad literature on expectations-based reference points by applying an expectations-based reference point to the disposition effect for the first time. KR's (2006, 2007, 2009) theoretical work has inspired several laboratory and empirical studies outside finance. For the classical endowment effect, researchers have mixed evidence: Ericson and Fuster (2011) report evidence consistent with the expectations-based reference point, while Heffetz and List (2014) find that varying the probability of the exchange opportunity does not affect a subject's tendency to trade (yet subjects tended to keep their assigned goods when those goods varied). They therefore conclude that the assignment, rather than the expectation, is what matters. For labor supply, the experimental studies by both Abeler et al. (2011) and Gill and Prowse (2012) reveal that subjects exert greater effort when they have higher expected payments. Assuming rational expectations, Crawford and Meng (2011) show that New York cab drivers' labor supply decisions are strongly affected by income and target hours proxied by past sample average. Risk attitudes are also shown to be affected when reference points are based on expectations. Using the data from a popular game show, Post et al. (2008) find that participants are less risk averse following unexpected big gains and losses but more risk averse following small gains and losses. Sprenger (2015) shows that using stochastic expectations as the reference point generally makes subjects less risk averse. Song's (2016) experimental study further shows that an expectations-based reference point exists and adjusts relatively quickly to the resolution of uncertainty.

This paper also contributes to the literature on market experience and behavior biases. List (2003) was the first to show in a field experiment that market experience reduces the endowment effect. Camerer et al. (1997) also show that cab drivers' targeting behavior was less pronounced among more experienced drivers. Feng and Seasholes (2005) find that individual sophistication and trading experience together eliminate the tendency to hold losses. In a laboratory study, Da Costa et al. (2013) show that undergraduate students demonstrated a stronger disposition effect than experienced investors. This paper provides a reasonable channel through a reference point adjustment lens.

The remainder of this paper is organized as follows: Section 2 builds a dynamic model of prospect theory preferences that accommodate rational expectations as the reference point under the principles of preferred personal equilibrium and disappointment aversion. Section 3 simulates the predictions of the model based on preferred personal equilibrium and presents the simulation results. Section 4 discusses the simulation results. Section 5 performs the robustness check of the main results of the model based on preferred personal equilibrium. Section 6 concludes this paper.

2. A Model of Prospect Theory Preferences with Expectations as the Reference Point

2.1. Model Setup

We consider a dynamic model of asset allocation between a risk-free asset and a stock, assuming loss aversion and diminishing sensitivity. Our model is exactly the same as that of BX (2009) with the exception that the reference point in our model is endogenously determined by rational expectations (whereas it is exogenously given in BX 2009). In particular, we consider a portfolio choice setting with T dates, $t = 0, 1, \ldots, T-1$. There are two assets: a risk-free asset, which earns a gross return of $R_f = 1$ in each period, and a risky asset, the price of which evolves along a binomial tree. Hence, the risky asset's gross return from t to $t+1$, $R_{t,t+1}$ is distributed according to the following:

$$R_{t,t+1} = \begin{cases} R_u > R_f & \text{with probability } \frac{1}{2}, \\ R_d < R_f & \text{with probability } \frac{1}{2}, \end{cases} \quad \text{i.i.d. across periods.} \tag{1}$$

We use u to denote an event wherein the gross return is R_u, and d to denote an event wherein the gross return is R_d. We assume $\frac{1}{2}R_u + \frac{1}{2}R_d$ to be strictly larger than $R_f = 1$, so expected stock return exceeds the risk-free rate.

We study the trading behavior of investors with prospect theory preferences. Specifically, at any date $t = 0, 1, \ldots, T-1$, the investor makes the optimal decision by comparing their final wealth W_t^T to their reference point W_t^R. Let $\Delta W_t = W_t^T - W_t^R$, denoting the expected gain/loss relative to their reference point. Following Tversky and Kahneman (1992) and BX (2009), we consider a specific form for the utility function, one that demonstrates loss aversion and diminishing sensitivity:

$$v(\Delta W) = \begin{cases} \Delta W^{\alpha} & \text{for } \Delta W \geq 0, \\ -\lambda(-\Delta W)^{\alpha} & \text{for } \Delta W < 0. \end{cases} \tag{2}$$

We have set the parameter values such that $\alpha = 0.88$ and $\lambda = 2.25$ following Tversky and Kahneman (1992). The loss-aversion coefficient is represented by λ, and $\lambda > 1$ implies loss aversion (i.e., the investor dislikes losses more than he enjoys gains); α governs the curvature of the value function above and below the reference point, and $\alpha = 0.88$ implies diminishing sensitivity, i.e., the investor's value function is concave above the reference point and convex below.

At each date from $t = 0$ to $t = T-1$, the investor must decide how to split his wealth between the risk-free and risky assets. Let x_t be the number of shares of the risky asset he holds at date t, and let $h_t = \{x_0, \ldots, x_{t-1}, R_{0,1}, \ldots, R_{t-1,t}\}$ represent the realized history at the beginning of date t. The stock position x_t is chosen based on previous history h_t, while the choice of x_t determines W_{t+1} per the following budget constraint:

$$W_{t+1} = W_t R_f + x_t P_t (R_{t,t+1} - R_f). \tag{3}$$

Given $\{x_\tau\}_{\tau=t}^{T-1}$, we can therefore further derive W_t^T for a realization of $R_t^T = \{R_{t,t+1}, \ldots, R_{T-1,T}\}$ based on Equation (3):

$$\begin{aligned} W_t^T &= (W_T \mid h_t, R_t^T) \\ &= W_t R_f^{T-t} + \sum_{\tau=t}^{T-1} x_\tau P_\tau R_f^{T-\tau-1}(R_{\tau,\tau+1} - R_f). \end{aligned} \tag{4}$$

At any history h_t, the investor's decision problem is to maximize the expected gain–loss utility in the final period as follows:

$$\max_{x_t} E[v(\Delta W_t)] = E[v(W_t^T - W_t^R)], \tag{5}$$

subject to Equation (4), the requirement that x_τ is optimal at every history h_τ, and a nonnegativity of wealth constraint, $W_T \geq 0$.

In the tradition of Benartzi and Thaler (1995) as well as BX (2009), this model imposes the assumption called "narrow framing" or "mental accounting." The T trading dates together form an "evaluation period," which is the horizon for evaluating investment performance and deriving gain–loss utility. Benartzi and Thaler (1995) empirically estimate an evaluation period of approximately one year, which this paper follows for its simulations. By analyzing the choice between a risk-free asset and stock, our model also implicitly assumes that investors evaluate gains and losses stock by stock (BX 2009). Barberis and Huang (2001) support this assumption, showing that treating the trading decision as if investors consider each stock separately better fits the empirical data.

2.2. Fixed Reference Point

2.2.1. Model Analysis. We first consider a case where the reference point is fixed over time, i.e., the date 0 expectation of final wealth:

$$W_t^R = \bar{W} = EW_T, \quad \forall\, t.$$

Given any fixed reference point $\bar{W}$, we can rewrite the investors' dynamic optimization problem as a static problem wherein investors directly choose their wealth in different possible states at the final date, as shown by Cox and Huang (1989) and BX (2009).

Following BX (2009), we rank the $t+1$ nodes at any date t as $j=1,\dots,t+1$, where $j=1$ corresponds to the highest node in the tree at that date and $j=t+1$ to the lowest. We also use $P_{t,j}=P_0R_u^{t-j+1}R_d^{j-1}$ to denote risky asset price in node j at date t, $\pi_{t,j}$ to denote the ex ante probability of reaching that node, and $q_{t,j}$ to denote the state price density for that node.

We thus rewrite the investors' dynamic optimization problem for a fixed reference point $\bar{W}$ as follows:

$$\max_{\{W_{T,j}\}_{j=1,\dots,T+1}} \sum_{j=1}^{T+1} \pi_{T,j} v(W_{T,j}-\bar{W}), \tag{6}$$

subject to the budget constraint

$$\sum_{j=1}^{T+1} \pi_{T,j} q_{T,j} W_{T,j} = W_0 \tag{7}$$

and a nonnegativity wealth constraint,

$$W_{T,j} \geq 0, \quad j=1,\dots,T+1. \tag{8}$$

The above property enables us to focus on final period wealth allocation when defining our equilibrium.

Definition 1. We call $\{\bar{W}^*, \{W_{T,j}^*\}_{j=1,\dots,T+1}\}$ a rational expectation equilibrium if the following two conditions are satisfied:

1. Given $\bar{W}^*$, $\{W_{T,j}^*\}_{j=1,\dots,T+1}$ is the solution of the above optimization problem (6)–(8).
2. $\bar{W}^*$ is determined by rational expectation:

$$\bar{W}^* = \sum_{j=1}^{T+1} \pi_{T,j} W_{T,j}^*. \tag{9}$$

Definition 1 captures the rational expectation equilibrium and the reference point corresponding to the transformed static problem. We base this definition upon KR's (2006) definition of personal equilibrium in their new reference-dependent model. Personal equilibrium imposes internal consistency in the sense when the reference point is fixed, the resulting optimal solution also generates the reference point itself. In particular, this solution requires two steps: (1) fixing the reference point and deriving an optimal solution and (2) ensuring that the optimal solution generates the reference point. There may exist multiple equilibria due to the self-fulfilling property of rational expectations. Following KR's (2006) preferred personal equilibrium, we further refine our definition of equilibrium by focusing on the most efficient equilibrium in the presence of multiple equilibria. One imagines that if the investor is able to select the reference point at date 0, he would always want to select the reference point yielding the highest expected utility.

For an arbitrary $\bar{W}$, let $V(\bar{W})$ be the optimal expected utility by solving the maximization problem (6)–(8). The following proposition characterizes the reference point for any candidate equilibrium. The proofs of the following and all subsequent results can be found in the appendix.

Proposition 1. *For $k=1,\dots,T+1$, denote*

$$\begin{aligned}\bar{W}(k) = &\left(\sum_{i=1}^{k} W_0 \pi_{T,i} q_{T,i}^{-1/(1-\alpha)}\right) \\ &\cdot \left(\left(1-\sum_{i=1}^{k}\pi_{T,i}\right)\sum_{i=1}^{k}\pi_{T,i}q_{T,i}^{-\alpha/(1-\alpha)} \right. \\ &\left. + \sum_{i=1}^{k}\pi_{T,i}q_{T,i}^{-1/(1-\alpha)}\sum_{i=1}^{k}\pi_{T,i}q_{T,i}\right)^{-1}\end{aligned} \tag{10}$$

and

$$\begin{aligned}U(k) = &\left(\sum_{i=1}^{k}\pi_{T,i}q_{T,i}^{-\alpha/(1-\alpha)}\right)^{1-\alpha}\left(W_0-\bar{W}(k)\sum_{i=1}^{k}\pi_{T,i}q_{T,i}\right)^{\alpha} \\ &- \lambda\bar{W}(k)^{\alpha}\sum_{i=k+1}^{T+1}\pi_{T,i}.\end{aligned} \tag{11}$$

In any rational expectation equilibrium, the equilibrium reference point $\bar{W}$ must be some $\bar{W}(k)$, and $\bar{W}(k)$ can be an equilibrium reference point if $U(k)=V(\bar{W}(k))$.

We base our construction of rational expectation equilibrium on BX's (2009) conclusion, which shows that an investor's optimal policy is to use a "threshold" strategy. In this strategy, the investor allocates a wealth greater than the reference level $\bar{W}^*$ upon the k date T nodes that offer the highest prices for risky assets as well as a wealth level of 0 for the remaining date nodes. For the reference point $\bar{W}(k)$, we first calculate the optimal levels of final wealth following conditions (6)–(8) given this threshold strategy. We then use condition (9) to pin down the expression of $\bar{W}(k)$, and this gives us (10). Equation (11) is then the optimal expected utility given the reference point expressed in (10). Based on these results, we can calculate the rational expectation equilibrium taking the following steps. First, for every $k: 1\leq k\leq T+1$, we calculate the corresponding $\bar{W}(k)$ using (10). Second, Equation (10) alone does not guarantee that the calculated $\bar{W}(k)$ is indeed a rational expectation reference point: If k^* leads to a rational expectation equilibrium, we must ensure that, given $\bar{W}(k^*)$ as reference point, the investor has no incentive to deviate to another threshold value k, i.e., the optimal expected utility given that a reference point of $\bar{W}(k^*)$ (denoted by $V(\bar{W}(k^*))$) exactly equals the expected utility generated by threshold k^* (denoted by $U(k^*)$). Third, in the case of multiple equilibria, we further derive the most efficient equilibrium that generates the highest expected gain–loss utility.

Following BX (2009), if k^* is indeed an equilibrium threshold strategy, then the optimal wealth allocation $W_{T,j}$ in node j at final date T is given by the following:

$$W_{T,j} = \bar{W}(k^*) + q_{T,j}^{-1/(1-\alpha)} \frac{W_0 - \bar{W}(k^*) \sum_{i=1}^{k^*} \pi_{T,i} q_{T,i}}{\sum_{i=1}^{k^*} \pi_{T,i} q_{T,i}^{-\alpha/(1-\alpha)}}$$

for $j \le k^*$, and $W_{T,j} = 0$ otherwise.

The optimal share holdings $x_{t,j}$ are given by

$$x_{t,j} = \frac{W_{t+1,j} - W_{t+1,j+1}}{P_0(R_u^{t-j+2} R_d^{j-1} - R_u^{t-j+1} R_d^{j})},$$

where we can calculate the intermediate wealth allocations by working backward from date T using

$$W_{t,j} = \frac{(1/2) W_{t+1,j} q_{t+1,j} + (1/2) W_{t+1,j+1} q_{t+1,j+1}}{q_{t,j}},$$

for all

$$0 \le t \le T-1, \quad 1 \le j \le t+1.$$

The key issue is determining whether the investor wants to purchase a stock at date 0: Notice that we can view BX's (2009) model as a special case within our model, as that model's reference point $W_0 R_f^T = W_0$ is exactly the same as $\bar{W}(T+1)$, as defined by Equation (10) (because of $\sum_{i=1}^{T+1} \pi_{T,i} = 1$ and $\sum_{i=1}^{T+1} \pi_{T,i} q_{T,i} = 1$).[6] We thus immediately derive the following corollary:

Corollary 1. *The range of the expected stock returns leading to no stock purchase is the same as that of BX* (2009), *which assumes* $\bar{W} = W_0$.

This argument has two parts. First, if it is optimal to not buy stock when the reference point is $\bar{W} = W_0$, then $\bar{W} = W_0$ is also part of the rational expectation equilibrium. If other rational expectation equilibria involving stock purchase exist, the equilibrium with $\bar{W} = W_0$ is the most efficient. This is because the expected utility without stock purchase is zero, while the expected utilities with stock purchase are negative because of gains and losses being roughly symmetric relative to their expected values. This leads to negative expected gain–loss utility due to loss aversion. Second, when it is optimal for the investor to purchase shares under the reference point $\bar{W} = W_0$, $\bar{W} = W_0$ cannot be an equilibrium reference point. We must thus determine the reference point endogenously from rational expectation, and the equilibrium must involve the purchase of stock.

The above argument leads to the question of why a loss-averse investor would wish to purchase that stock in the first place: Purchasing the stock will lead to roughly symmetric outcomes relative to the expectations-based reference point; thus, the expected utility tends to be negative because of loss aversion. By not purchasing the stock, the investor incurs no gains or losses. This argument, however, implicitly assumes that in the calculation of the rational expectation equilibrium, when the action is not purchasing stock, the reference point changes to $\bar{W} = W_0$. Our solution follows personal equilibrium in assuming that the reference point remains the same when deriving an optimal action. In other words, if purchasing the stock falls within a rational expectation equilibrium, the reference point is always the expected final wealth as generated by the decision to purchase the stock. As a result, not purchasing the stock would generate a sure loss, while purchasing the stock will at the very least generate possible gains. Not purchasing the stock may therefore be suboptimal.[7] This argument differs from that of the previous paragraph, as it pertains to deriving the rational expectation equilibrium, while the previous paragraph compares different rational expectation equilibria.

2.2.2. An Example. When rational expectations endogenously determine the reference point, BX's (2009) result that prospect theory often predicts the opposite of the disposition effect no longer holds true. To illustrate this basic intuition, we present a simple numerical example with $T = 2$ and the parameter values exactly the same as in the corresponding example of BX (2009). In particular, we set μ, the annual gross expected return, as 1.1, and σ, the standard deviation of stock returns, as 0.3. These values for μ and σ imply $(R_u, R_d) = (1.25, 0.85)$. We set other parameters as $W_0 = 40$, $P_0 = 40$, and $R_f = 1$ with an evaluation period of one year. As shown by BX (2009), when the reference point is $W_0 = 40$, the optimal trading strategy is given by

$$(x_0, x_u, x_d) = (4.0, 5.05, 3.06).$$

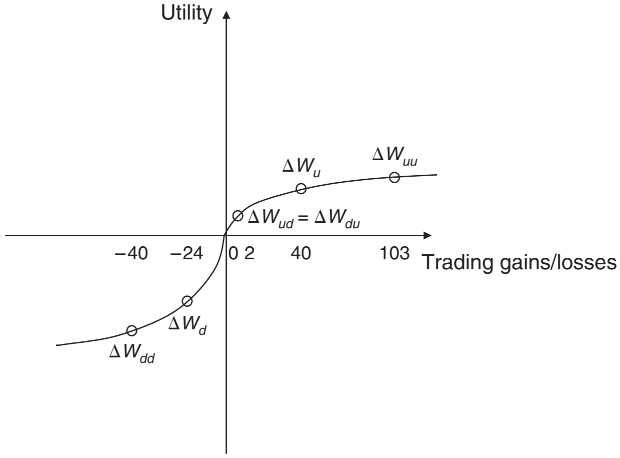
Initially, an investor buys 4.0 shares of the risky asset. After a good stock return at date 1, he increases his position to 5.05 shares. After a poor stock return at date 1, he decreases his position to 3.06 shares. There is thus no disposition effect under the reference point W_0.

When rational expectations (as defined in our model) determine the reference point, however, the new reference point is calculated at $\bar{W} = 52.8$. The optimal trading strategy under this new reference point changes to

$$(x_0, x_u, x_d) = (3.26, 2.51, 3.93).$$

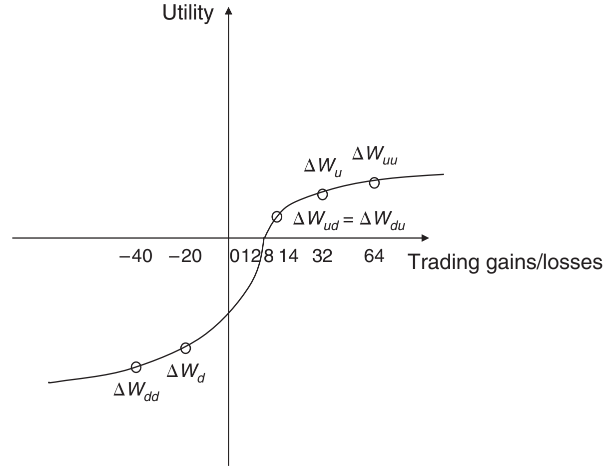
Initially, an investor buys 3.26 shares of the risky asset. After a good stock return at date 1, he decreases his position to 2.51 shares. After a poor stock return at date 1, he increases his position to 3.93 shares. The investor thus demonstrates strong disposition effect.

Figure 1. Gain–Loss Utility When Reference Point Is the Initial Wealth



Notes. The x axis is the change in wealth relative to the *initial wealth* (not the reference point) and represents trading gains and losses. For instance, ΔW_u is the change in wealth relative to the initial wealth at date 1 after $R_1 = R_u = 1.25$ is realized, and ΔW_{ud} is the change in wealth relative to the initial wealth at date 2 after $R_1 = R_u = 1.25$ and $R_2 = R_d = 0.85$ are realized. The y axis is the gain–loss utility. In this numerical example, the reference point is at 0, and the trading pattern implies the opposite of the disposition effect.

Figure 2. Gain–Loss Utility When Reference Point Is the Initial Expected Final Wealth



Notes. The x axis is the change in wealth relative to the *initial wealth* (not the reference point) and represents trading gains and losses. For instance, ΔW_u is the change in wealth relative to the initial wealth at date 1 after $R_1 = R_u = 1.25$ is realized, and ΔW_{ud} is the change in wealth relative to the initial wealth at date 2 after $R_1 = R_u = 1.25$ and $R_2 = R_d = 0.85$ are realized. The y axis is the gain–loss utility. In this numerical example, the reference point occurs at 4, and the trading pattern implies the disposition effect exactly.

What is the intuition for this result? Figures 1 and 2 illustrate the reason: For both figures, the x axis represents the change in wealth relative to initial wealth (not the reference point), which represents trading gains and losses. For instance, ΔW_u is the change in wealth relative to W_0 after realizing a good return at date 1, while ΔW_{ud} is the change in wealth relative to W_0 following a good return at date 1 and a bad return at date 2. The y axis is gain–loss utility.

Figure 1 illustrates a case where initial wealth is the reference point; thus, the kink occurs at 0. As BX (2009) clearly show, if an investor is currently in the losses/gains domain relative to the reference point, he will take a stock position such that the highest/lowest wealth leaves him at or near the reference point. Crucially, if the investor has a wealth farther from the reference point in either direction, then he needs to take a larger stock position for the highest (in the case of losses) or lowest (in the case of gains) wealth to reach the reference point. This nonmonotonic trading pattern mainly results from loss aversion, which generates a sharp change in marginal utility, which in turn implies first-order risk aversion. This makes the investor extremely risk averse around the reference point, and he demands fewer shares of the stock. When current wealth is far away from the reference point, however, loss aversion diminishes, and thus the investor demands more shares. Diminishing sensitivity complicates this intuition a little bit in the sense that it will strengthen the disposition effect when the reference point is the status quo wealth, but its effect is ambiguous when the reference point is lagged expected wealth. However, the simulation suggests that the effect of diminishing sensitivity on the disposition effect is subordinate to that of loss aversion in our setting.

In this example, $\Delta W_u = 40$ and $\Delta W_d = -24$. The trading gain at date 1 is therefore further from the reference point than the trading loss, so the investor tends to demand more shares of the stock if there is a trading gain. This pattern is the opposite of the disposition effect.

Compared to Figure 1, the key change in Figure 2 is that the kink moves from 0 to 12.8 because of the change in the reference point. In this case, $\Delta W_u = 32$ and $\Delta W_d = -20$. Since trading loss is farther away from the reference point, 12.8, than trading gain, the investor tends to demand more shares of the stock if there is a trading loss, a pattern consistent with the disposition effect.

In general, the reference point defined as rational expectations is higher than the status quo reference point if the expected stock returns are positive. As a result, trading gains are generally closer to the reference point than trading losses. Since loss-averse investors tend to demand more shares if they are farther from the reference point, they tend to demand more shares if there are trading losses.

This example also reveals the intuition behind investors' wish to purchase a stock in the first place when rational expectations define the reference point.

In fixing the reference point to $\bar{W} = 52.8$, not purchasing stock generates a sure loss of 12.8, while purchasing stock leads to a lottery in final wealth of $(0.25, 103.6; 0.5, 53.6; 0.25, 0)$. Expected gain–loss utility is -21 for the former and -16 for the latter case, so purchasing stock is optimal at date 0 under a rational expectation equilibrium. It is simple to check that not purchasing the stock at date 0 is not a rational expectation equilibrium, because given the reference point $\bar{W} = 40$, purchasing the stock generates an expected gain–loss utility of 1, while purchasing no stock generates neither gains nor losses.

2.3. Variable Reference Point

2.3.1. Model Analysis. One may think that a fixed reference point model is unrealistic: As time goes by, an investor can adjust the expectations and thus his reference point. In this section, we analyze the variable reference point model. The ability to adjust a reference point correlates to the investor's level of sophistication: the more experienced the investor, the more likely he can quickly adjust the reference point. Studying the adaptation of the reference point therefore relates to studies on the effects of market experience. Previous studies have shown that behavioral bias, including the disposition effect, reduces when people have greater experience (List 2003, Feng and Seasholes 2005, Da Costa et al. 2013). The adjustment in reference point provides a reasonable channel to explain this effect.

At date t, there are 2^t different possible histories. We denote the reference point at a certain history h_t as $\bar{W}(h_t)$. As in the fixed reference point model, we define our concept of equilibrium as follows:

Definition 2. We call $\{\bar{W}^*(h_t), W^*(h_t), x^*(h_t)\}$ a rational expectation equilibrium if the following three conditions are satisfied:

1. At any history h_t, given $\{\bar{W}^*(h_t), W^*(h_t)\}$, $x^*(h_t)$ is the solution of the optimization problem (5).
2. $W^*(h_t)$ is the current wealth, which is updated according to Equation (3).
3. $\bar{W}^*(h_t)$ is determined by a specific rational expectation condition, which we will explain later.

In this case, the investor's dynamic optimization problem is path dependent; hence, we can no longer treat the dynamic problem as a static problem and must take a backward procedure to solve it. We make the following assumption on parameter values:

Assumption 1. *Define* $g = (R_u - R_f)/(R_f - R_d) = (R_u - 1)/(1 - R_d)$. *Then,* $g < \lambda$.

Since we let $\lambda = 2.25$, the above assumption holds for all numerical examples considered in this paper. We base the following proposition on BX (2006), the working paper version of BX (2009), and characterize the optimal decision at $T-1$. We omit the proof, which appears in BX (2006).

Proposition 2. *If Assumption 1 holds, the investor's optimal date* $T-1$ *share holdings are given by*[8]

$$x_{T-1}(\Delta W) = \begin{cases} \min\left\{x_L(\Delta W), \dfrac{W_{T-1}}{P_{T-1}(1-R_d)}\right\} & \text{if } \Delta W < 0, \\ x_G(\Delta W) & \text{if } \Delta W \geq 0, \end{cases} \tag{12}$$

where $\Delta W = W_{T-1} - W^R_{T-1}$,

$$x_L(\Delta W) = \frac{-\Delta W}{P_{T-1}}\left(\frac{R_u - R_d}{(R_u - 1/(\lambda(1-R_d)))^{1/(1-\alpha)} + 1} - (1 - R_d)\right)^{-1}, \tag{13}$$

and

$$x_G(\Delta W) = \frac{\Delta W}{P_{T-1}}\left(\frac{R_u - R_d}{(R_u - 1/(1-R_d))^{1/(1-\alpha)} + 1} + (1 - R_d)\right)^{-1}. \tag{14}$$

Proposition 2 establishes the same trading pattern that the aforementioned numerical example in Section 2.2 does. Because of loss aversion, the optimal stock holding at date $T-1$ increases with the distance of current wealth from the reference point. In Definition 2, we do not fully specify our rational expectation condition, as there are many different assumptions about the reference point to cover within this condition. For instance, depending on how quickly the investor can adapt to the previous gains/losses and adjust his reference point, the reference point can be the current rational expectation of final wealth or rational expectations with different lags.

We denote the n-period-lagged reference point as $\bar{W}^*(h_t) = E[W_T \mid h_{\max\{t-n,0\}}]$ for an integer $n \geq 0$. We first show that if current expected final wealth is the reference point, i.e., $n = 0$ and $\bar{W}^*(h_t) = E[W_T \mid h_t]$, then the investor will not purchase any stock in a rational expectation equilibrium.

Corollary 2. *If* $\bar{W}^*(h_t) = E[W_T \mid h_t]$, *the investor does not buy any stock in a rational expectation equilibrium.*

We use the two-period version of our model to illustrate the intuition. At date 1, when current expected final wealth is the reference point, there are only two possible outcomes of final wealth in calculating the reference point. Assume that the reference point involves holding some stock shares. Given positive expected stock returns, expected final wealth will be higher than current wealth. As shown by BX (2009) and Proposition 2, the optimal strategy is thus to gamble until the better final outcome reaches the reference point; then both high and low final wealth are below the reference point so expected final wealth cannot equal the given reference point. Purchasing stock is thus not a rational expectation equilibrium. When the reference point is one-period-lagged expected final wealth, on the other hand, the investor at date 1 calculates the expected value of four possible levels of final wealth to obtain the reference point. In deriving the optimal strategy at each of the two nodes at date 1, however, the investor considers only two outcomes at date 2. The contradiction mentioned for current expected final wealth as the reference point does not necessarily exist.

The above corollary implies the need to use lagged rational expectations as reference points to obtain reasonable result. KR (2009) also use this approach, and there is an empirical reason for such a choice. Using data from the popular game show *Deal or No Deal*, Post et al. (2008) estimate the reference point adjustment and show that the reference point does not fully adjust to the current state, but is rather affected by lagged expected values. In an experimental stock market, Baucells et al. (2011) elicit subjects' reference points and find that the average of intermediate prices plays an important role. Using a similar design, Arkes et al. (2008) suggest that investors adapt their reference points more slowly to the most recent price when there is a loss rather than a gain. These studies indicate that investors are not very likely to fully adjust their reference points to the current price in stock trading.

In the following discussion, we explore two specifications that employ a lagged-expectation reference point via $n=1$ and $n=2$. Once we specify all reference points, we can then solve backward for the investor's dynamic optimization. For example, given Proposition 2's depiction of the investor's optimal date $T-1$ share holdings, using a one-period-lagged expectations-based reference point at date $T-2$, the investor decides how to allocate wealth at history $h_{T-1}=\{h_{T-2},u\}$ and $h_{T-1}=\{h_{T-2},d\}$ to maximize his expected utility through the reference point $\bar{W}(h_{T-2})$. Similarly, we can solve the problems at date $t\le T-3$, assuming that all future share holdings are optimal. Finally, we impose rational expectation conditions to pin down all reference points.

2.3.2. An Example. We use a numerical example where $T=3$ to illustrate the rational expectation equilibrium with a variable reference point. We assume a reference point of one-period-lagged expected final wealth with the same parameters as in Section 2.2.2. Table 1 reports the return realizations (top-left panel), stock price (top-right panel), optimal shares held (bottom-left panel), and wealth (bottom-right panel) at each note. We include the reference point that determines the optimal share holding at each note in the bracket of the bottom-right panel.

This example shows a clear disposition effect. Take date 2 as an example. Because the investor does not purchase any new shares at date 1, the average purchase price is always 40 in all periods, making the nodes uu, ud, and du all gains. Among the three gain nodes, the investor sells at nodes uu and du, while he holds the stock at the loss node dd. This is a clear disposition effect. Unsurprisingly, the reference point is also adjusted based on historical realized returns. After a good return at date 1, the reference point increases from 49.54 to 72.61, while it decreases from 49.54 to 26.48 following a bad return at date 1.

2.4. Disappointment Aversion

In our definition of equilibrium, the investor takes the reference point as given when making an investment decision. Another modeling approach, however, is to let investors determine both the reference point and their decision at the same time, which is similar in principle to equilibrium in such disappointment-aversion models as those of Bell (1985), Loomes and Sugden (1986), and Gul (1991), as well as the choice-acclimating personal equilibrium in KR (2007). Specifically, in this alternative approach, the decision problem with the fixed reference point case can be written as

$$\max_{\bar{W},\{W_{T,j}\}_{j=1,\ldots,T+1}} \sum_{j=1}^{T+1} \pi_{T,j} v(W_{T,j}-\bar{W}), \tag{15}$$

subject to the budget constraint

$$\sum_{j=1}^{T+1} \pi_{T,j} q_{T,j} W_{T,j} = W_0, \tag{16}$$

nonnegativity of wealth constraint

$$W_{T,j}\ge 0, \quad j=1,\ldots,T+1, \tag{17}$$

and the rational expectation constraint

$$\bar{W}=\sum_{j=1}^{T+1} \pi_{T,j} W_{T,j}. \tag{18}$$

Proposition 3. *The optimal solution to the maximization problem (15)–(18) must satisfy $\bar{W}^*=W_0R_f^T$ and the investor does not buy any stock in equilibrium.*

The proof of the above proposition is similar to that of Corollary 1 and is hence is omitted from this section. Obviously,

$$\bar{W}^*=W^*_{T,1}=\cdots=W^*_{T,T+1}=W_0R_f^T$$

satisfies constraints (16)–(18) and gives an expected utility of zero. Other combinations of

$$\{\bar{W},\{W_{T,j}\}_{j=1,\ldots,T+1}\}$$

satisfying constraints (16)–(18), in contrast, cannot yield higher expected utility under the gain–loss utility function. The same logic also applies to the variable reference point case.

Table 1. A Three-Period Example When Reference Point Is One-Period-Lagged Expected Final Wealth

Return realizations				Prices			
			uuu				67.90
		uu				56.92	
			uud				49.62
	u		udu		47.72		49.62
		ud				41.59	
			udd				36.26
1.00				40.00			
			duu				49.62
		du				41.59	
	d		dud		34.87		36.26
			ddu				36.26
		dd				30.39	
			ddd				26.49
Risky asset shares held				**Wealth**			
			—				94.00
		1.14				81.60	
			—			(72.61)	73.20
	1.47		—		68.00		73.60
		1.78			(49.54)	59.20	
			—			(72.61)	49.60
3.64				40.00			
			—	(49.54)			51.20
		1.79				36.80	
	2.31		—		21.20	(26.48)	24.00
			—		(49.54)		27.20
		2.81				10.80	
			—			(26.48)	0

Notes. This table reports the result of the most efficient rational expectation equilibrium for a three-period example assuming that the reference point is the one-period-lagged expected final wealth. The top-left panel shows return realization at each node in the tree. The top-right panel shows the corresponding price at each node. The bottom-left and bottom-right panels report, for each node, the optimal number of shares held in the risky asset and the optimal wealth, respectively. The reference point that determines the optimal share holding at each node is included in parentheses in the bottom-right panel. The investor's initial wealth is $40, the net risk-free rate is zero, and the initial price, annual expected return, and annual standard deviation of the risky asset are $40, 1.1, and 0.3, respectively.

We discussed the intuition previously. When the reference point and the optimal stock holding change at the same time, purchasing stock is always suboptimal. This is because when purchasing stock (i.e., the reference point is the expected final wealth), the distribution of gains and losses is roughly symmetric relative to the expected value, implying a negative expected gain–loss utility due to loss aversion. Not purchasing stock (i.e., the reference point is the initial wealth) generates neither gains nor losses.

This result illustrates a limitation of the expectations-based reference point as a general idea. One of the two major expectations-based reference point models, the disappointment-aversion model, leads to an unreasonable result of no trading because this modeling approach often generates a higher degree of risk aversion than the preferred personal equilibrium. This point is discussed by KR (2007, Footnote 19), but how this difference affects the effective application of expectations-based reference point models in important decision contexts has not been fully recognized. Our result suggests that the distinction between these two modeling approaches could be crucial for application purposes.[9]

3. Simulation

The numerical examples in the previous section are merely illustrative. To provide systematic evidence of how the reference point affects the disposition effect, this section formally simulates the trading pattern of loss-averse investors using the following four different specifications of the reference point:

- SQ: BX's (2009) status quo wealth
- EC: Initial expected final wealth
- L1: One-period-lagged expected final wealth
- L2: Two-period-lagged expected final wealth

For comparison purposes, our simulation uses BX's (2009) specifications: BX (2009) simulate the selling versus holding decision of 10,000 loss-averse investors, each trading four stocks T times over an evaluation period of one year. Each investor has an initial wealth of 40 to allocate to each stock. All stocks start with an initial price of 40, and the stock return follows a binomial distribution with an annual gross return of μ ranging from 1.03 to 1.13. The standard deviation, σ, is 0.3. Investors have a loss-aversion coefficient fixed at $\lambda = 2.25$ and a curvature of value function $\alpha = 0.88$. We take the risk-free rate to be $R_f = 1$. Assuming that the price will go up or down with equal probability, the values of R_u and R_d relate to μ and σ as follows:

$$R_u = \mu^{1/T} + \sqrt{(\mu^2 + \sigma^2)^{1/T} - (\mu^2)^{1/T}},$$
$$R_d = \mu^{1/T} - \sqrt{(\mu^2 + \sigma^2)^{1/T} - (\mu^2)^{1/T}}.$$

The simulation has three stages. Stage 1 solves for the most efficient rational expectation equilibrium. The first two specifications (SQ and EC) are relatively easy, as we can transform a dynamic investment decision problem into a static problem to obtain closed-form solutions (see Proposition 1). The latter two cases, L1 and L2, are much more complicated: We must first guess the reference point for each history, then solve backward for the dynamic investment decision problem. We reach equilibrium when the guessed reference points match the one-period-lagged or two-period-lagged expected final wealth.[10] Considering the complicated nature of the simulation process for the latter two cases, we report only the cases where $T = 2, 3, 4$. These are sufficient to illustrate the main predictions of the model. Stage 2 generates 40,000 realizations of the price sequence based on the given distribution and number of trading dates within a year. For each price sequence, we can easily obtain the optimal stock position and corresponding reference point based on the results of Stage 1. Stage 3 further calculates the statistics to show the model's predictions, which we describe in greater detail below.

3.1. The Disposition Effect

Odean (1998) constructs the classical measurement for the disposition effect: He first calculates trading gains and losses by comparing current purchase price to average purchase price, and then defines the proportion of gains realized (PGR) as the number of realized gains divided by the number of realized and paper gains. He defines the proportion of losses realized (PLR) as the number of realized losses divided by the number of realized and paper losses. If PLR < PGR, a disposition effect exists. Simple numerical examples can show that the PGR/PLR ratio is a more robust measure of the disposition effect than difference (PGR − PLR) as the ratio is not affected by such confounding factors as portfolio size and trading frequency.[11] This paper therefore reports ratio as well as PGR and PLR measures.

Table 2 reports the simulated PGR/PLR ratio and the PGR and PLR values in the brackets for SQ and EC. The left three columns assume a reference point of initial wealth (SQ), while the right three columns assume an expectations-based reference point, i.e., initial expected final wealth (EC).

Our first case, SQ, which assumes BX's (2009) status quo wealth as the reference point, successfully replicates BX's (2009) result. The PGR/PLR ratio is smaller than 1 in all cases with returns larger than 1.08, suggesting that there is essentially *no* disposition effect, or even that the *opposite* of the disposition effect is occurring.

When assuming initial expected final wealth as the reference point, interestingly, the PGR/PLR ratio is larger than 1 in most cases, indicating a strong disposition effect. In all cases, when $T = 2$ and $T = 3$, almost all gains are realized (PGR = 1.00 or PGR = 0.80), but no losses (PLR = 0.00), suggesting an infinite degree of disposition effect. When $T = 4$, there is a strong disposition effect for returns ranging from 1.06 to 1.10: As returns increase, the disposition effect generally disappears.

Prediction 1 (Constant Expectations-Based Reference Point). *Assuming initial expected final wealth as a constant reference point generates the disposition effect in most cases.*

As BX (2009) point out, simulated loss-averse investors are only willing to accept stocks with expected returns much higher than the risk-free rate. When initial expected wealth becomes the reference point, the region in which investors would choose to purchase the stock initially remains the same. This is consistent with the theoretical result in Corollary 1. We also observe that as the number of trading opportunities within a year increases, investors tend to accept lower stock returns at date 0. This is because more trading opportunities smooth the stock risk from the perspective of the initial date.

Table 3 reports the same PGR and PLR statistics for the two variable reference point specifications. The left two columns display results based on a reference point of one-period-lagged expected final wealth (L1) for $T = 3$ and $T = 4$, while the right column displays results based on a reference point of two-period-lagged expected final wealth for $T = 4$.

Table 3 reports a substantial disposition effect in many cases. Assuming expectations as the reference point, our model tends to give a robust prediction of the disposition effect, regardless of whether the reference point is variable or not. Furthermore, we can see that (compared to the case of EC) an adjusted

Table 2. Simulated PGR and PLR for Specifications SQ and EC

	(SQ) BX's (2009) status quo wealth as the reference point			(EC) Initial expected final wealth as the constant reference point		
μ	$T=2$	$T=3$	$T=4$	$T=2$	$T=3$	$T=4$
1.03	—	—	—	—	—	—
1.04	—	—	—	—	—	—
1.05	—	—	—	—	—	—
1.06	—	—	3.00 (0.67/0.22)	—	—	2.97 (0.67/0.22)
1.07	—	—	3.00 (0.67/0.22)	—	—	3.00 (0.67/0.22)
1.08	—	· (0.80/0.00)	3.00 (0.67/0.22)	—	· (0.80/0.00)	3.00 (0.67/0.22)
1.09	—	0.61 (0.40/0.67)	0.60 (0.40/0.67)	—	· (0.80/0.00)	3.00 (0.67/0.22)
1.10	0.00 (0.00/1.00)	0.60 (0.40/0.67)	0.60 (0.40/0.67)	· (0.00/1.00)	· (0.80/0.00)	3.00 (0.67/0.22)
1.11	0.00 (0.00/1.00)	0.60 (0.40/0.67)	0.60 (0.40/0.67)	· (1.00/0.00)	· (0.80/0.00)	0.71 (0.50/0.70)
1.12	0.00 (0.00/1.00)	0.60 (0.40/0.67)	0.33 (0.25/0.75)	· (1.00/0.00)	· (0.80/0.00)	0.51 (0.36/0.70)
1.13	0.00 (0.00/1.00)	0.60 (0.40/0.67)	0.33 (0.25/0.75)	· (1.00/0.00)	· (0.80/0.00)	0.51 (0.36/0.70)

Notes. This table reports the proportion of gains realized, the proportion of losses realized, and the ratio (PGR/PLR). PGR and PLR are taken from a simulation of 10,000 investors' decisions regarding holding or selling. Each investor trades four stocks. The PGR/PLR ratio is presented for different values of gross expected return over the year (μ) and different trading frequencies within the year (T). The left three columns present the case where the reference point is the initial wealth; the right three columns report the case where the initial expected final wealth is the constant reference point. Other parameter values are set to be $\sigma = 0.3$, $\lambda = 2.25$, $\alpha = 0.88$, $R_f = 1$, $W_0 = 40$, and $P_0 = 40$. A dash means that the investors are not willing to purchase the stock in the first place.

Table 3. Simulated PGR and PLR for Specifications L1 and L2

	(L1) One-period-lagged expected final wealth as the variable reference point		(L2) Two-period-lagged expected final wealth as the variable reference point
μ	$T=3$	$T=4$	$T=4$
1.03	—	—	—
1.04	—	—	—
1.05	—	—	—
1.06	—	3.10 (0.93/0.30)	6.57 (0.73/0.11)
1.07	—	2.62 (0.79/0.30)	· (0.67/0.00)
1.08	—	2.63 (0.79/0.30)	1.28 (0.64/0.50)
1.09	1.20 (080/0.67)	6.34 (0.63/0.10)	1.40 (0.38/0.27)
1.10	1.20 (080/0.67)	0.75 (0.50/0.67)	1.40 (0.38/0.27)
1.11	1.20 (080/0.67)	0.61 (0.43/0.70)	0.61 (0.39/0.64)
1.12	1.20 (080/0.67)	0.72 (0.43/0.60)	0.50 (0.33/0.67)
1.13	0.67 (0.50/0.74)	0.53 (0.43/0.80)	0.35 (0.28/0.80)

Notes. This table reports the proportion of gains realized, the proportion of losses realized, and the ratio (PGR/PLR). PGR and PLR are taken from a simulation of 10,000 investors' decisions regarding holding or selling. Each investor trades four stocks. The PGR/PLR ratio is presented for different values of gross expected return over the year (μ) and different trading frequencies within the year (T). The left two columns present the case where the reference point is one-period-lagged expected final wealth; the right column reports the case where the reference point is two-period-lagged expected final wealth. Other parameter values are set to be $\sigma = 0.3$, $\lambda = 2.25$, $\alpha = 0.88$, $R_f = 1$, $W_0 = 40$, and $P_0 = 40$. The dash means that the investors are not willing to purchase the stock in the first place.

reference point weakens the magnitude of the disposition effect: PGR/PLR is smaller in Table 3 than in the corresponding EC cases in Table 2. To obtain a more accurate measure, as the expected stock returns increase, let us define the switching return as the first return not demonstrating the disposition effect. For $T = 4$, the switching return is 1.10 in specification L1, 1.11 in specification L2, and 1.11 in specification EC: Clearly, the more quickly the reference point adapts to price realization, the less likely it becomes that we can observe the disposition effect. The existence of the disposition effect also depends on the number of trading opportunities T. To see this, notice that the switching return is 1.13 in model L1 when $T = 3$, and 1.10 when $T = 4$. Clearly, as the number of trading opportunities T increases, fewer cases demonstrate the disposition effect.

Prediction 2 (Variable Expectations-Based Reference Point). *When investors update their reference points according to one-period-lagged* (L1) *or two-period-lagged expected final wealth* (L2)*, the disposition effect still exists in many cases. The magnitude of the disposition effect, however, is weaker compared to cases that use a constant expectations-based reference point* (*EC*)*. In general, the quicker the investors adapt their reference points to price realization, the less likely it becomes that we can observe the disposition effect.*

3.2. Difference in Initial Position

The initial purchase decision is very important in BX's (2009) argument for why prospect theory mostly predicts the opposite of the disposition effect. In this section, we explore the predictions of initial position under different specifications. Table 4 reports the case where $T = 4$ for all four specifications; all other case patterns are similar.

Table 4. Initial Position

μ	SQ	EC	L1	L2
1.03	0	0	0	0
1.04	0	0	0	0
1.05	0	0	0	0
1.06	1.09	0.89	1.90	0.87
1.07	1.22	0.94	2.18	1.33
1.08	1.36	0.99	2.12	1.60
1.09	1.51	1.03	2.05	2.36
1.10	1.66	1.08	2.19	2.59
1.11	1.82	4.08	3.08	2.75
1.12	5.81	4.21	4.37	4.85
1.13	6.18	4.34	5.11	7.40

Notes. This table reports the initial positions for different specifications of the reference point. The initial positions are taken from a simulation of 10,000 investors' stock-trading decisions. Each investor trades four stocks. The initial positions are presented for different values of gross expected return over the year (μ) and $T = 4$. The cases with other values of T have a similar pattern. Other parameter values are set to be $\sigma = 0.3$, $\lambda = 2.25$, $\alpha = 0.88$, $R_f = 1$, $W_0 = 40$, and $P_0 = 40$.

Compared to a status quo reference point (SQ), an initial expected final wealth (EC) reference point generally leads investors to become more conservative in their initial stock purchase decisions. This is because the initial expected final wealth (when purchasing stock) is higher than the status quo wealth, so more prices will be coded as generating losses, hence reducing the attractiveness of the stock. When investors adjust the reference point based on historical realized returns, however, they become more aggressive and initially demand more shares compared to the SQ and EC cases. This is because in cases where prices are low, investors will adjust the reference point down to code some poor returns as gains. On average, there will be fewer large losses and gains relative to the reference point compared to cases that have a constant reference point. Since losses affect the level of utility more than gains because of loss aversion, stocks become more attractive when a variable reference point is used. Comparing the results of L1 and L2, we can also see that the more quickly investors adjust their reference points, the more attractive the stock, and thus the more shares investors demand at the initial date.

Prediction 3 (The Initial Position). *With a constant reference point, using initial expected final wealth as the reference point* (*EC*) *leads to the holding of fewer shares in the initial position than using the status quo wealth* (*SQ*) *does. With a variable reference point, investors tend to demand more shares at the initial date than they do with a constant reference point: The quicker the adjustment, the more shares investors initially demand.*

3.3. History-Dependent Effect with Variable Reference Point

With an updated expected final wealth reference point, price history affects not only current wealth, but also the reference point; thus, there is a natural history-dependent effect in the trading patterns we explore in this section. A binary returns distribution enables us to naturally define good and bad history using return realizations at date 1. Specifically, if the return at date 1 is R_u, we classify all realized prices following this node as a having good history; otherwise, we say that they have had a bad history.

For a constant reference point, as long as the current price is the same, we can show that the optimal stock position will remain the same regardless of which path leads to the current price. When investors adjust their reference points, however, the optimal stock position will differ following different histories, even when the current price is the same (because price history affects the reference point level). Specifically, let $P_{2,ij} = P_0 R_i R_j$ for $i = u, d$, $j = u, d$ denote the price at date 2 following a return R_i at date 1 and R_j at date 2. Let $x_{2,ij}$ denote the corresponding optimal stock position following each price history. The following prices are the

Table 5. Optimal Stock Position Following Good and Bad Price Histories

	$T=3$		$T=4$	
μ	Good history ($P_{2,ud}$)	Bad history ($P_{2,du}$)	Good history ($P_{2,ud}$)	Bad history ($P_{2,du}$)
1.03	—	—	—	—
1.04	—	—	—	—
1.05	—	—	—	—
1.06	—	—	0.52	0.89
1.07	—	—	0.60	1.05
1.08	1.54	1.71	0.61	0.98
1.09	1.67	1.75	0.64	0.94
1.10	1.78	1.79	2.48	2.13
1.11	1.87	1.83	2.77	2.78
1.12	4.69	2.20	4.83	2.93
1.13	5.44	3.57	4.62	2.60

Notes. This table reports the levels of $x_{2,ud}$ and $x_{2,du}$ assuming that investors update their reference points as one-period-lagged expected final wealth. These two positions share the same current price but different price histories. If the return at date 1 is R_u, all the price realizations following this node are classified as having a good history; otherwise, they have a bad one. The results are from 10,000 investors' simulated selling versus holding decisions. Each investor trades four stocks. The results are presented for different values of gross expected return over the year (μ) and different trading frequencies within the year. Other parameter values are set to be $\sigma=0.3$, $\lambda=2.25$, $\alpha=0.88$, $R_f=1$, $W_0=40$, and $P_0=40$. A dash means that the investors are not willing to purchase the stock in the first place.

same: $P_{2,du}=P_{2,ud}$. Therefore, with a constant reference point, $x_{2,du}=x_{2,ud}$. For variable reference points, however, this is not the case. Table 5 reports the levels of $x_{2,du}$ and $x_{2,ud}$, assuming investors update their reference points to one-period-lagged expected final wealth. These two positions share the same current price, but differ in price histories.

For both $T=3$ and $T=4$, and when the expected stock returns are low, given the same current price, investors tend to hold fewer shares of the stock after a good history. This pattern is reversed when the expected stock returns are high; i.e., investors take a larger position following a good history.

For this pattern's intuition, at price $P_{2,ud}$, a good return at date 1 generates a high reference point relative to the current state at date 2 (with a bad return at date 2), placing current wealth in the losses domain. For a similar reason, at price $P_{2,du}$, current wealth after a bad history falls in the gains domain relative to its reference point. Calculating the distance from the reference point in each case suggests that when expected stock returns are low, the reference point following a good history is not very high, so losses tend to be closer than gains to the reference point, and thus investors tend to hold a smaller position following a good price history. As expected stock returns rise, investors become more aggressive and take a larger position. When a good return is realized at date 1, given the larger initial stock position, the reference point is very high; this creates larger losses, leading to a larger stock position in the losses domain.

Prediction 4 (History-Dependent Effect on the Optimal Position). *With a variable reference point and given the same current price, the optimal number of shares held is smaller following a good price history for low expected stock returns. When expected stock returns increase, the pattern is reversed.*

3.4. Paper Gains/Losses and Realized Gains/Losses

Odean's (1998) Table III observes that realized gains/losses are smaller in absolute values than paper gains/losses, where gains and losses are defined relative to the average purchase price. This pattern implies that investors are more likely to realize small winners and losers than big ones, a pattern seldom explained by existing literature. To explore our model's prediction in this regard, Table 6 reports the simulated paper gains/losses and realized gains/losses under various specifications.

Panel A reports that for two cases with a constant reference point, regardless of whether the reference point is status quo or initial expected wealth, the average realized gains/losses are smaller in absolute value than the average paper gains/losses in most cases (except for those with very high expected stock returns). Panel B displays cases with a variable reference point, wherein we see the same pattern with a slow adjustment (L2). When the adjustment is relatively quick (L1), however, there is no substantial difference between paper and realized gains/losses. For expected returns ranging from 1.06 to 1.08, paper gains/losses are even smaller than realized ones in absolute value.

The intuition for this pattern comes from the prediction of loss aversion. Given that small trading gains and losses (relative to average purchase price) are on average closer to the reference point than large ones, loss aversion predicts more sales of the stocks at small trading gains and losses. While this explains why the status quo–based reference point is able to generate a similar pattern, quick adjustment of the reference point can change its location so much that small trading gains and losses are not necessarily close to the reference point at any given time.

Prediction 5 (The Paper Gains/Losses and Realized Gains/Losses). *In specifications SQ, EC, and L2, realized gains/losses are smaller than paper ones in absolute values, a pattern consistent with Odean's (1998) empirical finding. This pattern is not present or even reversed, however, in specification L1 when the reference point is updated relatively quickly.*

Table 6. Paper Gains/Losses and Realized Gains/Losses

μ	Paper gains	Realized gains	Paper losses	Realized losses	Paper gains	Realized gains	Paper losses	Realized losses
Panel A: Constant reference point								
	(SQ) BX's (2009) status quo wealth as the reference point				(EC) Initial expected final wealth as the constant reference point			
1.03	—	—	—	—	—	—	—	—
1.04	—	—	—	—	—	—	—	—
1.05	—	—	—	—	—	—	—	—
1.06	0.15	0.08	−0.15	−0.11	0.16	0.09	−0.15	−0.10
1.07	0.15	0.08	−0.15	−0.10	0.16	0.09	−0.14	−0.09
1.08	0.15	0.08	−0.15	−0.10	0.16	0.09	−0.14	−0.08
1.09	0.15	0.02	−0.15	−0.11	0.17	0.09	−0.13	−0.08
1.10	0.14	0.02	−0.15	−0.11	0.17	0.10	−0.13	−0.07
1.11	0.14	0.02	−0.14	−0.10	0.17	0.16	−0.14	−0.17
1.12	0.16	0.12	−0.12	−0.13	0.16	0.09	−0.13	−0.16
1.13	0.16	0.12	−0.12	−0.13	0.16	0.10	−0.13	−0.16
Panel B: Adjusted reference point								
	(L1) One-period-lagged expected final wealth as the variable reference point				(L2) Two-period-lagged expected final wealth as the variable reference point			
1.03	—	—	—	—	—	—	—	—
1.04	—	—	—	—	—	—	—	—
1.05	—	—	—	—	—	—	—	—
1.06	0.17	0.17	−0.16	−0.18	0.16	0.12	−0.14	−0.06
1.07	0.18	0.21	−0.15	−0.17	0.15	0.08	−0.15	·
1.08	0.18	0.21	−0.15	−0.17	0.16	0.09	−0.14	−0.10
1.09	0.19	0.15	−0.14	−0.06	0.17	0.07	−0.13	−0.09
1.10	0.19	0.17	−0.12	−0.07	0.17	0.07	−0.12	−0.09
1.11	0.18	0.18	−0.13	−0.12	0.17	0.07	−0.12	−0.10
1.12	0.18	0.18	−0.13	−0.13	0.16	0.07	−0.14	−0.12
1.13	0.17	0.16	−0.13	−0.14	0.18	0.11	−0.13	−0.15

Notes. This table reports the paper gains and losses and the realized gains and losses from a simulation of 10,000 investors' decisions regarding holding or selling. Each investor trades four stocks. The results are presented for different values of gross expected return over the year (μ) and $T = 4$. The cases with other values of T have a similar pattern. Other parameter values are set to be $\sigma = 0.3$, $\lambda = 2.25$, $\alpha = 0.88$, $R_f = 1$, $W_0 = 40$, and $P_0 = 40$ throughout this simulation. A dash means that the investors are not willing to purchase the stock in the first place. The dot means that no losses are realized in this case.

4. Discussion of Simulation Results

Our simulation results clearly show that using a reference point based on expected final wealth under the principle of preferred personal equilibrium (whether using initial expectations as a constant reference point or lagged expectations as a variable reference point) can generate results that display the disposition effect in most cases. Besides successfully predicting the disposition effect, our model also explores a new dimension of reference point adjustment to generate new predictions. Predictions 2 through 5 offer new predictions aside from the disposition effect, which distinguishes our model from other alternative theories on the disposition effect.

Adjustment of the reference point is also related to another important topic in the literature: market experience and bias reduction. List (2003) and others (e.g., Feng and Seasholes 2005, Da Costa et al. 2013) have shown that market experience can significantly reduce behavioral biases. This is due to either a selection bias in the sense that those with lower biases are more successful and thus are more likely to become experienced, or that people eventually learn about their bias and are therefore able to debias themselves. This paper suggests an adjusting reference point as a third channel. As investors become more experienced, they may realize that it is irrational to stick to their initial expected or status quo wealth in judging gains and losses. Our results show that the more quickly investors adjust their reference point, the less likely they are to demonstrate the disposition effect. Our model also predicts that speeding up reference point adjustment leads investors to be less conservative and demand more shares of stocks at the initial date. These predictions reasonably capture the characteristics of the experienced investors' behavior.

Our Prediction 5 requires more discussion. Despite Odean's (1998) empirical finding that the average of realized gains/losses is smaller in absolute value than that of paper gains, Ben-David and Hirshleifer (2012) find that the probability of selling monotonically increases for positive returns; i.e., investors are more likely to realize big winners than small ones. One difference between the two papers is that Odean (1998) uses the account-level transaction data in the United States from 1987 to 1993, while Ben-David and Hirshleifer (2012) use similar data but from 1990 to 1997. Another difference is that the former paper uses the whole sample, while the latter divides the sample up by holding period to do the analysis. A substantial heterogeneity in trading behavior possibly explains these different results, as Table 5 implies: Our predictions are largely consistent with Odean's (1998) when the reference point is not quickly adjusted, yet model L1's predictions support Ben-David and Hirshleifer (2012) when expected stock returns are from 1.08 to 1.11 ($T=3$, not reported here) and from 1.06 to 1.08 ($T=4$). When investors quickly update their reference points, it is possible that big winners fall closer to the updated reference point than small winners; hence, investors choose to realize the big winners. Assuming such heterogeneity, the pattern that ultimately appears depends on whether most investors update their reference points relatively quickly as well as on which subsample the estimation focuses on. Therefore, our model may provide a potential channel to reconcile these contradictory empirical findings in the literature based on reference point adjustment.

A number of authors in the literature have suggested a model of realization utility to explain the disposition effect (e.g., BX 2012), wherein investors derive utility from *realizing* gains and losses instead of from final total wealth (as assumed in our model). We would like to emphasize that our model and the model of realization utility are, in fact, complementary: In reality, investors probably derive utility from realizing gains and losses and from final total wealth. Our model aims to understand to what extent disposition effect can be explained using a standard prospect theory model. It is possible that our model cannot fully explain the reality, and other models such as the realization-utility model are needed to increase the explanatory power.

Our model differs from that of BX (2012), however, in terms of predictions. BX's (2012) model predicts a strong disposition effect in the sense that, unless forced to sell at a loss by a liquidity shock, the investor only sells stocks when they are trading at a price higher than that of his original purchase. In other words, selling at a loss is solely triggered by exogenous liquidity shock in BX's (2012) model, and selling at a loss also implies a complete exit from the market. This result comes from their model's basic setting: investors derive utility from realizing gains and losses, and are hence reluctant to sell stocks that have suffered losses based on their active decisions. In our model, however, selling at a loss can be an endogenous decision by the investor and there can be partial sales.[12]

5. Robustness Check

In this section, we provide several robustness checks of our model results based on preferred personal equilibrium.

5.1. Incorporating Consumption Utility

In our benchmark model, the investor's preference is simply based on gain–loss utility. In this section, we extend our model to include the consumption utility as well. Following Barberis et al. (2001) and KR (2006, 2007), which all incorporate consumption utility and gain–loss utility, we consider the following standard utility function:

$$u(W) + v(u(W) - u(W^R)),$$

where $u(W)$ measures the consumption utility from final wealth W, and $v(u(W) - u(W^R))$ measures the utility derived from gains/losses relative to the reference utility $u(W^R)$. Following the literature (e.g., Pagel 2016), we set $u(W) = W^{1-\theta}/(1-\theta)$ with $\theta = 4$. This is the standard CARA utility function with the coefficient of relative risk aversion set at 4. And the gain–loss utility function $v(\cdot)$ takes the following specific form:

$$v(x) = \begin{cases} \eta x & \text{for } x \geq 0, \\ \lambda \eta x & \text{for } x < 0. \end{cases}$$

For simplicity, we let the reference point be fixed as the date 0 expectation of final wealth $\bar{W}$. Then, we can similarly write down the optimization problem as

$$\max_{\{W_{T,j}\}_{j=1,\ldots,T+1}} \sum_{j=1}^{T+1} \pi_{T,j}[u(W_{T,j}) + v(u(W_{T,j}) - u(\bar{W}))], \tag{19}$$

subject to the budget constraint

$$\sum_{j=1}^{T+1} \pi_{T,j} q_{T,j} W_{T,j} = W_0 \tag{20}$$

and a nonnegativity wealth constraint

$$W_{T,j} \geq 0, \quad j = 1, \ldots, T+1. \tag{21}$$

It turns out that our previous results are robust to this inclusion of consumption utility. For example, we simulate a case with $T=2$, $\sigma=0.3$, $W_0=40$, $P_0=40$, $R_f=1$, $\lambda=2.25$, and $\eta=1$. In this case, the disposition effect always exists for an annual gross return of μ ranging from 1.03 to 1.13.

One possible criticism of our benchmark model's consideration of only gain–loss utility is that it suggests that investors would be better off not searching for information about the stock market. This is because forming an expectation of purchasing the stock will necessarily generate gains and losses, and with loss aversion the utility is negative, while expecting not to purchase the stock creates neither gains nor losses. This issue can also be partly solved by adding consumption utility. For example, if we let $\eta = 1/4$ and keep other parameters the same, the investor derives higher utility from purchasing the stock under the principle of preferred personal equilibrium than from not purchasing under the reference point of initial wealth when the annual gross return is higher than 1.06. This result is intuitive, as stock purchase indeed tends to improve consumption utility in many cases.

5.2. Exogenous Initial Reference Point

One may argue that the initial reference point can be exogenous (as opposed to the endogenous expectation of final wealth). KR (2009) also discuss a case wherein the decision maker's initial reference point is not entirely rational. We consider a two-period model where the exogenous date 0 reference point is set to $\bar{W}_0$, and the date 1 reference point is the expectation of final wealth. We thus write our equilibrium definition as follows:

Definition 3. For a given $\bar{W}_0$, we consider $\{\bar{W}_1^*, W^*(h_t), x^*(h_t)\}$ a rational expectation equilibrium if it satisfies the following three conditions:

1. At any history, h_t, x^* is the solution of the optimization problem (5).
2. $W^*(h_t)$ is the current updated wealth according to Equation (3).
3. $\bar{W}_1^*$ is determined by rational expectation condition $\bar{W}_1^* = E[W_2 \mid h_0]$.

We consider two possible initial reference points: $\bar{W}_0 = W_0$ and $\bar{W}_0 = 1.5W_0$. Interestingly, we find no presence of the disposition effect when $\bar{W}_0 = W_0$, while there is a disposition effect when $\bar{W}_0 = 1.5W_0$. The specification of the initial reference point thus tends to affect the appearance of the disposition effect.

5.3. Evaluation Period

Following Benartzi and Thaler (1995) as well as BX (2009), we assume an evaluation period of one year. To give an example of how different evaluation period lengths affect the disposition effect, we simulate a case with an evaluation period of half a year and $T = 3$. In all specifications of the reference point, investors require an annual expected stock return of at least 1.11 to purchase stock, suggesting that the shorter the evaluation period, the more risk averse the investor. With fixed initial expected final wealth as the reference point, there still remains a substantial disposition effect with an infinite PGR/PLR ratio, the same as the result in Table 2. Surprisingly, using one-period-lagged expected final wealth as the variable reference point creates a much stronger disposition effect than the corresponding case in Table 2: PGR/PLR is consistently around 1.2 for annual expected stock returns equal to or greater than 1.11, while there is no disposition effect in Table 2 when the annual expected stock return is greater than 1.11. In general, a shorter evaluation period makes investors more risk averse in their initial stock choice and leads to a stronger disposition effect after the stock purchase. This result is consistent with that of Benartzi and Thaler (1995), who find that narrow framing is a behavioral bias reinforcing the effect of loss aversion. Barberis et al. (2006) also show that loss aversion requires narrow framing to explain first-order risk aversion in purchasing small gambles.

5.4. Other Simplifications Relative to KR's (2006) New Reference-Dependent Model

KR (2006) develop a more general version of the reference-dependent model in which total utility is a weighted average of consumption and gain–loss utilities; consumption utility is also the unit for gain–loss comparison. This more general version keeps the essential feature of loss aversion while incorporating the effect of standard consumption utility. Strictly following this specification does not lead to qualitatively different conclusions for the purposes of this paper, as it always implies the same effect of loss aversion on risk attitude.

KR's (2006) original specification also uses a stochastic reference point, and the utility is a probability-weighted average of gain–loss utilities relative to different outcomes of that reference point. Masatlioglu and Raymond (2016) show that whether the reference point is stochastic or deterministic has important implications to an individual's risk attitude. Sprenger (2015) also shows in a lab experiment that stochastic reference points generally make investors less risk averse. The prediction that stock holding monotonically increases in distance to the reference point is essentially preserved, however, except that now the distance is a probability-weighted average of the distances to all possible outcomes. This does not affect the main predictions of our model.

6. Conclusion

In the title of this paper, we ask whether prospect theory can explain the disposition effect given an expectations-based reference point. The main message of this paper implies that some expectations-based reference point models can explain the disposition effect, but there are limitations in applying the general idea of an expectations-based reference point to certain specific settings. In particular, a model based on loss aversion in prospect theory can explain the disposition effect under the principle of preferred personal equilibrium. We also find that, quite surprisingly, when a model permits no lag in the expectations-based reference point, or determines the reference point solely by disappointment aversion, the model cannot explain why the investor bought the stock in the first place.

Under the principle of preferred personal equilibrium, the predictions of loss aversion on individual selling patterns can lead to the disposition effect in most cases with a reference point of lagged expected final wealth (for both constant initial expectation and updated expectations). This model also predicts that the disposition effect is more likely to be present when the reference point is not updated quickly enough, when the expected stock return is low, and when stock trading is infrequent. Our model also makes interesting predictions on other topics besides the disposition effect. Adjusting the reference point predicts a history-dependent effect on stock holding even when the current price remains the same. The quicker investors are able to adjust their reference points, the less conservative they are in initial purchase decisions. When reference point adjustment is slow, our model's investors are more likely to realize small wins and losses as opposed to large ones, an observation consistent with Odean's (1998) finding; but the opposite pattern appears when reference point adjustment is quick, which is consistent with Ben-David and Hirshleifer's (2012) empirical finding.

It is important to clarify that defining the reference point as lagged expected final wealth is a sufficient but unnecessary condition for loss aversion to generate the disposition effect. In theory, any reference point that is sufficiently higher than initial wealth can lead to the disposition effect. Given the abundant empirical evidence on expectations-based reference points outside of finance, however, assuming an expectations-based reference point seems a reasonable first step. While this paper assumes that expectations are rational, the theoretical framework is flexible enough to accommodate several forms of potentially irrational expectations. For instance, investors may linearly extrapolate from their past returns to form new expectations, or demonstrate overoptimistic beliefs and representative biases. Building more realistic expectation formations into our model will further improve its explanatory power.

As the first to introduce expectations-based reference points into investors' trading behaviors, this paper focuses on the disposition effect and its related individual trading patterns. The idea that expectations can affect the reference point has several important implications for asset prices and deserves a systematic and careful examination in future research.

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Appendix. Omitted Proofs

A.1. Proof of Proposition 1

The proof proceeds in two steps. First, we state several key properties of the solutions to the optimization problem (6)–(8). Second, based on these established properties, we solve the rational expectation equilibrium from Definition 1.

Define $\hat{w}_{T,j} = W_{T,j} - \bar{W}$ to be the investor's gain/loss relative to his reference point, and then the optimization problem (6)–(8) can be rewritten as

$$\max_{\{\hat{w}_{T,j}\}_{j=1,\ldots,T+1}} \sum_{j=1}^{T+1} \pi_{T,j} v(\hat{w}_{T,j}), \tag{A.1}$$

subject to the budget constraint

$$\sum_{j=1}^{T+1} \pi_{T,j} q_{T,j} \hat{w}_{T,j} = W_0 - \bar{W} \tag{A.2}$$

and a nonnegativity of wealth constraint

$$\hat{w}_{T,j} \geq -\bar{W}, \quad j = 1, \ldots, T+1. \tag{A.3}$$

The following properties have already been proven by BX (2009); hence, we just state these properties without proving them.

Lemma 1. *There exists at least one solution to the optimization problem (A.1)–(A.3).*

Lemma 2. *If the investor's optimal gain/loss $\hat{w}_{T,j}$ is different from zero at the end of some path, then it is different from zero at the end of all paths.*

Lemma 3. *If the optimal allocation is nonzero, there must be at least one path at the end of which the gain/loss is $-\bar{W}$, so that the investor is wealth constrained.*

Lemma 4. *It is not optimal to have a path with an unconstrained negative allocation $\hat{w}_{T,j}$ < (i.e., $-\bar{W} < \hat{w}_{T,j} < 0$).*

Lemma 5. *Any path with a constrained negative wealth allocation must have a state price density not lower than that of any path with a positive allocation.*

The above results imply that if $\hat{w}_{T,j}$ is not zero for all j, then $\hat{w}_{T,j}$ can be either strictly positive or constrained negative ($-\bar{W}$). Moreover, the optimal wealth allocation has a threshold property. There is a threshold in the state price density such that paths with state price densities higher than that threshold have a constrained negative allocation, while paths with state price densities lower than that threshold have a strictly positive allocation.[13] Since we have already ranked the nodes according to the state price density, the above threshold property implies that there is a threshold k in the node such that

$$\hat{w}_{T,j} = \begin{cases} >0 & \text{if } j \le k, \\ -\bar{W} & \text{if } j > k. \end{cases} \tag{A.4}$$

From Equation (A.2), we obtain

$$\sum_{j=1}^{k} \pi_{T,j} q_{T,j} \hat{w}_{T,j} = W_0 - \sum_{j=1}^{k} \pi_{T,j} q_{T,j} \bar{W}. \tag{A.5}$$

For $i, j \le k$, the first order conditions imply

$$\frac{v'(\hat{w}_{T,i})}{v'(\hat{w}_{T,j})} = \frac{q_i}{q_j} \Rightarrow \frac{\hat{w}_{T,j}}{\hat{w}_{T,i}} = \left(\frac{q_i}{q_j}\right)^{1/(1-\alpha)}. \tag{A.6}$$

Plugging Equation (A.6) into Equation (A.5) yields

$$\hat{w}_{T,i} = \frac{W_0 - \sum_{j=1}^{k} \pi_{T,j} q_{T,j} \bar{W}}{\sum_{j=1}^{k} \pi_{T,j} q_{T,j}^{-\alpha/(1-\alpha)}} q_{T,i}^{-1/(1-\alpha)}. \tag{A.7}$$

Finally, the rational expectation condition (9) requires that

$$\sum_{i=1}^{k} \pi_{T,i}(\hat{w}_{T,i} + \bar{W}) = \bar{W}. \tag{A.8}$$

Equation (A.8) is a linear equation about $\bar{W}$. Solving this equation yields Equation (10).

The above proof merits two remarks. First, if the investor's optimal gain/loss $\hat{w}_{T,j}$ is zero at the end of all paths, then by Equation (A.5), we must have $\bar{W} = W_0$. And this can be viewed as a special case of Equation (10) by letting $k = T+1$. Second, Equation (10) only derives the equilibrium reference point in a candidate equilibrium. To be an equilibrium, the investor's optimal threshold strategy has to be k under reference point $\bar{W}(k)$, which is true when $U(k) = V(\bar{W}(k))$.

A.2. Proof of Corollary 1

The key of the proof is to show that $\bar{W} = W_0$ can be a rational expectation equilibrium reference point if and only if it is optimal not to buy any stock when the reference point is $\bar{W} = W_0$. First, if it is optimal not to buy any stock when the reference point is $\bar{W} = W_0$, then it is straightforward to show that

$$\{\bar{W}^* = W_0, \{W^*_{T,j} = W_0\}_{j=1,\ldots,T=1}\}$$

constitutes a rational expectation equilibrium from Definition 1. Second, if it is optimal to buy some stock when the reference point is $\bar{W} = W_0$, then the rational expectation condition in Definition 1 cannot be satisfied because the expected stock return exceeds the risk-free rate.

If there exists a rational expectation equilibrium with reference point $\bar{W} = W_0$, then this equilibrium is also the most efficient rational expectation equilibrium. Obviously, under the rational expectation equilibrium

$$\{\bar{W}^* = W_0, \{W^*_{T,j} = W_0\}_{j=1,\ldots,T=1}\},$$

the date 0 expected utility is zero. Suppose that there exists another equilibrium $\{\bar{W}^*, \{W^*_{T,j}\}_{j=1,\ldots,T+1}\}$ with date 0 expected utility

$$\sum_{j=1}^{T+1} \pi_{T,j} v(W^*_{T,j} - \bar{W}^*).$$

Because of loss aversion and risk aversion, the above expression is less than $v(\sum_{j=1}^{T+1} \pi_{T,j} W^*_{T,j} - \bar{W}^*) = 0$. Therefore, the equilibrium

$$\{\bar{W}^* = W_0, \{W^*_{T,j} = W_0\}_{j=1,\ldots,T=1}\}$$

is the most efficient rational expectation equilibrium.

A.3. Proof of Corollary 2

Suppose that at date $T-1$, the reference points satisfy $\bar{W}^*(h_{T-1}) = E[W_T \mid h_{T-1}]$. We want to show that $W_{T-1} = \bar{W}^*_{T-1}$ in any rational expectation equilibrium. Suppose not, and $W_{T-1} > \bar{W}^*_{T-1}$. From Proposition 2, we can calculate $W_T(u)$ and $W_T(d)$ as follows:

$$W_T(u) = \bar{W}^*_{T-1} + 2\frac{W_{T-1} - \bar{W}^*_{T-1}}{q_u^{-\alpha/(1-\alpha)} + q_d^{-\alpha/(1-\alpha)}} q_u^{-1/(1-\alpha)} \quad \text{and}$$

$$W_T(d) = \bar{W}^*_{T-1} + 2\frac{W_{T-1} - \bar{W}^*_{T-1}}{q_u^{-\alpha/(1-\alpha)} + q_d^{-\alpha/(1-\alpha)}} q_d^{-1/(1-\alpha)},$$

where

$$q_u = \frac{2(R_f - R_d)}{R_f(R_u - R_d)} \quad \text{and} \quad q_d = \frac{2(R_u - R_f)}{R_f(R_u - R_d)}.$$

Obviously, both $W_T(u)$ and $W_T(d)$ are strictly larger than $\bar{W}^*_{T-1}$. Hence,

$$E_{T-1}[W_T] = \tfrac{1}{2} W_T(u) + \tfrac{1}{2} W_T(d) > \bar{W}^*_{T-1},$$

a contradiction.

Now suppose that $W_{T-1} < \bar{W}^*_{T-1}$. From Proposition 2 we can still calculate $W_T(u)$ and $W_T(d)$ as follows:

$$W_T(u) = \min\left\{\bar{W}^*_{T-1} + 2\frac{\bar{W}^*_{T-1} - W_{T-1}}{q_d(\lambda q_u/q_d)^{1/(1-\alpha)} - q_u}, 2W_{T-1}/q_u\right\}$$

and

$$W_T(d) = \max\left\{\bar{W}^*_{T-1} - 2\frac{\bar{W}^*_{T-1} - W_{T-1}}{q_d(\lambda q_u/q_d)^{1/(1-\alpha)} - q_u} \times \left(\frac{\lambda q_u}{q_d}\right)^{1/(1-\alpha)}, 0\right\}.$$

If

$$W_T(u) = \bar{W}^*_{T-1} + 2\frac{\bar{W}^*_{T-1} - W_{T-1}}{q_d(\lambda q_u/q_d)^{1/(1-\alpha)} - q_u}$$

and

$$W_T(d) = \bar{W}^*_{T-1} - 2\frac{\bar{W}^*_{T-1} - W_{T-1}}{q_d(\lambda q_u/q_d)^{1/(1-\alpha)} - q_u} \times \left(\frac{\lambda q_u}{q_d}\right)^{1/(1-\alpha)},$$

$E_{T-1}[W_T] < \bar{W}^*_{T-1}$ under Assumption 1 because

$$\frac{q_u}{q_d} = \frac{R_f - R_d}{R_u - R_f} = \frac{1}{g}.$$

This also contradicts the requirement that $\bar{W}^*_{T-1} = E_{T-1}[W_T]$.

Another possibility is that $W_T(u) = 2W_{T-1}/q_u$ and $W_T(d) = 0$. Then $\bar{W}^*_{T-1} = E_{T-1}[W_T]$ implies that

$$\tfrac{1}{2}W_T(u) + \tfrac{1}{2}W_T(d) = \bar{W}^*_{T-1} \Rightarrow \bar{W}^*_{T-1} = W_{T-1}/q_u.$$

But then

$$\begin{aligned}&\bar{W}^*_{T-1} + 2\frac{\bar{W}^*_{T-1} - W_{T-1}}{q_d(\lambda q_u/q_d)^{1/(1-\alpha)} - q_u}\\ &= W_{T-1}/q_u + 2\frac{W_{T-1}/q_u - W_{T-1}}{q_d(\lambda q_u/q_d)^{1/(1-\alpha)} - q_u}\\ &= W_{T-1}/q_u\left[1 + \frac{2(1-q_u)}{q_d(\lambda q_u/q_d)^{1/(1-\alpha)} - q_u}\right].\end{aligned}$$

Under Assumption 1, $\lambda q_u/q_d > 1$, and hence

$$\frac{2(1-q_u)}{q_d(\lambda q_u/q_d)^{1/(1-\alpha)} - q_u} < \frac{2(1-q_u)}{q_d - q_u} = 1.$$

The last equation is true because $q_d + q_u = 2$. Therefore, when $\bar{W}^*_{T-1} = W_{T-1}/q_u$,

$$\bar{W}^*_{T-1} + 2\frac{\bar{W}^*_{T-1} - W_{T-1}}{q_d(\lambda q_u/q_d)^{1/(1-\alpha)} - q_u} < 2W_{T-1}/q_u.$$

But this contradicts the assumption that $W_T(u) = 2W_{T-1}/q_u$. This leaves only one remaining possibility: $W_{T-1} = \bar{W}^*_{T-1}$. Therefore, in any rational expectation equilibrium, the investor does not buy any stock at date $T-1$.

At date $T-2$, given that the investor does not buy any stock at date $T-1$, the same logic implies that W_{T-2} is equal to the reference point $\bar{W}^*_{T-2}$ in any rational expectation equilibrium. By induction, we conclude that the investor does not buy any stock at all in a rational expectation equilibrium.

Endnotes

[1] Odean (1998) explicitly considers expected-utility explanations for asymmetry across winners and losers based on richer specifications of the investor's problem, finding that portfolio rebalancing, transaction costs, taxes, and rationally anticipated mean reversion cannot explain observed asymmetries. Weber and Camerer (1998) also find that incorrect beliefs concerning mean reversion cannot explain the disposition effect.

[2] The third feature of prospect theory, nonlinear probability weighting, roughly assumes that investors systematically overweigh small probabilities and underweigh large probabilities. For simplicity's sake, the literature on the disposition effect often does not discuss this feature, nor does this paper.

[3] One notable exception is the study by Health et al. (1999), who analyze the option of exercising decisions, wherein there is no natural purchase price to rely on. They find that historical high price plays an important role in driving decisions. In BX (2009), the assumed reference point is wealth from investing in risk-free assets, which is essentially a status quo assumption that takes interest rate into account.

[4] KR (2007) discuss the difference between the choice-acclimating personal equilibrium (CPE) and the preferred personal equilibrium. CPE is essentially similar to the disappointment-aversion models, and hence shares similar predictions.

[5] There are several other studies on the disposition effect that do not involve loss aversion. For instance, Ben-David and Hirshleifer (2012) propose a model that uses investor speculation to explain the disposition effect. Chang et al. (2016) combine real trading data with a laboratory experiment to show that the delegated portfolio demonstrates an antidisposition effect, proposing that cognitive dissonance is one source of the disposition effect. Strahilevitz et al. (2011) and Frydman and Camerer (2016) use the regret-devaluation mechanism to predict that more repurchasing will occur when there is a gain than when there is a loss, a conclusion that is consistent with the data.

[6] While the BX (2009) model also assumes a reference point of wealth from risk-free asset investment, their simulation assumes that $R_f = 1$.

[7] The disappointment aversion models in the literature assume that the reference point changes with the action when deriving an optimal solution (Bell 1985, Loomes and Sugden 1986, Gul 1991). The choice-acclimating personal equilibrium of KR (2007) exhibits a similar feature. For more detail, see Section 2.4.

[8] BX (2006) only require that the condition $g < \lambda^{1/\alpha}$ must be satisfied. This condition holds under Assumption 1, since $\alpha = 0.88$.

[9] We did not find empirical papers that test which model, preferred personal equilibrium or disappointment aversion, is more psychologically appropriate. In many existing studies (e.g., Abeler et al. 2011, Sprenger 2015), the two models make the same predictions and hence are indistinguishable. The essential difference between the two is whether investors can adjust their reference points quickly enough to incorporate the consequences of the actions. The evidence on the adjustment speed of the reference point is mixed. In general, the reference point tends to be sticky when the stakes are high and the loss is large (Post et al. 2008, Song 2016, DellaVigna et al. 2017).

[10] Since we do not have closed-form solutions for the L1 and L2 cases, it is also difficult to claim with certainty that our simulated results actually represent the most efficient equilibrium. We tried our best to solve this problem during the simulations: For example, we started from an initial guess with all reference points being W_0, and also tried other initial guesses to see whether we achieved similar results. Finally, we compared our L1 and L2 results with those of the SQ and EC cases to see whether the results strongly differed. Considering these efforts, we are quite confident that our simulation's results are the most efficient rational expectation equilibrium.

[11] We would like to thank an anonymous referee for providing an example illustrating this point: Assume three investors have the same tendency for a disposition effect, in the sense that their ratios of hazard rates for selling winners divided by losers is the same. Investors A and B have the same turnover rates, but different size portfolios. Investors A and C have portfolios of the same size, but with different turnover rates. Using numerical examples, one can show that the calculated difference PGR − PLR differs significantly across the three investors, yet the PGR/PLR ratio is the same.

[12] If we replace BX's (2012) exogenous liquidity shock with the wealth constraint in our model, then BX's (2012) results tend to imply that selling occurs only when the investor's wealth constraint is binding; hence, their model never realizes small losses. In contrast, our model allows for the realization of small losses, and a slow-adjusting reference point makes the likelihood of realizing small losses larger than that of realizing large losses.

[13] As argued by BX (2009), this threshold property crucially depends on the assumption that the probabilities of events u and d in each period are the same, and the property may fail if this assumption does not hold.

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