

Models of Reputations and Relational Contracts

Preliminary Lecture Notes

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1 Relational Contracts

In the previous moral hazard models, we require y —the dollar value of the agent’s contribution to the firm—to be observable, *ex ante describable* and *ex post verifiable*. However, for most principals, it is extremely difficult to measure y in a way that would allow the agent’s pay to be based on y through a compensation contract that could be enforced by a court, if necessary. One possible reason is that the agent’s contribution to firm value is not objectively measurable (advertising firms for example). Even if the agent’s contribution to firm value is not objectively measurable, however, it sometimes can be subjectively assessed by superiors who are well placed to observe the subtleties of the agent’s behavior and opportunities. Another possibility is that the agent’s contribution to firm value is “observable but not verifiable”. That is, the agent’s contribution is observable by the parties but not verifiable by a court.

In these situations, it is impossible to provide incentives to the agent in the single period setting. However, if the game is repeated for infinitely many times, we can use relational contracts to induce efforts. Relational contracts are informal agreements between the principal and agent about the compensation. Relational contracts cannot be enforced by a third party and so must be self-enforcing agreements. In the repeated setting, when the future relationship is sufficiently valuable, each player has no incentive to renege.

1.1 Toy Model Setup

The model discussed here is based on Baker, Gibbons, and Murphy (1994). There are two players, the Principal (P) who owns the firm; and the Agent (A), who works in the firm. The

worker chooses an unobservable action, a , that stochastically determines the his contribution to firm value, y . In particular, y equals either L or H , and the worker's action, $0 \leq a \leq 1$, equals the probability that $y = H$. (That is, higher actions produce higher probabilities of $y = H$; the action $a = 0$ guarantees that $y = L$ will occur.) The worker incurs an action cost $c(a)$, where $c(\cdot)$ is a strictly increasing and convex function with $c'(0) = 0$ and $c'(1) = \infty$.

We assume y is either not verifiable or not objectively measured. Therefore, it cannot be the basis of an enforceable contract. However, y can be subjectively measured. The timing of events in a stage game is the following. First, the firm offers the worker a compensation package (s, b) , where s is a base salary paid when the worker accepts the offer and b is a relational-contract bonus meant to be paid only when $y = H$. Second, the worker either accepts the compensation package or rejects it in favor of an alternative employment opportunity with payoff w_0 . Third, if the worker accepts then the worker chooses an action at cost $c(a)$. The firm does not observe the worker's action. Fourth, the firm and the worker observe the realization of the worker's contribution to firm value, y . Finally, if $y = H$ then the firm chooses whether to pay the worker the bonus b specified in the relational contract.

Both players are risk neutral. The firm's payoff when the worker's contribution is y and total compensation is w is the profit $y - w$. The worker's payoff from choosing an action a and receiving total compensation w is $w - c(a)$.

In this stage game, the unique subgame perfect equilibrium outcome is that the firm would choose not to pay a bonus, so the worker (anticipating the firm's decision) would choose not to supply any effort. So the firm (anticipating the worker's choice) would not pay a salary greater than L . Whether the worker is employed at this firm would then depend on whether w_0 is greater or less than L . If $w_0 \leq L$, then the worker will be employed at the

firm, but will not supply any effort. We will assume hereafter that $w_0 > L$, in which case the worker would not be employed at this firm in the single-period model. Meanwhile, denote a^{FB} to solve:

$$\max_a L + a(H - L) - c(a).$$

We require

$$L + a^{FB}(H - L) - c(a^{FB}) - w_0 > 0$$

such that it is socially optimal to employ the worker.

1.2 The Repeated Game

The key issue in the single-period setting is that given the contract is not enforceable by the third party, the firm always has incentives to renege and stop paying the bonus b . Anticipating this, the worker would choose not to supply any effort. However, if the repeated setting, the future concern may force the firm to honor the relational contract.

We will consider the game where the single-period game is repeated for infinitely many times. The firm is long-lived with discount rate δ while each worker lives for only one period. However, the new workers can observe all the histories (i.e., the output y and whether the firm honors the contract or not).

Since the worker is short-lived, if the worker accepts the contract and believes the firm is going to honor the contract, the worker's optimal action solves

$$\max_a s + ab - c(a).$$

The assumptions on $c(\cdot)$ guarantees that there is a unique maximizer $a^*(b)$ satisfying $c'(a) = b$. The worker will accept the contract if and only if

$$s + ba^*(b) - c(a^*(b)) \geq w_0.$$

The firm's expected payoff is

$$E\pi = L + a^*(b)(H - L) - s - ba^*(b).$$

Therefore, the highest expected payoff that the firm can receive in each period is:

$$E\pi(b) = L + a^*(b)(H - L) - w_0 - c(a^*(b)).$$

In analyzing this repeated game, we focus on grim trigger strategies: the worker accepts the contract as long as the expected payoff exceeds the outside option and the firm honors the contract in all of the previous periods. Once the firm does not honor the contract, the worker rejects the contract and the firm will always not honor the contract afterwards. We are interested in deriving the highest bonus achievable using such trigger strategies. Notice that at state H , the firm's discounted expected payoff if it honors the contract is:

$$(1 - \delta)(H - b - s) + \delta(1 - \delta)E\pi_2 + \delta^2(1 - \delta)E\pi_3 + \dots .$$

If the firm does not honor the contract, the firm does not pay the bonus b in the current period and its discounted expected payoff is:

$$(1 - \delta)(H - s) + \delta(1 - \delta)0 + \delta^2(1 - \delta)0 + \dots .$$

By the one-shot deviation principle, the firm will honor the contract at state H if and only if:

$$(1 - \delta)b \leq \delta(1 - \delta)E\pi_2 + \delta^2(1 - \delta)E\pi_3 + \dots .$$

Since $E\pi_t \leq E\pi(b)$, the highest b is achieved by letting $E\pi_t = E\pi(b)$ for all t , which yields:

$$\frac{1 - \delta}{\delta}b \leq E\pi(b).$$

Let $r = \frac{1 - \delta}{\delta} \in (0, \infty)$. Obviously, the higher r (or lower δ), the harder to deter the firms to deviate. If r is sufficiently low ($r_L \leq \frac{E\pi(H-L)}{H-L}$), the first best action a^{FB} can be achieved by setting $b = b^{FB} = H - L$. For r in the intermediate region (r_M), the highest achievable bonus falls as r increases, because the higher value of r makes future profits less valuable, so the firm is more tempted to renege. Finally, if r is sufficiently large r_H , no incentives can be provided through relational contracts.

Remark 1 *Another solution to the moral hazard problems in repeated setting is efficiency wage discussed by Shapiro and Stiglitz (1984). In this model, both the firm and the worker are long-lived. And the firm only pays the basic wage (no bonus). There are two key elements in this model: 1) the wage is above the market clearing level; and 2) there is unemployment and if the worker is found to be shirking, the worker will be fired. It is the combination of*

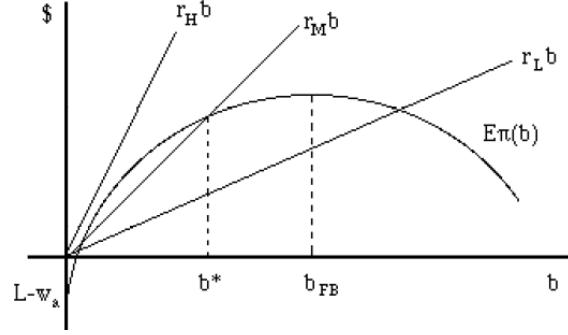


Figure 1: Highest bonus supported by grim trigger strategies.

unemployment and high wages that provide incentives to the worker.

1.3 General Model

1. Principal and agent are both risk neutral;
2. Infinite time horizon;
3. At the beginning of date t the principal offers:
 - a fixed salary w_t and
 - a contingent bonus $b_t : \phi \rightarrow \mathbb{R}$, where ϕ is the set of observed performance outcomes.
4. The agent decides whether to accept or reject the offer;
5. If the agent rejects, the principal and the agent receives their outside option utilities $(\bar{\pi}, \bar{u})$, where $\bar{s} = \bar{\pi} + \bar{u}$;
6. If the agent accepts, he observes a cost parameter $\theta_t \in \Theta = [\underline{\theta}, \bar{\theta}]$, with CDF $P(\cdot)$. θ_t is distributed independently across periods;

7. Then the agent chooses an effort $e_t \in [0, \bar{e}]$ and incurs a cost of $c(e_t, \theta_t)$ with $c(e_t, \theta_t) \geq 0$ and $c(0, \theta_t) = 0$ for all θ_t ;
8. Then output $y_t \in [\underline{y}, \bar{y}]$ is drawn from the cdf $F(y|e)$;
9. Finally the principal decides on whether to pay the bonus b_t .

Remarks:

- The fixed wage w_t can be enforced by the courts;
- The bonus payment b_t is voluntary. Thus, in equilibrium the principal must have an incentive to pay b_t ;
- The agent observes all the relevant information: $\{e_t, \theta_t, y_t\}$;
- The principal observes $\psi_t \subset \{e_t, \theta_t, y_t\}$:
 - all information is observable but not verifiable (MacLeod and Malcomson 1989):

$$\psi_t = \{e_t, \theta_t, y_t\}$$
;
 - hidden action: $\psi_t = \{\theta_t, y_t\}$;
 - hidden information: $\psi_t = \{e_t, y_t\}$.

The principal's payoff is given by:

$$\pi_t = (1 - \delta)E \left[\sum_{\tau \geq t} \delta^{\tau-t} \{d_\tau(y_\tau - W_\tau) + (1 - d_\tau)\bar{\pi}\} \right],$$

where $d_\tau = 1$ if the agent accepts the contract and $= 0$ otherwise; W_τ is the payment of the principal to the agent at period τ .

The agent's payoff is given by:

$$u_t = (1 - \delta)E \left[\sum_{\tau \geq t} \delta^{\tau-t} \{d_\tau(W_\tau - c(e_\tau, \theta_\tau)) + (1 - d_\tau)\bar{u}\} \right].$$

The expected surplus is $s_t = \pi_t + u_t$.

1.4 Self-enforcing Stationary Relational Contract

A relation contract describes for any period t and any history

- the compensation the principal should offer (and which should be paid);
- whether the agent should accept or reject the offer; and in the event of acceptance,
- the action the agent should take as a function of his realized costs θ_t .

Such a contract is *self-enforcing* if it describes a perfect public equilibrium (PPE) of the repeated game. A PPE is a Perfect Bayesian Equilibrium in which strategies are contingent on public histories only.

Stationary contracts: $W_t = w + b(\psi_t)$ and $e_t = e(\theta_t)$. It is without loss of generality to consider stationary contracts: if an optimal contract exists, then there are stationary contracts that are optimal.

Given a stationary contract, denote

$$\pi = E_{\theta,y}[y - W(\psi)|e = e(\theta)],$$

$$u = E_{\theta,y}[W(\psi) - c(e, \theta)|e = e(\theta)],$$

$s = \pi + u$ and $W(\psi) = w + b(\psi)$. Then, a self-enforcing contract must satisfy the following conditions:

1. The contract must be individually rational, i.e., each party must get at least its outside option utility: $u \geq \bar{u}$ and $\pi \geq \bar{\pi}$;
2. The contract must be incentive compatible, i.e., it must be optimal for the agent to choose $e(\theta)$:

$$e(\theta) \in \operatorname{argmax}_e E_y[W(\psi|e)] - c(e, \theta);$$

3. The dynamic enforcement constraint requires that at the end of each period no party has an incentive to renege on its payment obligations, i.e.,

$$\frac{\delta}{1 - \delta}(\pi - \bar{\pi}) \geq \sup_{\psi} b(\psi),$$

and

$$\frac{\delta}{1 - \delta}(u - \bar{u}) \geq -\inf_{\psi} b(\psi).$$

The dynamic enforcement constraint implies that:

$$\frac{\delta}{1 - \delta}(s - \bar{s}) \geq \sup_{\psi} b(\psi) - \inf_{\psi} b(\psi) = \sup_{\psi} W(\psi) - \inf_{\psi} W(\psi).$$

Proposition 1 *An effort schedule $e(\theta)$ that generates expected surplus s can be implemented with a stationary relational contract if and only if there is a payment schedule $W : \phi \rightarrow \mathbb{R}$ such that for all θ :*

$$e(\theta) \in \operatorname{argmax}_e \{E_y[w(\psi)|e] - c(e, \theta)\} \quad (IC)$$

and

$$\frac{\delta}{1-\delta}(s - \bar{s}) \geq \sup_{\psi} W(\psi) - \inf_{\psi} W(\psi) \quad (DE).$$

Proof. We have shown already that these conditions are necessary for a relational contract to be self-enforcing. To see that they are also sufficient consider a payment schedule $W(\psi)$ and an effort profile $e(\theta)$ that satisfies (IC) and (DE). Let

$$b(\psi) = W(\psi) - \inf_{\psi} W(\psi),$$

and

$$w = \bar{u} - E_{\theta,y} [b(\psi) - c(e, \theta)|e = e(\theta)].$$

Consider a stationary contract with w , $b(\psi)$ and $e(\theta)$ and the agent chooses $d = 0$ if the principal does not pay $b(\psi)$. This contract can be self-enforcing and implement $e(\theta)$. ■

1.4.1 Perfect Information

Suppose that the cost parameter θ_t , the agent's action e_t and the outcome y_t are observable by the principal and the agent, but cannot be verified to the courts. For simplicity we will also assume that θ_t is a constant. This is the case considered by MacLeod and Malcolmson (1989). In this case we have:

Proposition 2 *In the perfect information case, the stationary action e that gives rise to expected social surplus s can be implemented with a stationary relational contract if and only*

if

$$\frac{\delta}{1-\delta}(s - \bar{s}) \geq c(e) \quad (DE).$$

To see that this condition is also sufficient consider the following stationary relational contract. The agent gets the fixed wage $w = \bar{u}$ at the beginning of each period. If he chooses action e the principal pays an additional bonus $b = c(e)$. If he chooses any other action $\tilde{e} \neq e$, the principal pays no bonus and the agent chooses $d = 0$ in all future periods. If the agent took action e and the principal did not pay $b \geq c(e)$, the agent chooses $d = 0$ in all future periods. This is a subgame perfect equilibrium.

The above proposition has the following implications. The closer the discount factor to 1 the larger is the set of implementable actions. The larger the social surplus, the easier it is to implement a given action. The higher the cost of an action, the more difficult it is to implement this action.

1.4.2 Moral Hazard

Assume that the cost parameter θ_t is observable but the agent's action e_t is not: $W(\theta, y) = w + b(\theta, y)$. Let effort be a continuous variable with $c_e > 0$ and $c_{ee} > 0$. Finally, assume that $F(y|e)$ satisfies the monotone likelihood ratio property, i.e., $\frac{f_e(y|e)}{f(y|e)}$ is monotonically increasing in y , and Convexity of the Distribution Function Condition (CDFC) is satisfied as well: $F(y|e = c^{-1}(x; \theta))$ is convex in x for any θ .

Proposition 3 *The optimal contract implements an effort schedule $e(\theta) \leq e^{FB}(\theta)$. For each θ , the payments $W(\theta, y)$ are one-step: $W(\theta, y) = \underline{W}$ for all $y \leq \hat{y}(\theta)$ and $= \bar{W} = \underline{W} + \frac{\delta}{1-\delta}(s - \bar{s})$ otherwise.*

Since both parties are risk neutral, it is optimal to make the agent a residual claimant. In other words, $W(\theta, y) = w + y$, where $w = \bar{u} - \max_{e(\cdot)} E_{\theta, y}[y - c(e, \theta) | e = e(\theta)]$. This contract is not self-enforcing if

$$\bar{y} - \underline{y} > \frac{\delta}{1 - \delta}(s - \bar{s}).$$

In that case, the payments $W(\theta, y)$ are one-step: $W(\theta, y) = \underline{W}$ for all $y \leq \hat{y}(\theta)$ and $= \bar{W} = \underline{W} + \frac{\delta}{1 - \delta}(s - \bar{s})$ otherwise.

1.4.3 Hidden Information

Assume that the principal can observe the agent's effort level e_t , but he does not observe the cost parameter θ_t .

Proposition 4 *With hidden information, an effort schedule $e(\theta)$ that generates expected surplus s can be implemented by a stationary contract if and only if $e(\theta)$ is non-increasing and*

$$\frac{\delta}{1 - \delta}(s - \bar{s}) \geq c(e(\underline{\theta}), \underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial c(e(\theta), \theta)}{\partial \theta} d\theta.$$

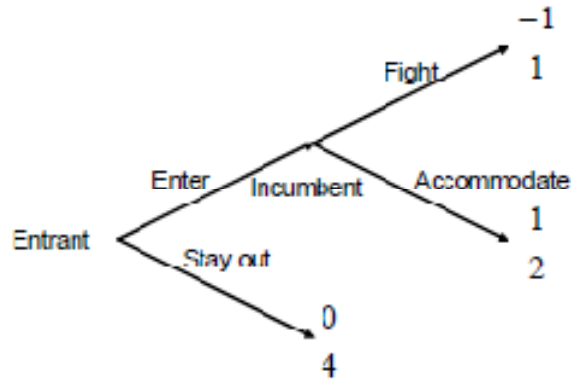


Figure 2: The stage game for the chain store paradox.

2 Reputations

2.1 Chain Store Paradox

The theoretical discussions of reputations is motivated by the classical chain store paradox (see, e.g., Kreps, Milgrom, Roberts, and Wilson (1982) and Kreps and Wilson (1982)). The stage game for the chain store paradox is given by Figure 2. There are two Nash equilibria in the above stage game: (enter, accommodate) and (stay out, fight). Latter violates backward induction.

2.1.1 A Two-Period Example

Chain store: the incumbent plays the game twice, against two different entrants (E_1 and E_2), with the second entrant E_2 observing outcome of first interaction. Incumbent receives total payoffs.

“Chain store paradox”: the only backward induction (subgame perfect) outcome is that both entrants enter, and incumbent always accommodates.

But, now suppose incumbent could be tough, t : such an incumbent receives a payoff

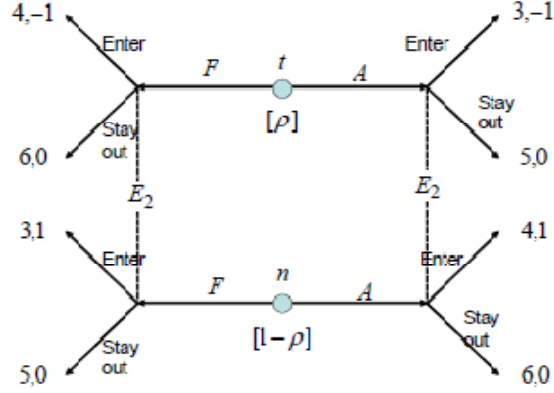


Figure 3: A signaling game representation of the subgame reached by E_1 entering.

of 2 from fighting and only 1 from accommodating. Other incumbent is normal, n . Both entrants' assigns probability $\rho \in (0, \frac{1}{2})$ to the incumbent being t . In the second market, normal incumbent accommodates and tough fights. Conditional on entry in the first market, the subsequent subgame can be represented by the following signalling game:

Note first that there are no pure strategy equilibria. There is a unique mixed strategy equilibrium: n plays F with probability α and A with probability $1 - \alpha$; t plays F for sure. E_2 enters for sure after observing A in the first period, and plays E with probability β after observing F in the first period.

E_2 is willing to randomize only if his posterior after F that the incumbent is t equals $\frac{1}{2}$. Since that posterior is given by Bayes' rule:

$$\Pr[t|F] = \frac{\rho}{\rho + (1 - \rho)\alpha},$$

solving $\Pr[t|F] = \frac{1}{2}$ yields $\alpha = \frac{\rho}{1-\rho}$. $\alpha < 1$ since $\rho < \frac{1}{2}$. Type n is willing to randomize if

$$4 = \beta 3 + (1 - \beta)5,$$

which implies that $\beta = \frac{1}{2}$. In the first period, the first entrant E_1 faces a probability of fighting given by $\rho + (1 - \rho)\alpha = 2\rho$. Hence, if $\rho < \frac{1}{4}$, E_1 faces F with sufficiently small probability (less than $\frac{1}{2}$) that he enters. However, if $\rho \in (\frac{1}{4}, \frac{1}{2})$, the probability of F is sufficiently high such that E_1 stays out.

To summarize, for $\rho < \frac{1}{4}$, E_1 enters with probability 1. After the entry, t fights for sure and n fights with probability $\alpha = \frac{\rho}{1-\rho}$. In the second period, E_2 enters for sure after observing A in the first period, and plays E with probability $\beta = \frac{1}{2}$ after observing F in the first period. For $\rho \in (\frac{1}{4}, \frac{1}{2})$, E_1 does not enter for sure and in the second period, E_2 enters for sure.

2.1.2 Infinite Horizon

Suppose now infinite horizon with the incumbent discounting at rate $\delta \in (0, 1)$ and a new potential entrant in each period. The incumbent is a long-lived player with payoff $(1 - \delta) \sum_{t=0}^{\infty} \delta^t u_t$ and the entrants are short-lived players. The incumbent has two different types n and t . Type t incumbent always fights with the entry while the stage payoff for the type n incumbent is given above.

Naturally, we can show the following is an equilibrium when $\delta > \frac{1}{3}$: the n incumbent fights all entrants as well, because the first time it fails to do so, it is revealed to be a type n player and then all subsequent entrants enter and the n incumbent accommodates from then on. However, this is NOT the only equilibrium of the infinite-horizon model. The problem is that even if the incumbent is revealed to be type n , there are many subgame perfect equilibrium in the subsequent game if the discount rate is sufficiently high. For example, if $\delta > \frac{1}{3}$, the following is also an equilibrium: “ t always fights. n accommodates the first

entry, and then fights all subsequent entry if it has not accommodated two or more times in the past. Once the incumbent has accommodated twice, it accommodates all subsequent entry.” The multiplicity of equilibria suggests that it might be more convenient to try to characterize the payoff set of equilibria without determining all of them explicitly. It turns out that characterizing the payoff set also helps us eliminate some equilibria, which cannot achieve the payoffs in the set.

Notice that if type t has prior probability greater than $\frac{1}{2}$, trivially there is never any entry and the normal type has payoff 4. Therefore, we only need to focus on the case where $\rho < \frac{1}{2}$.

Theorem 1 *Suppose $\rho < \frac{1}{2}$. The normal type incumbent must receive a payoff of at least $1 + 3\delta$ in any pure strategy Nash equilibrium.*

Proof. Suppose the normal type incumbent always plays F . Then there is no entry and the payoff is $4 > 1 + 3\delta$. Suppose the normal type does not always play F . Then there exists a first period τ where the incumbent n starts to accommodate at time τ . In a pure strategy equilibrium, if the incumbent does not accommodate at time τ , it must mean that this incumbent is tough and vice versa. After observing F in period τ , entrants conclude that the firm is the t type, and there is no further entry. An easy lower bound on the normal incumbent’s equilibrium payoff is then obtained by observing that the normal incumbent’s payoff must be at least the payoff from mimicking the t type in period τ . The payoff from such behavior is given by:

$$(1 - \delta) \sum_{t=0}^{\tau-1} \delta^t \times 4 + (1 - \delta)\delta^\tau \times 1 + (1 - \delta) \sum_{t=\tau+1}^{\infty} \delta^t \times 4 = (1 - \delta^\tau)4 + (1 - \delta)\delta^\tau + \delta^{\tau+1}4.$$

The above expression is simplified to $4 - 3\delta^\tau(1 - \delta)$. Since $\delta^\tau \leq 1$, a lower bound of the above expression is $4 - 3(1 - \delta) = 1 + 3\delta$. ■

The above theorem implies that for $\delta > \frac{1}{3}$, if the incumbent accommodates at time τ , it cannot be the case that all entrants enter and the incumbent accommodates in every period in the subsequent game. Otherwise, the payoff is $(1 - \delta^\tau)4 + \delta^\tau 2 < 4 - 3\delta^\tau(1 - \delta)$ for $\delta > \frac{1}{3}$.

As δ converges to 1, $1 + 3\delta$ converges to 4. Notice that the upper bound of equilibrium payoff is 4 as well. Therefore, as δ goes to 1, the normal type incumbent's payoff any pure strategy Nash equilibrium converges to 4. This is in contrast to the complete information repeated game, where the lower bound of payoff is 2 even if δ converges to 1 (an example of reputation effect).

2.2 Bad Reputations

In this section, we will show that the reputation effect may not be always good. Sometimes, the reputational concern of the long-run player to look good in the current period undermines commitment power and results in the loss of all surplus. This is called “bad reputations”.

2.2.1 The Stage Game

There are two players in the stage games. A motorist (the principal) has a car which is in need of repair. The motorist knows that the car requires one of two repairs with

	θ_t	θ_e
t	u	$-w$
e	$-w$	u

Figure 4: Payoffs for the motorist.

equal probability: an engine replacement or a mere tune-up; however the motorist lacks the expertise to determine which repair is necessary. The motorist therefore considers bringing the car to a certain mechanic (the agent) who possesses the ability to diagnose the problem and perform the necessary repair. We will model this by supposing that the mechanic, if hired, privately observes the state $\theta \in \{\theta_e, \theta_t\}$, indicating respectively whether an engine replacement is necessary, or a tune-up will suffice. Conditional on this information, the mechanic then chooses a repair $a \in \{e, t\}$, indicating engine replacement, or tune-up.

P's payoffs from the two possible repairs in the two different states are given by: (we will assume that $w > u > 0$)

P also has an outside option which gives a constant payoff normalized to zero. The agent has two possible types. If the agent is good, his payoffs are identical to those of the motorist. If the agent is bad, he always prefers to choose e . The prior that the agent is bad is μ .

The stage game is an extensive-form game. First, the state is drawn by nature and revealed to the agent but the principal. P then decides whether to hire A or not. If A is hired, the good type has to choose between e and t .

We will use the action profile $\{1, 1\}$ to denote the outcome in which the appropriate action is always chosen under both states. $\{1, 0\}$ corresponds to always providing e and $\{0, 0\}$ is always providing the wrong action.

Proposition 5 *The stage game has a unique sequential equilibrium. In this equilibrium, the good agent always chooses $\{1, 1\}$ and P hires A if and only if $\mu \leq \frac{2u}{u+w} < 1$.*

2.2.2 Bad Reputations: An Illustrating Example

Suppose the stage game is repeated. A is a long-lived player who discounts the future at rate δ . P is a short-lived player. Assume only the actions in the first period is observable by subsequent short-lived players. This means that a period t public history only includes the actions in the first period. Beginning from the second period, since the actions do not affect the subsequent outcomes, one would expect the unique sequential equilibrium is a repetition of the sequential equilibrium in the stage game.

Proposition 6 *Denote μ_2 to be the prior at the beginning of the second period. If $\mu_2 \leq \frac{2u}{u+w} < 1$, then in the unique sequential equilibrium, P always hires A and the good agent always chooses $\{1, 1\}$. If $\mu_2 > \frac{2u}{u+w}$, A will never be hired.*

Now let's consider a good A 's incentive to choose the appropriate action in the first period. Obviously, if $\mu > \frac{2u}{u+w}$, A is not hired in the first period. But if $\mu \leq \frac{2u}{u+w}$ such that A is hired and the state is θ_e , the good A has incentives to choose t to signal that he is good. Especially when μ is sufficiently high, choosing t will decrease μ_2 to zero and guarantee that A is always hired in the subsequent periods. However, P will not hire A anticipating this reputational concern. This leads to the following no-trade result:

Proposition 7 *If $\mu \in (\frac{u}{w}, \frac{2u}{u+w}]$ and $\delta > \frac{u+w}{2u+w}$, the unique sequential equilibrium is such that the good agent chooses $\{0, 1\}$ and P never hires A in the first period.*

Proof. Notice in the subgame reached by P hiring A, the good A always chooses action t at state t . Suppose at state θ_e , A chooses action e with probability $\gamma \in [0, 1]$. This implies that if $\gamma > 0$, after observing e , μ_2 would become:

$$\varphi(\mu) = \frac{\mu}{\mu + \frac{1}{2}(1 - \mu)\gamma} > \mu.$$

Since $\mu > \frac{u}{w}$, $\varphi(\mu) > \frac{2u}{2u+(w-u)\gamma} \geq \frac{2u}{u+w}$ because $\gamma \leq 1$. This implies that if e is chosen, A is never hired after the second period. The payoff of choosing e in the first period at state θ_e thus is $(1 - \delta)u$. However, the payoff of choosing t in the first period at state θ_e is $(1 - \delta)(-w) + \delta u > (1 - \delta)u$ since $\delta > \frac{u+w}{2u+w}$. Therefore, we must have $\gamma = 0$. But if $\gamma = 0$, the expected payoff for P is: $\frac{1}{2}(u - w) < 0$. As a result, A will never be hired in the first period. ■

However, if $\mu \leq \frac{u}{w}$, there exists a sequential equilibrium where the good agent chooses $\{0, 1\}$ and P never hires A in the first period. But there also exists another equilibrium where the good agent chooses $\{1, 1\}$ and A is hired in the first period.

2.2.3 Bad Reputations: General Result

Now suppose actions in every single period are observable by subsequent short-lived players. Let $\bar{V}(\mu, \delta)$ be the supremum of discounted average Nash equilibrium payoffs for the good agent. Then Ely and Välimäki (2003) show that:

Theorem 2

$$\lim_{\delta \rightarrow 1} \bar{V}(\mu, \delta) = 0, \forall \mu.$$

2.3 Building Reputations Through Costly Investments

2.3.1 Reputational Capital

Klein and Leffler (1981) argue that, in a free market, sellers who supply goods with poor performance will lose their reputation and hence future sales. This will deter the provision of poor performance. Hence, reputation can be viewed as an asset and the seller with higher reputational capital charges a higher price that Klein and Leffler interpret as a return to this capital. The value of this capital becomes zero whenever it is commonly believed that the seller supplies a good with low performance.

There are two different approaches to model reputational capital in the repeated-games literature. In the interpretative approach, the notion of reputation is used to interpret an equilibrium strategy profile in repeated games. For example, in the trigger strategy of the repeated prisoner's dilemma game, a player's reputation for cooperation is destroyed if the player deviates in the past. Reputation establishes a link between past behavior and expectations of future behavior, and this link is an equilibrium phenomenon, holding in some equilibria but not in others. However, the introduction of reputation involves no modification of the basic repeated game and adds nothing to the formal analysis.

The models we have studied so far are usually called the "adverse selection" approach to model reputations. Reputation in these models is represented as belief about certain "types". In order to do this, we should perturb from a game of complete information in which the players are "normal," and switch to a game of incomplete information. The idea that a player has an incentive to build, maintain, or milk his reputation is captured by the incentive that player has to manipulate the beliefs of other players about his type. The updating of these

beliefs establishes links between past behavior and expectations of future behavior. We say “reputations effects” arise if these links give rise to restrictions on equilibrium payoffs or behavior that do not arise in the underlying game of complete information.

As an asset, reputational capital requires costly investments to build and maintain. Following Mailath and Samuelson (2006) and Mailath and Samuelson (2001), we will investigate the incentives of making costly investments to build reputations.

2.3.2 Model Setup

There are two players in the stage game. A long-lived firm and a short-lived consumer. In each period t , the long-lived player chooses an effort level $a_t \in \{H, L\}$. The consumer then receives an idiosyncratic realization of a signal. The signal has two possible values, \bar{z} and \underline{z} . The marginal distribution is $\pi(\bar{z}|a_i) = \rho_i$, for $i = H, L$ and $1 > \rho_H > \rho_L = 1 - \rho_H > 0$. The signal \bar{z} generates a value of 1 to the consumer and the signal \underline{z} has value 0. There is incomplete information about the firm’s type. The normal type has the option of choosing high (with a cost of c) or low effort. But there is a single commitment type who will always choose low effort. The price is set to be the same as the consumers’ expected payoffs. In particular, if the firm is thought to be normal with probability μ_0 , and if the normal firm is thought to choose high effort with probability α , the the price will be

$$P(\mu_0\alpha) = \mu_0\alpha\rho_H + (1 - \mu_0\alpha)\rho_L.$$

In the above setting, the consumer is passive and the key question is whether the firm has incentive to build reputations given the payoff function P . Obviously, there exist equilibria

in which the normal firm always exert high effort. For example, if the cost of investment c is sufficiently small, the following is an equilibrium: “The firm continues to exert high effort as long as signal \bar{z} is realized for all the previous periods and the price is $\mu_t \rho_H + (1 - \mu_t) \rho_L$. If signal \underline{z} is realized, the firm immediately switches to low effort forever and the price is ρ_L .”

2.3.3 Markov Perfect Equilibrium

An implausible feature of the above equilibrium is the following. Consider two different histories: \bar{z} and $\bar{z}\bar{z}\underline{z}$. The equilibrium implies that the firm should invest under the first history but not under the second one. However, the posterior beliefs under these two histories are both equal to:

$$\frac{\mu_0 \rho_H}{\mu_0 \rho_H + (1 - \mu_0) \rho_L}.$$

Also, using punishment triggered by the signal \underline{z} causes multiple equilibria. As a result, we introduce Markov perfect equilibrium to overcome the above issues.

A Markov strategy for the normal firm is a mapping $\alpha : [0, 1] \rightarrow [0, 1]$, where $\alpha(\mu_0)$ is the probability of choosing action H when the posterior is μ_0 . The posteriors are updated according to Bayes' rule:

$$\varphi(\mu_0|\bar{z}) = \frac{[\rho_H \alpha(\mu_0) + \rho_L (1 - \alpha(\mu_0))] \mu_0}{[\rho_H \alpha(\mu_0) + \rho_L (1 - \alpha(\mu_0))] \mu_0 + \rho_L (1 - \mu_0)}$$

and

$$\varphi(\mu_0|\underline{z}) = \frac{[(1 - \rho_H) \alpha(\mu_0) + (1 - \rho_L) (1 - \alpha(\mu_0))] \mu_0}{[(1 - \rho_H) \alpha(\mu_0) + (1 - \rho_L) (1 - \alpha(\mu_0))] \mu_0 + (1 - \rho_L) (1 - \mu_0)}.$$

Given any public history $H^t \in \mathcal{H}^t = \{\bar{z}, \underline{z}\}^t$ and the prior μ_0 , we can compute the posteriors $\varphi(\mu_0|h^t)$. A Markov strategy α will imply strategy $\sigma^\alpha : \mathcal{H} \rightarrow [0, 1]$ such that $\sigma^\alpha(h_t) = \alpha(\varphi(\mu_0|h^t))$.

Definition 1 *The strategy α is a Markov perfect equilibrium if σ^α is maximizing for the normal firm for any prior μ_0 .*

2.3.4 The Impossibility of Building Reputations

Proposition 8 *There is a unique Markov perfect equilibrium in pure strategies. In this equilibrium, the normal firm exerts low effort with probability one for all μ_0 .*

Proof. The strategy $\alpha(\mu_0) = 0$ for all μ_0 is clearly an equilibrium. We need to argue that this is the only pure-strategy Markov perfect equilibrium. Suppose there are other equilibria with $\alpha(\mu_0) = 1$ for some μ_0 . Fix such an equilibrium and let $V_0(\mu_0)$ denote the value function of the normal firm. Thus, we have:

$$V_0(\mu_0) = (1-\delta)(p(\mu_0)-c) + \delta(\rho_H - \rho_L)(V_0(\varphi(\mu_0|\bar{z})) - V_0(\varphi(\mu_0|\underline{z}))) + \delta(\rho_L V_0(\varphi(\mu_0|\bar{z})) + (1-\rho_L)V_0(\varphi(\mu_0|\underline{z}))).$$

Notice that V_0 is bounded by ρ_L and $\rho_H - c$. Denote $V_0(\mu_0; L)$ to be the value of a one-period deviation to choosing low effort and then reverting to the equilibrium strategy of α . We have:

$$V_0(\mu_0; L) = (1-\delta)p(\mu_0) + \delta(\rho_L V_0(\varphi(\mu_0|\bar{z})) + (1-\rho_L)V_0(\varphi(\mu_0|\underline{z}))).$$

$\alpha(\mu_0) = 1$ implies that $V_0(\mu_0) \geq V_0(\mu_0; L)$ and hence:

$$\delta(\rho_H - \rho_L)(V_0(\varphi(\mu_0|\bar{z})) - V_0(\varphi(\mu_0|\underline{z}))) \geq (1 - \delta)c.$$

If μ_0 is 0 or 1, $\varphi(\mu_0|\bar{z}) = \varphi(\mu_0|\underline{z})$, ensuring $\alpha(\mu_0) = 1$ is suboptimal. For $\mu_0 \in (0, 1)$, first notice that $\delta(\rho_H - \rho_L)(V_0(\varphi(\mu_0|\bar{z})) - V_0(\varphi(\mu_0|\underline{z}))) \geq (1 - \delta)c$ implies $\alpha(\varphi(\mu_0|\bar{z})) = 1$.

Because if $\alpha(\varphi(\mu_0|\bar{z})) = 0$, we have:

$$V_0(\varphi(\mu_0|\bar{z})) = (1 - \delta)\rho_L + \delta(\rho_L V_0(\varphi(\mu_0|\bar{z})) + (1 - \rho_L)V_0(\varphi(\mu_0|\bar{z}))) \implies V_0(\varphi(\mu_0|\bar{z})) = \rho_L.$$

However, $V_0(\varphi(\mu_0|\underline{z})) \geq \rho_L$. This leads to a contradiction.

Denote $x = \varphi(\mu_0|\bar{z})$. $\alpha(x) = 1$ implies that:

$$\delta(\rho_H - \rho_L)(V_0(\varphi(x|\bar{z})) - V_0(\varphi(x|\underline{z}))) \geq (1 - \delta)c.$$

Notice that $\varphi(x|\underline{z}) = \mu_0$. Therefore, we get

$$\delta(\rho_H - \rho_L)(V_0(\varphi(x|\bar{z})) - V_0(\mu_0)) \geq (1 - \delta)c,$$

where

$$\varphi(x|\bar{z}) = \frac{\mu_0 \rho_H^2}{\mu_0 \rho_H^2 + (1 - \mu_0) \rho_L^2}.$$

Define a sequence $\{\mu^k\}$ such that $\mu^k = \frac{\mu_0 \rho_H^{2k}}{\mu_0 \rho_H^{2k} + (1 - \mu_0) \rho_L^{2k}}$. Then we have:

$$\delta(\rho_H - \rho_L)(V_0(\mu^1) - V_0(\mu^0)) \geq (1 - \delta)c.$$

Also, the above inequality implies that $\alpha(\mu^1) = 1$. Replace μ_0 by μ^1 and we have:

$$\delta(\rho_H - \rho_L)(V_0(\mu^2) - V_0(\mu^1)) \geq (1 - \delta)c.$$

Repeating this process yields:

$$V_0(\mu^k) \geq V_0(\mu^0) + \frac{(1 - \delta)kc}{\delta(\rho_H - \rho_L)},$$

which gives $\lim_{k \rightarrow \infty} V_0(\mu^k) = \infty$. But this is impossible since V_0 is bounded above by $\rho_H - c$.

Therefore, it cannot be that $\alpha(\mu_0) = 1$. ■

The possibility of an inept type potentially provides the normal firm an incentive to exert high effort and build reputations. However, the normal firm cannot always exert high effort. Otherwise, the consumers will eventually become almost certain that the firm is normal. As the posterior of a normal firm approaches to one, the effect of signals on the belief becomes smaller and smaller. At some point, the normal firm will find it optimal to revert to low effort. Anticipating this, the firm will exert low effort even earlier, causing the equilibrium to unravel. The only pure-strategy equilibrium calls for low effort. This result implies that, to guarantee that building reputations is an equilibrium, some other elements are needed. For example, we can consider mixed strategy equilibrium or introduce an exogenous probability of replacement. The role of replacement is to introduce lower and upper bounds on beliefs

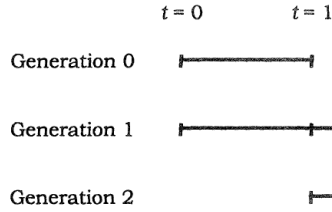


Figure 5: An OLG Economy.

such that k in the above proof will not converge to infinity. Mailath and Samuelson (2001) show that if there is a small probability of replacement, then there exists a pure strategy Markov perfect equilibrium where the normal firm always exerts high effort if the cost c is sufficiently small.

2.4 Markets for Reputations

In this section, we will discuss another way to guarantee reputation building. The idea is to allow the firms to trade reputations. The normal firms have higher incentives to build reputations if they anticipate that good reputations can be traded at a higher price. The model is based on Tadelis (1999).

2.4.1 Model Setup

A two-period OLG economy. At the beginning of period 0, there is a unit mass of “old” firms, half of whom are good and half of whom are normal. There is also a unit mass of young firms, again with half being good and half being normal. Each firm is distinguished by a name.

Good firms provide a service that is successful with probability $\rho > 1/2$ and a failure otherwise. A normal firm faces a choice in each period. It can exert high effort, at cost

$c > 0$, in which case its output is a success with probability ρ and a failure with probability $1 - \rho$. Alternatively, it can exert low effort at no cost, in which case its output is surely a failure.

A success has a value of 1 to a consumer, and a failure has value zero. Firms sell their outputs to consumers who cannot distinguish good firms from normal ones and cannot condition their price on whether the service will be a success or failure, and who are sufficiently numerous to bid prices for the firms' products up to their expected value.

At the end of period 0, all old firms disappear to be replaced by a generation of new firms. Firms that were previously new become old. These continuing firms have the option of retaining their name or abandoning it to either invent a new one with zero cost or buy a name, at the market price. Each new firm can invent a new name or buy a name. Consumers don't observe whether the name is owned by a continuing firm or a new firm. Consumers in the second period observe the name of each firm. For each old name, consumers observe whether the name was associated with a success or a failure in the previous period. However, they do not observe whether the name is owned by a continuing firm or a new firm.

2.4.2 Markets for Names

In this model, names are intrinsically worthless. However, the interperiod market for names may be active in some equilibria.

Consider a class of equilibria: every continuing firm experiencing a failure in period 0 abandons its name, every young firm whose service was successful retains its name, and no name carrying a failure are purchased. We can generate a continuum of equilibria and every equilibrium has the following feature:

Proposition 9 *Names carrying a success are traded in any equilibrium.*

Proof. Notice that in period 1, no normal firm will exert effort and only the good firms will exert effort. If no names are traded in equilibrium, then all names associated with old firms are abandoned and only successful young firms retain their names. If normal young firms exert low effort, then $\Pr(G|S) = 1$. In period 1, the consumers are willing to pay a price ρ for the firms with successful names but for the new names, the price is strictly less than ρ . Therefore, a new firm is willing to buy successful names from the old firms, which leads to a contradiction. If a fraction x of normal young firms exert high effort, then in period, the consumers are willing to pay a price $p_s = \frac{\rho}{1+x}$ for the firms with successful names but for the new names, the price is $p_n = \frac{\rho(1-\frac{1}{2}\rho)}{2-\frac{1}{2}(1+x)\rho}$. $p_s > p_n$ for $x < 1$. As long as $p_s > p_n$, a new firm is willing to buy successful names from the old firms, which leads to a contradiction. The last thing is to rule out $x = 1$ as a “no-name-trading” equilibrium. If not, then a normal young firm has incentive to deviate to low effort. This deviation saves the cost of high effort without affecting the payoffs. Thus, $x = 1$ is also not an equilibrium. Therefore, we can conclude that names carrying a success must be traded. ■

We seek to characterize equilibria in which all normal firms exert high effort in period 0. In such a configuration, there are measure ρ successful names in the names market associated with old firms (we assume the continuing firms will retain their successful names). We examine an equilibrium in which all of the ρ successful names are sold and θ is the fraction of these names purchased by good firms. By Bayes’ updating,

$$\Pr(G|S) = \frac{\frac{1}{2}\rho + \theta\rho}{2\rho} \quad \text{and} \quad \Pr(G|N) = \frac{1 - \frac{1}{2}\rho - \theta\rho}{2 - 2\rho}.$$

Therefore, we have

$$p_s = \rho \frac{1 + 2\theta}{4} \quad \text{and} \quad p_n = \rho \frac{2 - \rho - 2\rho\theta}{4 - 4\rho}.$$

The price of a name is then

$$p_s - p_n = \rho \frac{2\theta - 1}{4 - 4\rho}.$$

For c sufficiently small (i.e., there exists $\min\{1, \frac{2-\rho}{2\rho}\} \geq \theta > \frac{1}{2}$, such that $c \leq \rho(p_s - p_n)^1$), there exists equilibria in which high effort is exerted for every normal firm in period 0. Multiple equilibria exist since for c sufficiently small, there may be a continuum of θ satisfying $c \leq \rho(p_s - p_n)$.

The market for names plays an important role in creating the incentives for normal firms to exert high effort. Suppose that there was no market for names. It is then clear that old normal firms will not exert high effort in period 0, because there is no future reward for doing so. Also we have shown that there cannot be an equilibrium for all young normal firms to exert high effort in period 0. However, with such a market for names, it is possible that *all* normal firms exert high effort in period 0.

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¹We require $\theta \leq \frac{2-\rho}{2\rho}$ since $\rho\theta \leq (1-\rho)/2 + 1/2$.

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