SUPPLEMENT TO
"Random Authority"

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In this Online Appendix, we provide a complete proof of Proposition 2, and extend the analysis of optimal authority allocation to asymmetric organizations. Throughout the Online Appendix, we refer often to results in the main text and its Appendix using the numbering established there (e.g., Proposition 2). The numbers of equations and figures in this Online Appendix are all prefixed by A to distinguish them from those in the main text and its Appendix, and we always use this prefix in referring to them.

1 Proof of Proposition 2

Proposition 2 is proved by showing the following three lemmas:

Lemma A.1 *In symmetric organizations, the optimal authority allocation ρ must be symmetric or be a corner solution (i.e., if it is optimal to have $\rho_3 > 0$, then the optimal authority allocation must satisfy $\rho_4 = \rho_3$ or $\rho_4 = 0$).*

Lemma A.2 *Symmetric mixed authority is suboptimal in symmetric organizations (i.e., if $\rho_3 = \rho_4$, then it is optimal to set $\rho_3 = \rho_4 = 0$).*

Lemma A.3 *In symmetric organizations, authority allocation with $\rho_3 > 0$ and $\rho_4 = 0$ or $\rho_4 > 0$ and $\rho_3 = 0$ is always suboptimal.*

These lemmas imply Proposition 2 naturally. From Lemma A.1, there are two possibilities for the optimal random authority structure in symmetric organizations: (i) it must be symmetric when introducing both mixed authority structures, $\{M1\}$ and $\{M2\}$; and (ii) there exists only one mixed authority structure. Lemma A.2 rules out symmetric mixed authority arrangements in symmetric organizations. Lemma A.3 implies that introducing only $M1$ or $M2$ will be suboptimal as well.[1] It is immediately clear that including mixed authority is always suboptimal in symmetric organizations.

[1] Lemma A.3 has already been proved numerically by [1]. We provide a formal proof here.

PROOF OF LEMMA A.1

Proof. From Proposition 1, by plugging $\rho_4 = 1 - \rho_1 - \rho_2 - \rho_3$ into π_{RA} and taking the partial derivative over ρ_3, we obtain

$$\frac{\partial \pi_{RA}}{\partial \rho_3} = \frac{3\delta^4(1+4\delta)(2\lambda-1)^2\lambda(2\delta+\lambda)^2(1-\rho_1-\rho_2-2\rho_3)h_1(\rho_1,\rho_2,\lambda,\delta)}{(\lambda+\delta+2\lambda\delta)h_2^2(\rho_1,\rho_2,\rho_3,\lambda,\delta)}. \tag{A.1}$$

Since h_1 is independent of ρ_3, we have: (i) when $h_1 \geq 0$, then the maximum of π_{RA} is achieved at $\rho_3^\star = \frac{1-\rho_1-\rho_2}{2}$; and (ii) when $h_1 < 0$, then π_{RA} is maximized at a corner solution (i.e. $\rho_3^\star = 0$ or $\rho_3^\star = 1 - \rho_1 - \rho_2$). Thus if mixed authority is included in the optimal authority allocation, it must be symmetric (i.e., $\rho_3^\star = \rho_4^\star = \frac{1-\rho_1-\rho_2}{2}$) or be a corner solution (i.e., $\rho_3^\star = 0$ or $\rho_4^\star = 0$). ■

PROOF OF LEMMA A.2

Proof. Suppose by contradiction that there exists some (λ, δ) such that the optimal random authority is given by $\rho^\star = (\rho_1, \rho_2, \rho_3, \rho_4)$, which satisfies $\rho_3 = \rho_4 > 0$. Consider another randomized structure with a new probability vector $\tilde{\rho}^\star(\varepsilon) = (\rho_1+\varepsilon, \rho_2+\varepsilon, \rho_3-\varepsilon, \rho_4-\varepsilon)$, where $\varepsilon > 0$ is sufficiently small. Under symmetric mixed authority (i.e., $\rho_3 = \rho_4$), $S_{RA}^1 = S_{RA}^2$ and let $S_{RA} = S_{RA}^1 = S_{RA}^2$. From Proposition 1, we obtain:

$$\pi_{RA} = \left(\sum_{i=1}^{4} \rho_i a_i\right) S_{RA} + \sum_{i=1}^{4} \rho_i b_i, \tag{A.2}$$

where

$$\begin{aligned}
a_1 &= -\frac{2(1+2\delta)}{(1+4\delta)}, \quad a_2 = \frac{2\delta^2\left(2\delta^2+6\lambda^2\delta+(4\lambda-1)\lambda^2\right)}{(\lambda+\delta)^2(\lambda+2\delta)^2}, \\
a_3 &= a_4 = \frac{\lambda\left(\lambda^3+2(2+\lambda)\lambda^2\delta+\lambda(5+8\lambda+4\lambda^2)\delta^2+2(1+7\lambda+4\lambda^3)\delta^3+8\delta^4\right)}{(\lambda+\delta)^2(\lambda+\delta+2\lambda\delta)^2}, \\
b_1 &= -\frac{4\delta}{1+4\delta}, \quad b_2 = -\frac{4\delta(\delta+4\delta^2+\lambda^2+4\delta\lambda^2)}{(1+4\delta)(\lambda+2\delta)^2}, \\
b_3 &= b_4 = -\frac{2\delta\left(\delta+2\lambda^2+112\delta\lambda^2+4\delta^2(1+4\lambda^2)\right)}{(1+4\delta)(\lambda+\delta+2\lambda\delta)^2},
\end{aligned}$$

Similarly, the total expected profits π_{RA}^{ε} under $\tilde{\rho}^\star(\varepsilon)$ can be rewritten as:

$$\begin{aligned}
\pi_{RA}^{\varepsilon} = &\big[(\rho_1+\varepsilon)a_1+(\rho_2+\varepsilon)a_2+(\rho_3-\varepsilon)a_3+(\rho_4-\varepsilon)a_4\big]S_{RA}^{\varepsilon} \\
&+\big[(\rho_1+\varepsilon)b_1+(\rho_2+\varepsilon)b_2+(\rho_3-\varepsilon)b_3+(\rho_4-\varepsilon)b_4\big],
\end{aligned} \tag{A.3}$$

From Eq. (A.2) and (A.3), we can get:

$$\pi_{RA}^{\varepsilon} - \pi_{RA} = \left(\sum_{i=1}^{4} \rho_i a_i\right)(S_{RA}^{\varepsilon} - S_{RA}) + \big\{(a_1 + a_2 - a_3 - a_4)S_{RA}^{\varepsilon} + (b_1 + b_2 - b_3 - b_4)\big\}\varepsilon, \quad \text{(A.4)}$$

Obviously, as $\varepsilon \to 0$, we have $S_{RA}^{\varepsilon} \to S_{RA}$. Define by $S_{RA}^{'} = \lim_{\varepsilon\to 0} \frac{S_{RA}^{\varepsilon} - S_{RA}}{\varepsilon}$.

Rearranging Eq. (A.4) yields

$$\begin{aligned} \lim_{\varepsilon\to 0} \frac{\pi_{RA}^{\varepsilon} - \pi_{RA}}{\varepsilon} &= \left(\sum_{i=1}^{4} \rho_i a_i\right) \lim_{\varepsilon\to 0} \frac{S_{RA}^{\varepsilon} - S_{RA}}{\varepsilon} + \Big\{(a_1 + a_2 - a_3 - a_4) \lim_{\varepsilon\to 0} S_{RA}^{\varepsilon} + (b_1 + b_2 - b_3 - b_4)\Big\} \\ &= \left(\sum_{i=1}^{4} \rho_i a_i\right) S_{RA}^{'} + \big\{(a_1 + a_2 - a_3 - a_4)S_{RA} + (b_1 + b_2 - b_3 - b_4)\big\} \quad \text{(A.5)} \end{aligned}$$

Thus, to verify that $\tilde{\rho}^{\star}(\varepsilon)$ strictly dominates $\rho^{\star}$, it suffices to show that $\lim_{\varepsilon\to 0} \frac{\pi_{RA}^{\varepsilon} - \pi_{RA}}{\varepsilon} > 0$.

From the definition of S_{RA}, we have

$$S_{RA} = \frac{\delta(\lambda + \delta)(2\lambda - 1)\big(\sum_{i=1}^{3} \rho_i \omega_i\big)}{\big(\sum_{i=1}^{3} \rho_i \omega_{i+3}\big)}, \quad \text{(A.6)}$$

where

$$\begin{aligned} \omega_1 &= (\lambda + 2\delta)^2(\lambda + \delta + 2\lambda\delta)^2, \quad \omega_2 = \delta(1 + 4\delta)(\lambda + \delta + 2\lambda\delta)^2, \\ \omega_3 &= (1 + 4\delta)(\lambda + 2\delta)^2(\lambda + \delta\lambda + \lambda^2) + 2\delta\lambda^2(1 + 4\delta)(\lambda + 2\delta)^2, \\ \omega_4 &= (\lambda + 2\delta)^2(\lambda + \delta + 2\lambda\delta)^2 \left(3\lambda^2 + 2\lambda\delta(1 + 4\lambda) + (8\lambda - 1)\delta^2\right), \quad \text{(A.7)} \\ \omega_5 &= \delta^2(1 + 4\delta)(\lambda + \delta + 2\lambda\delta)^2 \left((5\lambda - 1)\lambda + (8\lambda - 1)\delta\right), \\ \omega_6 &= 4\lambda^2\delta^2(1 + 4\delta)(\lambda + 2\delta)^2(\lambda + \delta + 4\lambda\delta + \lambda^2) + (1 + 4\delta)(\lambda + 2\delta)^2 \\ &\times \left(3\lambda^4 + 8\delta\lambda^3(1 + \lambda) + \delta^2\lambda(-1 + 9\lambda + 16\lambda^2) + \delta^3(-1 + 4\lambda + 8\lambda^2)\right), \end{aligned}$$

From the definition of S'_{RA}, we obtain

$$S'_{RA} = -\frac{3\lambda(1 - 2\lambda)^2\delta^2(1 + 4\delta)(\lambda + \delta)\left(\lambda^2 + (3 + 2\lambda)\lambda\delta + (2 + 4\lambda)\delta^2\right)^2 \left(\sum_{i=1}^{3} \rho_i \varpi_i\right)}{\left(\sum_{i=1}^{3} \rho_i \varpi_{i+3}\right)^2}, \quad \text{(A.8)}$$

where

$$
\begin{aligned}
\varpi_1 &= \lambda^4 + \lambda^2\delta(-2+5\lambda+2\lambda^2) + \lambda\delta^2(-5+3\lambda+8\lambda^2) + \delta^3(-3-5\lambda+6\lambda^2) - 4\delta^4, \\
\varpi_2 &= \delta^2\big[\lambda(3\lambda-1) + \delta(-2+\lambda+6\lambda^2) - 4\delta^2\big], \\
\varpi_3 &= \lambda^4 + \delta\lambda^2(-2+5\lambda+2\lambda^2) + 2\delta^2\lambda(-3+3\lambda+4\lambda^2) + \delta^3(-5-4\lambda+12\lambda^2) - 8\delta^4 \\
\varpi_4 &= (\lambda+2\delta)^2(\lambda+\delta+2\lambda\delta)^2\big[3\lambda^2 + 2\delta\lambda(1+4\lambda) + \delta^2(8\lambda-1)\big], \\
\varpi_5 &= \delta^2(1+4\delta)(\lambda+\delta+2\lambda\delta)^2\big[\lambda(5\lambda-1) + \delta(8\lambda-1)\big], \\
\varpi_6 &= (\lambda+2\delta)^2(1+4\delta)\big[3\lambda^4 + 8\delta\lambda^3(1+\lambda) + \delta^2\lambda(-1+9\lambda+20\lambda^2+4\lambda^3) \\
&+ \delta^3(-1+4\lambda+12\lambda^2+16\lambda^3)\big],
\end{aligned} \tag{A.9}
$$

Plugging Eq. (A.6), (A.7), (A.8) and (A.9) into Eq. (A.5), we obtain

$$
\lim_{\varepsilon\to 0}\frac{\pi^{\varepsilon}_{RA}-\pi_{RA}}{\varepsilon} = \frac{\chi_3(\rho_1,\rho_2,\rho_3)\times\chi_4(\rho_1,\rho_2,\rho_3)}{(1+4\delta)(\lambda+\delta)(\lambda+2\delta)^2(\lambda+\delta+2\lambda\delta)^2\left(\sum_{i=1}^{3}\rho_i\varpi_{i+3}\right)^2\left(\sum_{i=1}^{3}\rho_i\omega_{i+3}\right)}, \tag{A.10}
$$

where

$$
\begin{aligned}
\chi_3 = 3\lambda(\delta-2\lambda\delta)^2\Big\{&\rho_1(\lambda+\delta+2\lambda\delta)(\lambda+2\delta)^2\big[3\lambda^2+2\lambda(1+4\lambda)\delta+\delta^2(8\lambda-1)\big] \\
&+\rho_2\delta^2(1+4\delta)(\lambda+\delta+2\lambda\delta)^2\big[\lambda(5\lambda-1)+\delta(8\lambda-1)\big]+\rho_3(\lambda+2\delta)^2(1+4\delta) \\
&\big[3\lambda^4+8(1+\lambda)\lambda^3\delta+\delta^2\lambda(-1+9\lambda+20\lambda^2+4\lambda^3)+\delta^3(-1+4\lambda+12\lambda^2+16\lambda^3)\big]\Big\},
\end{aligned}
$$

and

$$
\begin{aligned}
\chi_4 = {}& 2(1+4\delta)\big[\lambda^2+\lambda\delta(3+2\lambda)+(2+4\lambda)\delta^2\big]^2\times K_1\times K_2 \\
&+4\delta(\lambda+\delta)(\lambda+2\delta)(\lambda+\delta+2\lambda\delta)\big[2(1-\lambda)\lambda+\delta(3+4\lambda-4\lambda^2)+8\delta^2\big]\times K_3\times K_4.
\end{aligned} \tag{A.11}
$$

The coefficients K_i $(i = 1, 2, 3, 4)$ in χ_4 can be explicitly expressed as

$$\begin{aligned}
K_1 &= \rho_1\big[\lambda^4 + \delta\lambda^2(-2+5\lambda+2\lambda^2) + \delta^2\lambda(-5+3\lambda+8\lambda^2) + \delta^3(-3-5\lambda+6\lambda^2) - 4\delta^4\big] \\
&+ \rho_2\delta^2\big[\lambda(3\lambda-1) + \delta(-2+\lambda+6\lambda^2) - 4\delta^2\big] \\
&+ \rho_3\big[\lambda^4 + \lambda^2\delta(-2+5\lambda+2\lambda^2) + 2\lambda\delta^2(-3+3\lambda+4\lambda^2) + \delta^3(-5-4\lambda+12\lambda^2) - 8\delta^4\big] \\
K_2 &= \rho_1(1+2\delta)\big[\lambda^3 + 2\delta\lambda^2(2+\lambda) + \delta^3(2+4\lambda) + \delta^2\lambda(5+6\lambda)\big]^2 \\
&+ \rho_2(1+4\delta)\delta^2(\lambda+\delta+2\lambda\delta)^2\big[\lambda^2(4\lambda-1) + 6\delta\lambda^2 + 2\delta^2\big] \\
&+ \rho_3\lambda(1+4\delta)(\lambda+2\delta)^2\Big(\lambda^3 + 2\delta\lambda^2(2+\lambda) + \delta^2\lambda(5+8\lambda+4\lambda^2) + 2\delta^3(1+7\lambda+4\lambda^3) + 8\delta^4\Big) \\
K_3 &= \rho_1(1+2\delta)\big[\lambda^3 + 2\delta\lambda^2(2+\lambda) + \delta^2\lambda(5+6\lambda) + \delta^3(2+4\lambda)\big] \\
&+ \rho_2(1+4\delta)\delta^2(\lambda+\delta+2\lambda\delta) + \rho_3(1+4\delta)\Big(\lambda^3 + 4\delta\lambda^2 + \delta^2\lambda(5+2\lambda) + \delta^3(2+4\lambda)\Big)
\end{aligned}$$

and

$$\begin{aligned}
K_4 &= \rho_1(\lambda+2\delta)^2(\lambda+\delta+2\lambda\delta)^2\big[3\lambda^2 + 2\delta\lambda(1+4\lambda) + \delta^2(8\lambda-1)\big] \\
&+ \rho_2\delta^2(1+4\delta)(\lambda+\delta+2\lambda\delta)\big[\lambda(5\lambda-1) + \delta(8\lambda-1)\big] \\
&+ \rho_3(1+4\delta)\big[3\lambda^4 + 8\delta\lambda^3(1+\lambda) + \delta^2\lambda(-1+9\lambda+20\lambda^2+4\lambda^3) + \delta^3(-1+4\lambda+12\lambda^2+16\lambda^3)\big].
\end{aligned}$$

Note that given $\lambda \in (1/2, 1)$ and $\delta > 0$, ω_4, ω_5, ω_6 and χ_3 are strictly positive. Hence, the sign of $\lim_{\varepsilon\to 0} \frac{\pi^{\varepsilon}_{RA} - \pi_{RA}}{\varepsilon}$ is the same as the sign of χ_4.

Finally, we need to check the sign of χ_4. Plugging $\rho_2 = 1 - \rho_1 - 2\rho_3$, we can see that $\chi_4(\rho_1, \rho_2, \rho_3) > 0$.[2] Therefore, the new authority allocation $\tilde{\rho}^\star = (\rho_1 + \varepsilon, \rho_2 + \varepsilon, \rho_3 - \varepsilon, \rho_4 - \varepsilon)$ always outperforms the original one for any sufficiently small $\varepsilon > 0$, yielding a contradiction! ■

PROOF OF LEMMA A.3

Proof. Suppose by contradiction that only introducing some possibility of M1 can be optimal.[3] In other words, there exists some (λ, δ) such that the optimal authority allocation $\rho^\star = (\rho_1, \rho_2, \rho_3, \rho_4)$ satisfies $\rho_3 > 0, \rho_4 = 0$. Consider another authority allocation with a new probability vector $\hat{\rho}^\star(\varepsilon) = (\rho_1 + \varepsilon, \rho_2 + \varepsilon, \rho_3 - 2\varepsilon, 0)$ for sufficiently small $\varepsilon > 0$. From Proposition 1, we obtain:

$$\pi_{RA} = \left(\sum_{i=1}^{3} \rho_i a_i\right) S^1_{RA} + \left(\sum_{i=1}^{3} \rho_i b_i\right) S^2_{RA} + \sum_{i=1}^{3} \rho_i c_i, \tag{A.12}$$

[2] The details of calculations can be obtained upon request.

[3] The case for M2 is analogous and thus omitted.

where

$$
\begin{aligned}
a_1 &= -\frac{1+2\delta}{1+4\delta}, \quad a_2 = -\frac{\delta^2(2\delta^2+\lambda^2-6\delta\lambda^2+4\lambda^3)}{(\delta+\lambda)^2(2\delta+\lambda)^2}, \quad a_3 = -\frac{(1+2\delta)\lambda(2\delta+\lambda)}{(\delta+\lambda+2\delta\lambda)^2},\\
b_1 &= \frac{1+2\delta}{1+4\delta}, \quad b_2 = -\frac{\delta^2(2\delta^2+\lambda^2-6\delta\lambda^2+4\lambda^3)}{(\delta+\lambda)^2(2\delta+\lambda)^2}, \quad b_3 = -\frac{4\delta^2\lambda(\delta^2+\lambda^2+\delta\lambda+2\delta\lambda^3)}{(\delta+\lambda)^2(\delta+\lambda+2\delta\lambda)^2},\\
c_1 &= -\frac{4\delta}{1+4\delta}, \quad c_2 = -\frac{4\delta(\delta+\lambda^2)}{(2\delta+\lambda)^2}, \text{ and } \quad c_3 = -\frac{2\delta(\delta+2\lambda^2+4\delta\lambda^2)}{(\delta+\lambda+2\delta\lambda)^2}.
\end{aligned}
\tag{A.13}
$$

Denote $S^{1,\varepsilon}_{RA} = S^1_{RA}|_{\rho=\hat{\rho}^\star(\varepsilon)}$ and $S^{2,\varepsilon}_{RA} = S^2_{RA}|_{\rho=\hat{\rho}^\star(\varepsilon)}$, and from Eq. (A.12) we can get:

$$
\begin{aligned}
\pi^\varepsilon_{RA} = \big[(\rho_1+\varepsilon)a_1 + (\rho_2+\varepsilon)a_2 + (\rho_3-2\varepsilon)a_3\big]S^{1,\varepsilon}_{RA}\\
+ \big[(\rho_1+\varepsilon)b_1 + (\rho_2+\varepsilon)b_2 + (\rho_3-2\varepsilon)b_3\big]S^{2,\varepsilon}_{RA}\\
+ \big[(\rho_1+\varepsilon)c_1 + (\rho_2+\varepsilon)c_2 + (\rho_3-2\varepsilon)c_3\big].
\end{aligned}
\tag{A.14}
$$

Hence,

$$
\begin{aligned}
\pi^\varepsilon_{RA} - \pi_{RA} = \left(\sum_{i=1}^3 \rho_i a_i\right)(S^{1,\varepsilon}_{RA} - S_{RA}) + \left(\sum_{i=1}^3 \rho_i b_i\right)(S^{2,\varepsilon}_{RA} - S_{RA})\\
+ \left\{(a_1+a_2-2a_3)S^{1,\varepsilon}_{RA} + (b_1+b_2-2b_3)S^{2,\varepsilon}_{RA} + (c_1+c_2-2c_3)\right\}\varepsilon.
\end{aligned}
\tag{A.15}
$$

Obviously, as $\varepsilon \to 0$, $S^{1,\varepsilon}_{RA} \to S^1_{RA}$ and $S^{2,\varepsilon}_{RA} \to S^2_{RA}$. Denote the limits by

$$
\partial S^1_{RA}/\partial\varepsilon = \lim_{\varepsilon\to 0}\frac{S^{1,\varepsilon}_{RA} - S^1_{RA}}{\varepsilon}, \quad \partial S^2_{RA}/\partial\varepsilon = \lim_{\varepsilon\to 0}\frac{S^{2,\varepsilon}_{RA} - S^2_{RA}}{\varepsilon}.
$$

Rearranging Eq. (A.15) yields

$$
\begin{aligned}
\lim_{\varepsilon\to 0}\frac{\pi^\varepsilon_{RA} - \pi_{RA}}{\varepsilon} &= \left(\sum_{i=1}^3 \rho_i a_i\right)\lim_{\varepsilon\to 0}\frac{S^{1,\varepsilon}_{RA} - S_{RA}}{\varepsilon} + \left(\sum_{i=1}^3 \rho_i b_i\right)\lim_{\varepsilon\to 0}\frac{S^{2,\varepsilon}_{RA} - S_{RA}}{\varepsilon}\\
&+ \lim_{\varepsilon\to 0}\left\{(a_1+a_2-2a_3)S^{1,\varepsilon}_{RA} + (b_1+b_2-2b_3)S^{2,\varepsilon}_{RA} + (c_1+c_2-2c_3)\right\}\\
&= \left(\sum_{i=1}^3 \rho_i a_i\right)\partial S^1_{RA}/\partial\varepsilon + \left(\sum_{i=1}^3 \rho_i b_i\right)\partial S^2_{RA}/\partial\varepsilon\\
&+ \left\{(a_1+a_2-2a_3)S^1_{RA} + (b_1+b_2-2b_3)S^2_{RA} + (c_1+c_2-2c_3)\right\}.
\end{aligned}
\tag{A.16}
$$

From the definition of S^j_{RA}, we have

$$S^1_{RA} = \frac{\delta(\lambda+\delta)(2\lambda-1)\big(\rho_1 w_1 + \rho_2 w_2 + \rho_3 w_3\big)}{\big(\rho_1 w_5 + \rho_2 w_6 + \rho_3 w_7\big)},$$
$$S^2_{RA} = \frac{\delta(\lambda+\delta)(2\lambda-1)\big(\rho_1 w_1 + \rho_2 w_2 + \rho_3 w_4\big)}{\big(\rho_1 w_5 + \rho_2 w_6 + \rho_3 w_8\big)}, \tag{A.17}$$

where

$$\begin{aligned}
w_1 &= (\lambda+2\delta)^2(\lambda+\delta+2\lambda\delta)^2, \quad w_2 = \delta(1+4\delta)(\lambda+\delta+2\lambda\delta)^2,\\
w_3 &= (1+4\delta)(\lambda+2\delta)^2(\lambda+\delta\lambda+\lambda^2), w_4 = 2\delta(1+4\delta)\lambda^2(2\delta+\lambda)^2,\\
w_5 &= (\delta+\lambda)(\lambda+2\delta)^2(\lambda+\delta+2\lambda\delta)^2[3\lambda+(8\lambda-1)\delta],\\
w_6 &= \delta^2(1+4\delta)(\lambda+\delta+2\lambda\delta)^2[(5\lambda-1)\lambda+(8\lambda-1)\delta],\\
w_7 &= (1+4\delta)(\lambda+\delta)(\lambda+2\delta)^2\big[3\lambda^3+\delta\lambda^2(5+8\lambda)+\delta^2(-1+4\lambda+8\lambda^2)\big],\\
w_8 &= 4\lambda^2\delta^2(1+4\delta)(\lambda+2\delta)^2(\lambda+\delta+4\lambda\delta+\lambda^2).
\end{aligned} \tag{A.18}$$

From the definition of $\partial S^1_{RA}/\partial\varepsilon$ and $\partial S^2_{RA}/\partial\varepsilon$, we obtain

$$\partial S^1_{RA}/\partial\varepsilon = \frac{\upsilon_0(\rho_1\upsilon_1+\rho_2\upsilon_2+\rho_3\upsilon_3)}{(\rho_1 w_5+\rho_2 w_6+\rho_3 w_7)^2}, \quad \partial S^2_{RA}/\partial\varepsilon = -\frac{\upsilon_0(\rho_1\upsilon_4+\rho_2\upsilon_5+\rho_3\upsilon_6)}{(\rho_1 w_5+\rho_2 w_6+\rho_3 w_8)^2}, \tag{A.19}$$

where

$$\begin{aligned}
\upsilon_0 &= 3\delta^2(1+4\delta)\lambda(\delta+\lambda)(2\lambda-1)[\lambda^2+\delta\lambda(3+2\lambda)+\delta^2(2+4\lambda)]^2,\\
\upsilon_1 &= \lambda^3(2\lambda-1)+\delta(13\lambda-7)\lambda^2+\delta^2\lambda(-13+16\lambda+8\lambda^2)+\delta^3(-7-3\lambda+8\lambda^2+4\lambda^3)-8\delta^4,\\
\upsilon_2 &= \lambda^3+\delta\lambda^2(3+5\lambda)+\delta^2\lambda(1+16\lambda)+\delta^3(-3+11\lambda-4\lambda^2)-8\delta^4,\\
\upsilon_3 &= \lambda^4+\delta\lambda^2(-2+9\lambda)+2\delta^2\lambda(-3+8\lambda+2\lambda^2)+\delta^3(-5+4\lambda+4\lambda^2)-8\delta^4,\\
\upsilon_4 &= \lambda^3-4\lambda^5\delta\lambda^2(3+5\lambda-16\lambda^2-8\lambda^3)+\delta^2\lambda(3+10\lambda-4\lambda^2-32\lambda^3)\\
&+ \delta^3(1+5\lambda+24\lambda^2-28\lambda^3)+16\delta^4\lambda,\\
\upsilon_5 &= \lambda^3+\delta\lambda^2(3+5\lambda)+3\delta^2\lambda(1+2\lambda+4\lambda^2)+\delta^3(1+\lambda-8\lambda^2+20\lambda^3)-16\delta^4\lambda,\\
\upsilon_6 &= 2\lambda^4+4\delta\lambda^3(2+\lambda)+2\delta^2\lambda(-1+4\lambda+8\lambda^2)+2\delta^3(-1-8\lambda+12\lambda^2)-16\delta^4.
\end{aligned} \tag{A.20}$$

Plugging Eq. (A.13), (A.17), (A.18), (A.19) and (A.20) into Eq. (A.16), we obtain

$$\lim_{\varepsilon\to 0}\frac{\pi^\varepsilon_{RA}-\pi_{RA}}{\varepsilon} = \frac{3\lambda\delta^2(2\lambda-1)^2\chi_5(\rho_1,\rho_2,\rho_3)}{(1+4\delta)[\lambda^3+2\delta\lambda^2(2+\lambda)+\delta^3(2+4\lambda)+\delta^2\lambda(5+6\lambda)]\chi_6^2(\rho_1,\rho_2,\rho_3)}, \tag{A.21}$$

where

$$\chi_6 = (\rho_1 w_5+\rho_2 w_6+\rho_3 w_7)\times(\rho_1 w_5+\rho_2 w_6+\rho_3 w_8).$$

It is easy to observe that

$$\text{sign}\{\lim_{\varepsilon\to 0}\frac{\pi_{RA}^{\varepsilon}-\pi_{RA}}{\varepsilon}\}=\text{sign}\{\chi_5(\rho_1,\rho_2,\rho_3)\}.$$

Therefore, we only need to verify that $\chi_5(\rho_1,\rho_2,\rho_3) > 0$, which is true.[4] Hence, the new structure $\hat{\rho}^\star(\varepsilon)$ outperforms the original one $\rho^\star$ for any $\varepsilon \to 0$. Contradiction! ■

2 Optimal Authority Allocation in Asymmetric Organizations

In asymmetric organizations, the optimal authority structure may involve randomization among all possible deterministic authority allocations, making it difficult to explicitly characterize the optimal authority allocation. However, the intuition in symmetric organizations should still apply, and thus random authority can be strictly optimal under some parameter values. There are two major findings in this section. First, we show that mixed authority can be included as part of the optimal authority allocation in asymmetric organizations. Second, due to the existence of mixed authority, we show by example that the optimal degree of delegation may not be monotonic in incentive misalignment.

Observation A.1 *In asymmetric organizations, for any $\sigma_1^2 \neq \sigma_2^2$, then there exists (λ,δ) such that the optimal authority allocation satisfies either $\rho_3 > 0$ or $\rho_4 > 0$.*[5]

Observation A.1 reinforces the result in [1], which numerically claims that mixed authority (i.e., partial centralization) can be optimal when the degree of information asymmetry is sufficiently large. We theoretically prove that mixed authority can be part of the optimal authority allocation even under tiny asymmetry.

We have already proved that the optimal degree of delegation can change non-monotonically with the coordination need in symmetric organizations. However, in symmetric organizations, the optimal degree of delegation monotonically decreases with the incentive misalignment. In asymmetric organizations, the optimal degree of delegation may be non-monotonic in the incentive misalignment as illustrated by the following numerical example.

Denote $\alpha = \sigma_2^2/\sigma_1^2$. Then the optimal authority allocation satisfies:

$$\begin{aligned}
C^*|_{(\alpha,\lambda,\mu)=(1.5,0.56,0.6)} &= 0.71\circ\{P\}+0.29\circ\{M1\},\\
C^*|_{(\alpha,\lambda,\mu)=(1.5,0.58,0.6)} &= 0.83\circ\{P\}+0.17\circ\{M1\},\\
C^*|_{(\alpha,\lambda,\mu)=(1.5,0.9,0.6)} &= \{F\}.
\end{aligned}$$

[4]The exact expressions of χ_5 can be obtained upon request.
[5]The proofs are available upon request.

In this example, the coordination need is quite large, and there exists some degree of information distribution asymmetry in organizations. When the incentives are sufficiently aligned, including mixed authority can improve organizational performance, mainly due to the fact that mixed authority is a better fit to the information structure. However, as λ increases, the benefits from including mixed authority to enhance communication will be offset by the adaptation loss caused by the functional manager. This makes project authority more attractive. Finally, when incentive misalignment is sufficiently severe (λ very large), enforcing functional authority will become necessary in order to implement effective coordination.

References

[1] Rantakari, H. (2008): "Governing Adaption," *Review of Economic Studies*, 75, 1257–1285.