SUPPLEMENT TO
“ A Theory of Organizational Dynamics: Internal Politics and Efficiency”

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In the paper, we consider a stylized model of organizational dynamics. In this Online Appendix, we extend our baseline model in various important dimensions. Throughout the Online Appendix, we often refer to equations and figures in the main text and its Appendix using the numbering established there (e.g., Equation (4) and Figure 1). The numbers of equations and figures in this Online Appendix are all prefixed by A to distinguish them from those in the main text and its Appendix, and we always use this prefix in referring to them.

1 Exponential Distribution of Quality

The existence of the glass-ceiling equilibrium is a direct result of the assumption that v has bounded support. Apparently, if the support is unbounded, Condition (33) can never be satisfied, and hence all equilibria would be “power-switching.” In this section, we replace the uniform distribution assumption by assuming that a player’s quality, v, is distributed according to an exponential distribution with parameter λ.

We can similarly derive the equilibrium conditions under both majority voting and unanimity voting. The detailed derivations of the following and all subsequent equilibrium conditions are omitted, and can be obtained upon request. Under majority voting, the equilibrium cutoffs $(v_3^r, v_3^l, v_2^r, v_2^l)$ should satisfy the following system of four equations:

$$\begin{aligned}
\frac{e^{-\lambda v_3^r}}{2\lambda}+\frac{e^{-\lambda v_3^l}}{2\lambda} &= \tau \\
\frac{e^{-\lambda v_2^r}}{2\lambda}+\frac{e^{-\lambda v_2^l}}{2\lambda} &= \tau \\
2v_3^r-v_2^r-v_3^l &= \frac{B}{6} \\
\frac{1}{2}e^{-\lambda v_2^r}(v_2^l-v_3^r)+e^{-\lambda v_2^l}\left(2v_2^l-\frac{3}{2}v_2^r-\frac{1}{2}v_3^l\right) &= \frac{B}{6}\left(e^{-\lambda v_2^r}+e^{-\lambda v_2^l}\right).
\end{aligned} \tag{A.1}$$

Under unanimity voting, the equilibrium cutoffs $(v_3^r, v_3^l, v_2^r, v_2^l, v_1^r, v_1^l)$ should satisfy the following system of six equations:

$$\begin{aligned}
\frac{e^{-\lambda v_3^r}}{2\lambda} + \frac{e^{-\lambda v_3^l}}{2\lambda} &= \tau \\
\frac{e^{-\lambda v_2^l}}{2}\left(v_2^l - v_1^r + \frac{1}{\lambda}\right) + \frac{e^{-\lambda v_1^l}}{2\lambda} &= \tau \\
\frac{e^{-\lambda v_1^l}}{2}\left(v_1^l - v_2^r + \frac{1}{\lambda}\right) + \frac{e^{-\lambda v_2^l}}{2\lambda} &= \tau \\
2v_3^r - v_2^r - v_3^l &= \frac{B}{6} \\
v_1^1 + v_2^l - v_2^r - v_3^r &= \frac{B}{3} \\
2v_1^l - v_1^r - v_2^l &= \frac{B}{2}.
\end{aligned} \tag{A.2}$$

In the baseline model, the club's degree of incongruity can be measured by a single variable $c = B/(12\sqrt{a\tau})$. It is impossible to define a similar variable in this exponential distribution case. Therefore, in the following numerical example, we fix $\tau = 0.1$, $\lambda = 1$ and explore the impact of B on the long-term welfare of the organization.

Figure A.1 illustrates the comparison of long-term welfare under different cases. It is not surprising to see that the harmonious equilibrium still dominates majority voting in terms of long-term welfare. Under majority voting, as evident from Equation (A.1), admission standards for candidates of both types are distorted: one is favored, the other is discriminated against. This intuition does not depend on the existence of the glass-ceiling equilibrium, hence is independent of the assumption of quality having bounded support. This divergence of admission standards leads to lower long-term welfare under majority voting than in the harmonious equilibrium. Since the glass-ceiling equilibrium does not exist, long-term welfare changes continuously in B: there is no drop in welfare as shown in Figure 2.

Under unanimity voting, the presence of internal politics can enhance welfare relative to the harmonious equilibrium as in the baseline model. This is not surprising either. Under unanimity voting, incumbent members of both types have relatively balanced power and try to set relatively high standards for candidates of both types, mitigating the intertemporal free-riding problem. Moreover, under unanimity voting, it is the power-switching equilibrium that yields greater long-term welfare than the harmonious equilibrium, and hence the assumption that quality has unbounded support should not change this result.

2 Fixed Per-capita Rent

To make it easy to compare welfare, we assume in the baseline model that there is a fixed amount of total rent B, no matter how large the majority is. This section considers another case where the per-capita rent is fixed to be $\frac{B}{3}$. Then the total rent is $2B/3$ in contentious states and B in homogenous states. To make welfare comparison easy, in the following we do not include the amount of total rent in the welfare calculation.

For $i = 1, 2, 3$ and $j = l, r$, let $x_i^j = \frac{\bar{v} - v_i^j}{a}$, $y_i^j = \sqrt{\frac{a}{4\tau}} x_i^j$ and $c = \frac{B}{12\sqrt{a\tau}}$, just as before. Then the power-switching equilibrium conditions under majority voting are:

$$\begin{aligned} (y_3^r)^2 + (y_3^l)^2 &= 1 \\ (y_2^r)^2 + (y_2^l)^2 &= 1 \\ y_2^r + y_3^l - 2y_3^r &= 0 \\ \frac{1}{2} y_3^r (y_2^r + y_2^l) - (y_2^l)^2 &= c(3y_2^r + y_2^l)/3. \end{aligned} \tag{A.3}$$

Comparing Equation (A.3) and Equation (32) in the paper, we can see that the first two equations are the same. The last two equations become different due to the change of rent specification.

Similar to the baseline model, there also exists a glass-ceiling equilibrium under majority voting. The conditions for such an equilibrium become

$$\begin{aligned} (y_3^r)^2 + (y_3^l)^2 &= 1 \\ (y_2^r)^2 + (y_2^l)^2 &= 1 \\ y_2^r + y_3^l - 2y_3^r &= 0 \\ y_2^l = 0. \end{aligned} \tag{A.4}$$

It is straightforward to solve $(y_3^r, y_3^l) = (\frac{4}{5}, \frac{3}{5})$, and $(y_2^r, y_2^l) = (1, 0)$ in this glass-ceiling equilibrium.

The equilibrium conditions under unanimity voting are

$$
\begin{aligned}
(y_3^r)^2 + (y_3^l)^2 &= 1 \\
y_2^r + y_3^l - 2y_3^r &= 0 \\
y_3^r - y_2^l + y_2^r - y_1^l &= 2c \\
y_2^l + y_1^r - 2y_1^l &= 2c \\
(y_2^r)^2 + (y_2^l)^2 &= 1 + (y_2^r - y_1^l)^2 \\
(y_1^r)^2 + (y_1^l)^2 &= 1 + (y_1^r - y_2^l)^2.
\end{aligned} \tag{A.5}
$$

We solve the most efficient equilibrium under each voting rule. Figure A.2 depicts the relative frequency of homogeneous states over contentious states, q_3/q_2, under both majority voting and unanimity voting. Comparing Figure A.2 and Figure 4, we can see that when the per-capita rent is fixed, the fraction of time spent in the homogeneous states will be greater. Since there is no "dilution" of rent among majority-type members, the incumbent members are more likely to favor candidates of the same type than in the baseline model,

In terms of long-term welfare, Figure A.3 exhibits exhibits a pattern similar to that shown in Figure 2 in the paper. Under majority rule, although Equation (A.3) is different from Equation (32), the solutions to these two systems of equations share a similar pattern: either y_i^r or y_i^l is larger than $\frac{\sqrt{2}}{2}$ while the other is smaller than $\frac{\sqrt{2}}{2}$. In other words, the equilibrium admission standards under majority voting are biased in opposite directions relative to those in the harmonious equilibrium. As a result, the long-term welfare under majority voting is always lower than that in the harmonious equilibrium.

As in the baseline model, unanimity voting can dominate the harmonious equilibrium in terms of long-term welfare when the degree of incongruity c is relatively low, because internal politics under unanimity voting leads to higher admission standards for both types of candidates and thus offsets the intertemporal free riding in the harmonious equilibrium.

Under unanimity voting, it is also interesting to notice that when the degree of incongruity is small, the long-term welfare is higher in the case of fixed total rent than in the case of fixed per-capita rent. When the degree of incongruity is small, the presence of internal politics increases welfare. Hence, a more unequal distribution of rent increases the long-term welfare by intensifying internal politics. Compared with the case with fixed per-capita rent, a fixed total rent leads to a more unequal distribution of rent and thus higher long-term welfare. But when the degree of incongruity is large, the long-term welfare is higher in the case of fixed per-capita rent. In particular, when the degree of incongruity is in the range from 0.43 to 0.6, long-term welfare is greater in the case of fixed per-capita rent than in the harmonious equilibrium, which in turn is greater than in the case of fixed total rent. When the degree of incongruity is large, internal politics decreases welfare, because both types of incumbent members will set standards that are too stringent to admit candidates of the opposite type. Thus, a more unequal distribution of rent decreases the long-term welfare. Consequently, the long-term welfare is greater in the case of fixed per-capita rent than in the case of fixed total rent when the club is sufficiently incongruous. This finding suggests that if the organization can commit ex ante to the form of rent distribution, the form of the most desirable rent distribution will depend on the degree of incongruity.

3 Different Discount Factor

For simplicity, we assume in the paper that there is no discounting and the effective discount factor is the probability of staying in the club, $\frac{2}{3}$. To relax this assumption, we now assume that, in any period, exiting by incumbent members does not always occur. Specifically, at the beginning of each period, with probability p, the three incumbents know that there is an exit in this period, and they have to search for a candidate within this period. With the complementary probability, there is no exit and the game moves to the next period. Probabilistic exit occurs when the incumbent member receives a random chance of promotion or an outside option that dominates staying in the club. Regardless of whether there is exit or not, there is rent B to be distributed in every period.

Obviously, in this extension, the effective discount factor becomes $1 - \frac{1}{3}p$, which can be arbitrarily close to 1 as p approaches 0. The first best solution thus maximizes $3E[v|v \geq v^*] - p\tau F(v^*)/(1 - F(v^*))$, and it is straightforward to obtain the interior solution $v^* = \bar{v} - \sqrt{2pa\tau/3}$. In the harmonious equilibrium, the expected value to an incumbent member if a candidate with quality $\hat{v}$ is admitted is:

$$\frac{2}{3}\left(1 + \left(1 - \frac{2}{3}p\right) + \left(1 - \frac{2}{3}p\right)^2 + ...\right)\hat{v} = \frac{\hat{v}}{p}.$$

This implies that in an interior solution, $\hat{v} = \bar{v} - \sqrt{2pa\tau}$.

Now we analyze the equilibrium with internal politics. Denote π_i^R to be the right-type incumbent's searching payoff at state i when there is a random exit, and $\tilde{\pi}_i^R$ to be the right-type incumbent's expected searching payoff at state i. Obviously,

$$\tilde{\pi}_i^R = p\pi_i^R + (1 - p)(\tilde{\pi}_i^R + \mu_i),$$

where μ_i is the rent received by the right-type incumbent. With probability p, there is an exit and the searching payoff is π_i^R; with the complementary probability, there is no exit and the searching payoff is $\tilde{\pi}_i^R + \mu_i$.

Under majority voting, Equation (21) becomes

$$\frac{v_3^r}{p} + \frac{2}{3}\left[\frac{B}{3} + \tilde{\pi}_3^R\right] = \pi_3^R - \tau,$$

which implies

$$\frac{v_3^r}{p} + \frac{2}{3}\left[\frac{B}{3p} + \pi_3^R\right] = \pi_3^R - \tau. \tag{A.6}$$

Similarly, Equations (22)-(24) should be rewritten as:

$$\begin{aligned}
\frac{v_3^l}{p} + \frac{2}{3}\left[\frac{B}{2p} + \pi_2^R\right] &= \pi_3^R - \tau; && \text{(A.7)}\\
\frac{v_2^r}{p} + \frac{2}{3}\left[\frac{5B}{12p} + \frac{1}{2}\pi_2^R + \frac{1}{2}\pi_3^R\right] &= \pi_2^R - \tau; && \text{(A.8)}\\
\frac{v_2^l}{p} + \frac{2}{3}\left[\frac{B}{4p} + \frac{1}{2}\pi_1^R + \frac{1}{2}\pi_2^R\right] &= \pi_2^R - \tau; && \text{(A.9)}
\end{aligned}$$

and Equations (25)-(27) should be rewritten as:

$$\begin{aligned}
\pi_3^R &= \frac{\overline{v} - v_3^r}{3a}\left[\frac{B}{3p} + \pi_3^R\right] + \frac{\left[\overline{v}^2 - (v_3^r)^2\right]}{4ap} + \frac{v_3^r - \underline{v}}{2a}[\pi_3^R - \tau]\\
&\quad + \frac{\overline{v} - v_3^l}{3a}[\frac{B}{2p} + \pi_2^R] + \frac{\left[\overline{v}^2 - \left(v_3^l\right)^2\right]}{4ap} + \frac{v_3^l - \underline{v}}{2a}[\pi_3^R - \tau]; && \text{(A.10)}\\
\pi_2^R &= \frac{\overline{v} - v_2^r}{3a}\left[\frac{5B}{12p} + \frac{1}{2}\pi_2^R + \frac{1}{2}\pi_3^R\right] + \frac{\left[\overline{v}^2 - (v_2^r)^2\right]}{4ap} + \frac{v_2^r - \underline{v}}{2a}[\pi_2^R - \tau]\\
&\quad + \frac{\overline{v} - v_2^l}{3a}\left[\frac{B}{4p} + \frac{1}{2}\pi_1^R + \frac{1}{2}\pi_2^R\right] + \frac{\left[\overline{v}^2 - \left(v_2^l\right)^2\right]}{4ap} + \frac{v_2^l - \underline{v}}{2a}[\pi_2^R - \tau]; && \text{(A.11)}\\
\pi_1^R &= \frac{\overline{v} - v_2^r}{3a}\pi_1^R + \frac{\left[\overline{v}^2 - (v_2^r)^2\right]}{4ap} + \frac{v_2^r - \underline{v}}{2a}[\pi_1^R - \tau]\\
&\quad + \frac{\overline{v} - v_2^l}{3a}[\frac{B}{2p} + \pi_1^R] + \frac{\left[\overline{v}^2 - \left(v_2^l\right)^2\right]}{4ap} + \frac{v_2^l - \underline{v}}{2a}[\pi_1^R - \tau]. && \text{(A.12)}
\end{aligned}$$

Rearranging the above seven equations yields (we similarly define $x_i^{b'} \equiv (\overline{v} - v_i^{b'})/a$)

$$
\begin{aligned}
(x_3^r)^2 + (x_3^l)^2 &= 4p\tau/a; \\
(x_2^r)^2 + (x_2^l)^2 &= 4p\tau/a; \\
x_2^r + x_3^l - 2x_3^r &= \frac{B}{6a}; \\
(x_2^r + x_2^l)\frac{B}{a} &= 3x_2^r x_3^r + 3x_2^l x_3^l + 6x_2^l x_2^r - 12(x_2^l)^2.
\end{aligned}
\tag{A.13}
$$

Notice that this model is equivalent to our baseline model with effective cost of delay $\tau' = p\tau$. Let $y_i^{b'} = x_i^{b'}\sqrt{a/(p\tau)}/2$, for $i = 1, 2, 3, 4$ and $b' = l, r$, and $c' = B/(12\sqrt{ap\tau})$. Then we derive the following four equations which are exactly the same as Equation (32) in the paper (after replacing c with c'):

$$
\begin{aligned}
(y_3^r)^2 + (y_3^l)^2 &= 1 \\
(y_2^r)^2 + (y_2^l)^2 &= 1 \\
y_2^r + y_3^l - 2y_3^r &= c' \\
y_2^r y_3^r + y_2^l y_3^l + 2y_2^l y_2^r - 4(y_2^l)^2 &= 2c'(y_2^r + y_2^l).
\end{aligned}
\tag{A.14}
$$

Under unanimity voting, we can derive the same equilibrium conditions about $y_3^r, y_3^l, y_2^r, y_2^l, y_1^r, y_1^l$ using the above definitions of $y_i^{b'}$ and c'.

Finally, using some algebra, we recalculate the long-term welfare in each case. Similar to the baseline model,

$$
U = 3Ev + \frac{3}{2}a + p\tau - 2\sqrt{pa\tau}\gamma, \tag{A.15}
$$

where γ summarizes the total long-term expected welfare loss for the club in each case. Furthermore, the expressions of γ are exactly the same as the ones in the baseline model. Equation (A.15) implies that as p approaches zero, $\sqrt{pa\tau}$ goes to zero, and the long-term welfare in each case converges to the same value. However, as p enters into Equation (A.15) in the same way, the comparisons of the long-term welfare do not directly depend on p, and we still only need to compare γ.

Therefore, Proposition 5 in our main context still holds if we replace c with the newly defined variable c'. Since $c' = c/\sqrt{p} > c$ for all $p < 1$, the range of c for which unanimity voting can yield greater long-term welfare than the harmonious equilibrium (that is, the range in which internal politics can improve welfare) shrinks compared to the baseline model. Moreover, this range is smaller as p becomes lower. The reason for this result is the following. As the discounting factor increases, the intertemporal free-riding problem becomes less severe since the effective cost of delay $p\tau$ becomes smaller. As a result, internal politics becomes less beneficial.

4 Larger Committee Size: $n = 5$

In the baseline model we consider a club with three members. In this section, we extend our baseline model to a club with five members ($n = 5$), in order to show that the main results of the baseline model can be generalized to larger committees.[1] Increasing the club size from three to five effectively increases the discount factor from 2/3 to 4/5, thus the analysis below also provides a different way of increasing the effective discount factor than from the way of introducing probabilistic exit considered in the preceding section.

It is straightforward to verify that, just as before, majority voting is dominated by the harmonious equilibrium when $n = 5$, because the equilibrium admission standards under majority voting are biased in opposite directions relative to those in the harmonious equilibrium. To save space, we omit the details of the derivation of this result.

Let $y_i^{b'} = x_i^{b'}\sqrt{a/\tau}/2$, for $i = 1, 2, 3, 4$ and $b' = l, r$, and $c' = B/(12\sqrt{a\tau})$. Under unanimity voting, we can derive ten equilibrium conditions about $y_5^r, y_5^l, y_4^r, y_4^l, y_3^r, y_3^l, y_2^r, y_2^l, y_1^r, y_1^l$

$$\begin{aligned}
(y_5^r)^2 + (y_5^l)^2 &= 1 \\
(y_4^r)^2 + (y_4^l)^2 &= 1 + (y_4^r - y_1^l)^2 \\
(y_3^r)^2 + (y_3^l)^2 &= 1 + (y_3^r - y_2^l)^2 \\
(y_2^r)^2 + (y_2^l)^2 &= 1 + (y_2^r - y_3^l)^2 \\
(y_1^r)^2 + (y_1^l)^2 &= 1 + (y_1^r - y_4^l)^2 \\
2y_4^r + y_5^l - 3y_5^r &= \frac{3}{5}c \\
y_4^l + y_3^r - y_4^r - y_5^r &= \frac{4}{5}c \\
2y_2^l + y_1^r - 3y_1^l &= 0 \\
y_2^r + y_3^l - y_1^l - y_2^l &= 2c \\
\frac{25}{3}(x_5^r - x_1^l) - \frac{5}{3}(x_5^l + x_4^l - x_1^r - x_2^r) &= \frac{11}{2}c.
\end{aligned} \tag{A.16}$$

We can similarly use γ to summarize the total long-term expected welfare loss for the club. Here, γ^u changes to

$$\gamma^u \equiv 6q_5/(y_5^r + y_5^l) + q_4(1 + 5(y_1^l)^2 + 5(y_4^l)^2)/(y_1^l + y_4^l) + q_3(1 + 5(y_2^l)^2 + 5(y_3^l)^2)/(y_2^l + y_3^l).$$

And in the harmonious equilibrium $\hat{\gamma} = \frac{3}{2}\sqrt{2}$.

Figure A.4 illustrates the the comparison of welfare losses γ between unanimity voting and the

[1] From the analysis of the model with $n = 5$, it is clear that solving the model with an arbitrary n is too tedious. However, it is not hard to see that the insights of our baseline model should still hold.

harmonious equilibrium. Similar to the baseline model, unanimity voting yields greater long-term welfare than the harmonious equilibrium when c is relatively low. Moreover, the range of c for which this property holds obviously becomes larger (we require $c \leq 0.42$ in the $n = 3$ case). The reason for this result is the following. On the one hand, larger club size increases the discounting factor, and hence the intertemporal free-riding problem becomes less severe as in the previous section. On the other hand, the incentives of politicking become less severe with larger club size as the total amount of rent is fixed. As in the fixed per-capita rent case, less politicking due to larger club sizes increases the long-term welfare when the degree of incongruity is large.

5 Equilibrium Characterization under Unanimity Voting

Based on the derivation procedure of Proposition 4 in the Appendix, we can solve for all possible equilibria under unanimity voting. The proposition below provides a complete characterization of all equilibria under unanimity voting. (Details of the derivation are available upon request.)

Proposition A.1 *Under unanimity voting rule, there are five different kinds of equilibria.*

1. *When $0 < c \equiv B/12\sqrt{a\tau} < 2/3$, a "pro-minority power-switching" equilibrium exists in which candidates of both types are admitted in each state with positive probabilities. However, candidates of the majority type in the contentious state have a lower probability of admission (higher admission standard) than those of the minority type, and are less likely to be admitted as c increases. They are never admitted when c goes to 2/3.*

2. *When $10/29 < c < 2.839$, a "pro-majority power-switching" equilibrium exists in which, as in the pro-minority power-switching equilibrium, candidates of both types are admitted in each state with positive probabilities and candidates of the majority type in the contentious state have a higher probability of admission (lower admission standard) than those of the minority type.*

3. *When $c > 10/29$, the glass-ceiling equilibrium exists and it is the same as in the majority case. In the long run, the system switches between state 3 and state 2 (resp. state 1 and state 0) if the initial state is 3 or 2 (resp. 1 or 0).*

4. *When $c > 2/3$, a "minority tyranny" equilibrium exists in which, in the contentious state, only candidates of the minority type are admitted. In the long run, the club only switches between state 2 and state 1.*

5. *When $c > 2.839$, an "exclusive" equilibrium exists in which the incumbent members in homogeneous states 3 and 0 admit only candidates of their own type. In the long run, the club stays at either state 3 or state 1.*

Proposition A.1 characterizes the range of c each of the five equilibria exists. The following table then characterizes the set of equilibria for each value of c.

c	$\left(0, \frac{10}{29}\right)$	$\left(\frac{10}{29}, \frac{2}{3}\right)$	$\left(\frac{2}{3}, 2.839\right)$	(> 2.839)
Pro-minority Power-switching	✓	✓		
Pro-majority Power-switching		✓	✓	
Glass Ceiling		✓	✓	✓
Minority Tyranny			✓	✓
Exclusive				✓

Figure A.5 depicts the long-term welfare comparison of different equilibria under unanimity voting. The welfare in the most efficient equilibrium depicted in Figure 2 of the main text is obtained from taking the lower envelope of Figure A.5.

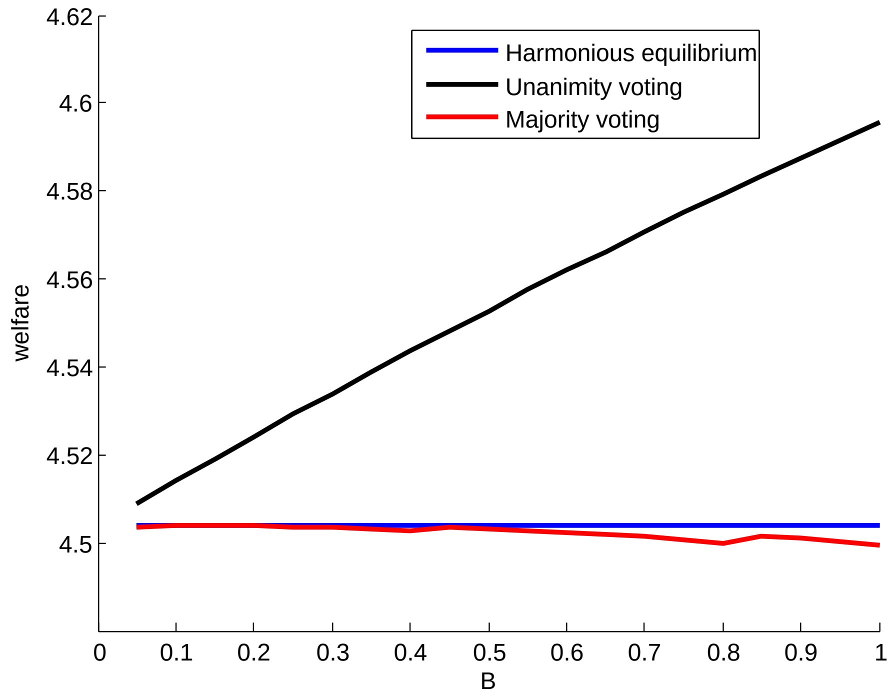

Figure A.1: Long-Term Welfare Comparison Under Exponential Distribution

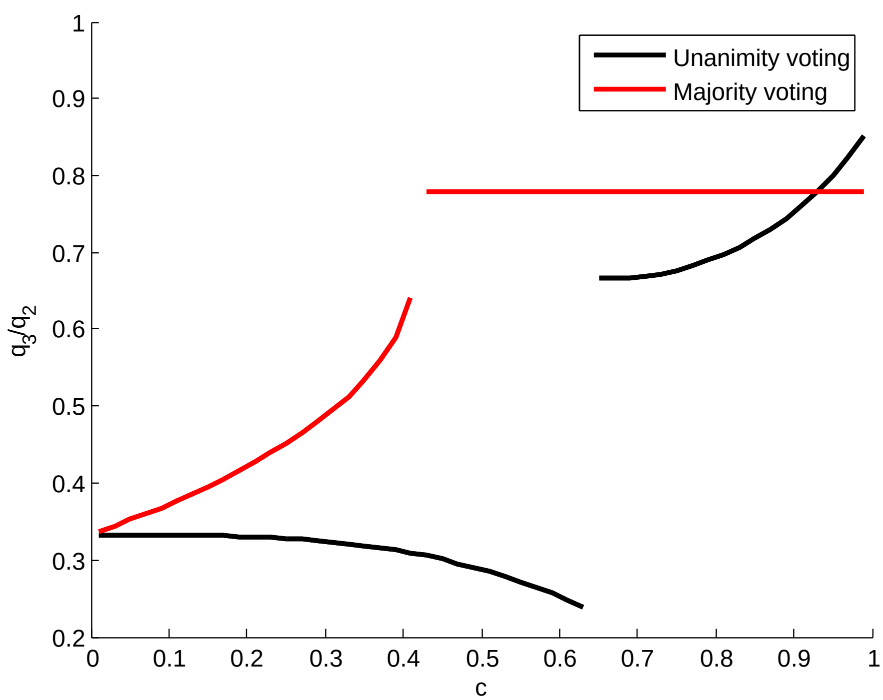

Figure A.2: Relative State Frequency Under Fixed Per-capita Rent

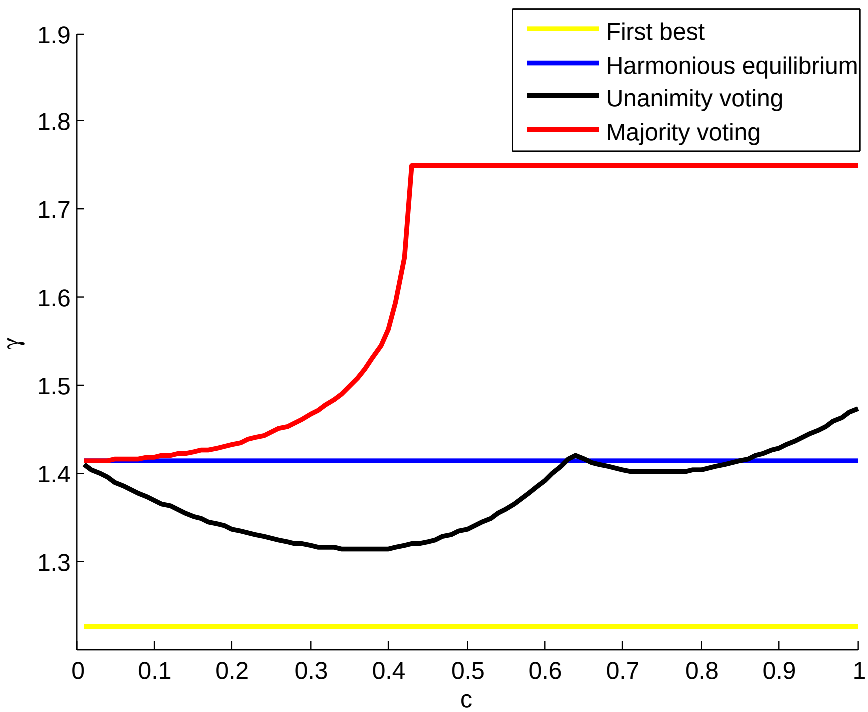

Figure A.3: Long-Term Welfare Loss Comparison Under Fixed Per-capita Rent

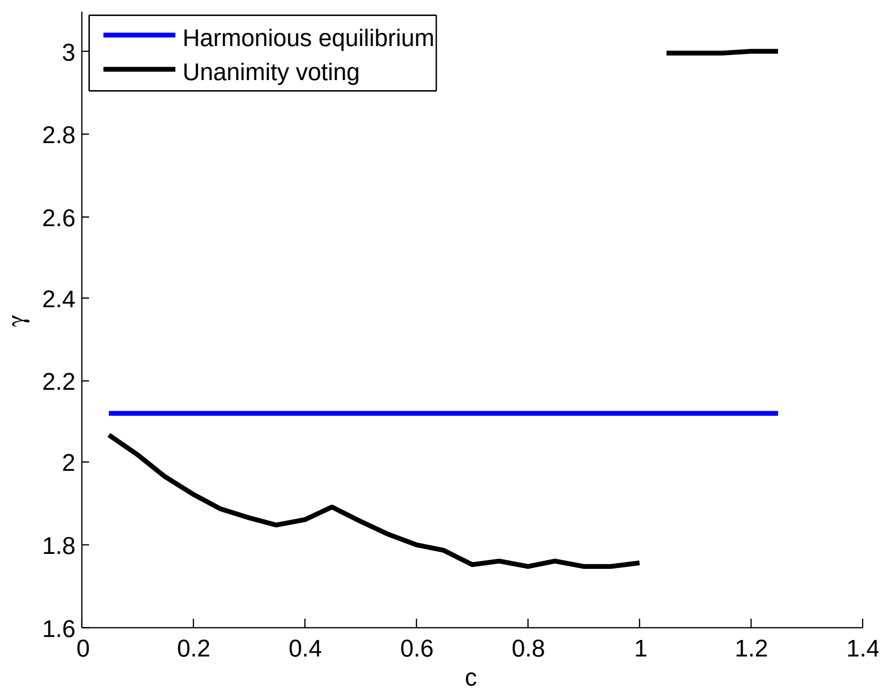

Figure A.4: Long-Term Welfare Loss When $n = 5$

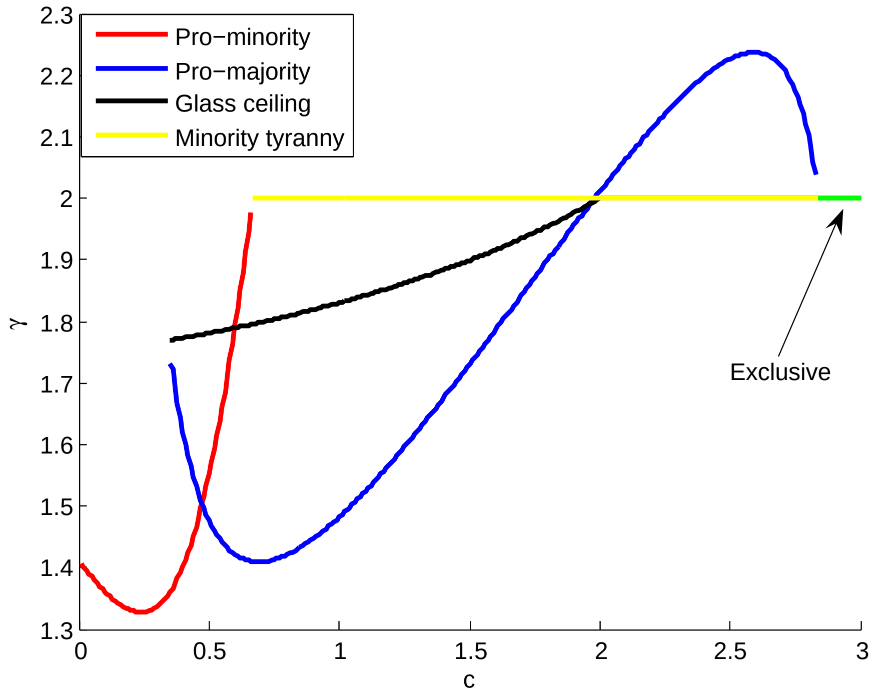

Figure A.5: Long-Term Welfare Comparison of Different Equilibria under Unanimity Voting