# Strategic Information Transmission Models

# Preliminary Lecture Notes

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# 1 Cheap Talk

In game theory, cheap talk is communication between players which does not *directly* affect the payoffs of the game. This is in contrast to Spencer's signaling model in which sending certain messages may be costly for the sender depending on the state of the world. The classic example is of an expert trying to explain the state of the world to an uninformed decision maker. The decision maker, after hearing the report from the expert, must then make a decision which affects the payoffs of both players.

The classical model of cheap talk is introduced by Crawford and Sobel (1982). Real examples of cheap talk include:

1. "Monetary mystique": A central bank is unwilling to make precise statements about its policy objectives.

2. Security analyst recommendations.

3. Rating Agency.

## 1.1 Model Setting

There are two players, a Sender (S) and a Receiver (R) of information. S holds some private information about a payoff-relevant state.

The timing of the game is specified in Figure 1.

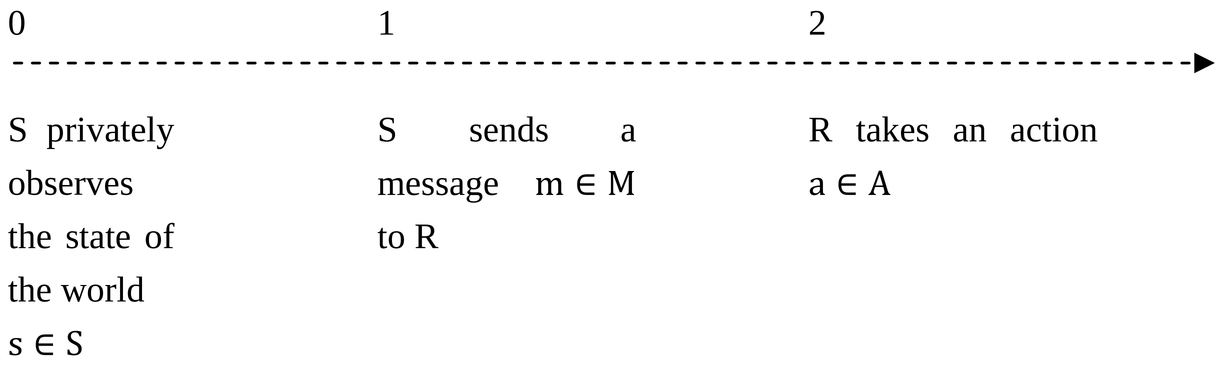


Figure 1: Timeline of the Cheap Talk Game

*Payoffs:*

Payoffs are $U^S(a,s)$ and $U^R(a,s)$. In particular, we will use quadratic utility functions:

$$U^S(a,s) = -[a-(s+b)]^2 \quad \text{and} \quad U^R(a,s) = -[a-s]^2,$$

where $b > 0$ measures how nearly the S's interests coincide with the R's. Notice that the signal $m$ is irrelevant to the payoff functions ("talk is cheap"). Also the message space $M$ is independent of the state $s$.

Given state $s$, the sender's mostly preferred action is $a^S(s) = b + s$ and the receiver's mostly preferred action is $a^R(s) = s$. Both $a^S$ and $a^R$ are increasing in $s$: both players' interests are aligned. However, there are conflicts as well since $a^S > a^R$.

## 1.2 A Motivating Example

State space $S = \{0,1\}$, message space $M = \{0,1\}$, action space $A = [0,1]$. The prior is such that each state happens with equal probability ($\frac{1}{2}$).

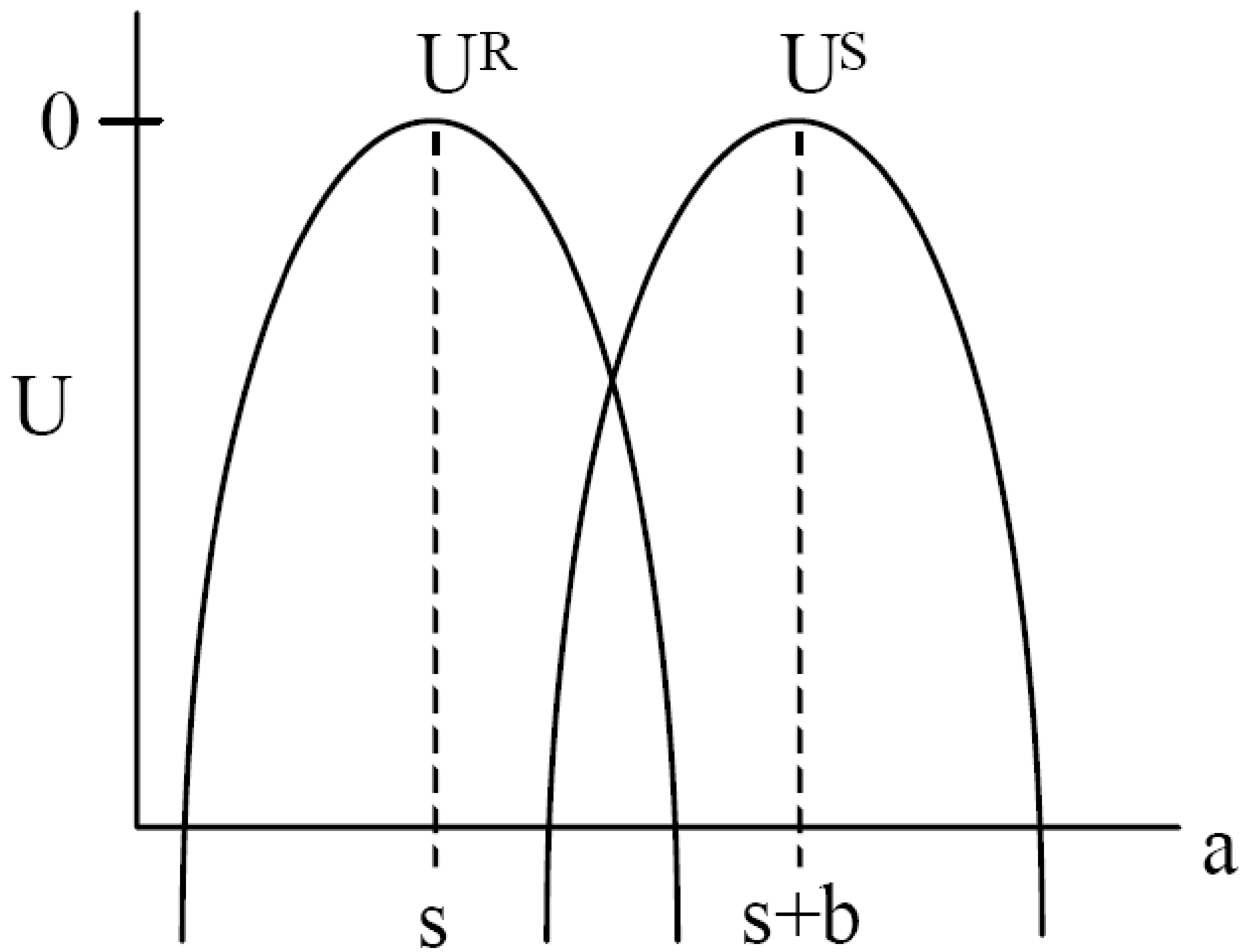


Figure 2: Illustration of the Utility Functions

Strategies: A strategy is a plan of action covering every contingency that might arise.

For S, a strategy is a function from types to actions. Let $q(m|s)$ be the probability that S sends message $m$ when the true state is $s$.

For R, a strategy must specify an action $a(m) \in A$ for each message $m \in M$.

Beliefs are updated by Bayes' rule. If R conjectures that S chooses $m$ according to the strategy $q(m|s)$, then after receiving message, R's posterior belief about state $s$ is derived from Bayes' rule:

$$p(s|m) = \frac{\frac{1}{2}q(m|s)}{\frac{1}{2}(q(m|s) + q(m|1-s))}. \tag{1}$$

*Definition of equilibrium:*

**Definition 1** *The strategies $\{q(m|s), a(m)\}$ form a perfect Bayesian equilibrium if:*

1. *for each $s \in \{0,1\}$, $\sum_{m=0,1} q(m|s) = 1$ and if $m^\star \in M$ is in the support of $q(\cdot|s)$, then*

$$m^\star \in argmax_m U^S(a(m), s);$$

2. *for each $m \in M$,*

$$a(m) \in argmax_a \sum_s U^R(a,s)p(s|m),$$

*where $p(\cdot|m)$ is given by equation 1 if $q(m|0) + q(m|1) > 0$.*

Depending on the values of $b$, there are different types of equilibria.

1. If $b \in (\frac{1}{4}, \frac{1}{2})$, there are multiple equilibria:

(a) (babbling equilibrium) In a babbling equilibrium, no information is conveyed from the sender to the receiver. There are many ways to construct a babbling equilibrium: 1) $q(m|s) = \frac{1}{2}, \forall m, s$ and $a(m) = \frac{1}{2}, \forall m$ or 2) $q(1|s) = 1, \forall s$ and $a(1) = \frac{1}{2}$, $a(0) = 0$;

(b) (fully separating equilibrium) $q(m|s) = 1$ if $m = s$, $= 0$ otherwise and $a(m) = m, \forall m$;

(c) (partial separating equilibrium) $q(1|1) = 1, q(1|0) = \frac{1-2b}{2b}$ and $a(0) = 0, a(1) = 2b$.

Comparison of the expected utilities:

For the R:

$$\text{fully separating}(0) > \text{partially separating}(-\tfrac{1}{2}(1-2b)) > \text{babbling}(-\tfrac{1}{4}).$$

For the S:

$$\text{fully separating } (-b^2) > \text{partially separating}(-\tfrac{1}{2}(1-b)^2 - \tfrac{1}{2}b^2)$$
$$> \text{babbling}(-\tfrac{1}{2}(\tfrac{1}{2}-b)^2 - \tfrac{1}{2}(\tfrac{1}{2}+b)^2).$$

Which equilibrium is realized crucially depends on the receiver's beliefs (e.g., the receiver thinks how trustworthy the sender is). This is also interpreted as social norms.

2. If $b \leq \frac{1}{4}$, only babbling and fully separating can be equilibrium.

3. If $b > \frac{1}{2}$, the "unique" equilibrium is the babbling equilibrium.

**Remark 1** *From this simple example, we can see that there are three kinds of indeterminacy in cheap-talk models: multiple off-the-path responses, multiple meanings of messages, and multiple equilibrium associations between types and actions. The first indeterminacy is the result of different possible specifications of behavior off the path of equilibrium (i.e., the receiver may have different responses when the sender sends a message which is NOT supposed to appear in equilibrium). The second kind of indeterminacy, multiple meanings of messages, arises in any non-babbling outcome. One can take any equilibrium outcome and form a new equilibrium outcome by permuting the messages. Usually, we do not care about the actual choice of messages but only about the payoff-relevant association of actions to types. Therefore, this kind of multiplicity of equilibrium is not a problem. The third kind of indeterminacy is the focus of our attention. Since babbling equilibria always exist, if there also exists an equilibrium with meaningful communication, there must be multiple equilibrium type-action distributions.*

### 1.3 Continuous State Space

State space $S = [0, 1]$, action space $A = [0, 1]$, message space $M$ is finite but has a sufficiently large number of elements.[1] The prior is such that $s \sim U[0, 1]$.

Similarly, let $q(m|s)$ be the probability that S sends message $m$ when the true state is $s$ and $a(m) \in A$ be the action chosen by the R facing message $m \in M$.

Belief is updated by Bayes' rule:

[1] The finiteness of the message space is NOT an important assumption. The result does not change if we assume $M = [0, 1]$.

$$f(s|m) = \frac{q(m|s)f(s)}{\int_0^1 q(m|t)f(t)dt} = \frac{q(m|s)}{\int_0^1 q(m|t)dt}. \quad (2)$$

**Definition 2** *The strategies $\{q(m|s), a(m)\}$ form a perfect Bayesian equilibrium if:*

1. *for each $s \in [0,1]$, $\sum_{m \in M} q(m|s) = 1$ and if $m^\star \in M$ is in the support of $q(\cdot|s)$, then*

$$m^\star \in argmax_m U^S(a(m), s);$$

2. *for each $m \in M$,*

$$a(m) \in argmax_a \int_0^1 U^R(a,s)f(s|m)ds,$$

*where $f(\cdot|m)$ is given by equation 2 if $m \in M$ is sent with strictly positive probability.*

Obviously, there always exists a babbling equilibrium: $a(m) = \frac{1}{2}, \forall m$ and S sends each message with equal probability $q(m|s) = \frac{1}{|M|}$. We are interested in an informative equilibrium where at least two different actions $a < a'$ are induced in equilibrium.

Some observations:

1. There does not exist a fully-separating equilibrium;

2. There exists $t \in S$, such that $U^S(a,t) = U^S(a',t)$;

3. $a^S(t) \in (a, a')$ and $a^S(t) - a = a' - a^S(t)$;

4. No $t' > t$ induces $a$ and no $t' < t$ induces $a'$.

First consider a candidate equilibrium where only two messages $m = 0, 1$ and two actions $a_1 < a_2$ are induced in equilibrium. If $a(0) = a_1$ and $a(1) = a_2$, then the above observations imply that there exists a cutoff $t$ such that $t' > t$ sends message $m = 1$ and $t' < t$ sends message $m = 0$. Bayesian updating immediately implies: $\mathbb{E}(s|m = 0) = \frac{1}{2}t$ and $\mathbb{E}(s|m = 1) = \frac{1}{2}(1 + t)$. $a(m) \in \text{argmax}_a \int_0^1 U^S(a, s)f(s|m)ds$ implies that $a(0) = \frac{1}{2}t$ and $a(1) = \frac{1}{2}(1 + t)$. Then, we should have:

$$\frac{1}{2}t + b = \frac{1}{2}(1 - t) - b \Longrightarrow t = \frac{1}{2} - 2b.$$

This is indeed an equilibrium if $b < \frac{1}{4}$.

The expected utility for the receiver is

$$EU^R = \int_0^{\frac{1}{2}-2b} -(\frac{1}{4} - b - s)^2 ds + \int_{\frac{1}{2}-2b}^{1} -(\frac{3}{4} - b - s)^2 ds = -\frac{2}{3}[(\frac{1}{4} + b)^3 + (\frac{1}{4} - b)^3].$$

The expected utility for the sender is

$$EU^S = \int_0^{\frac{1}{2}-2b} -(\frac{1}{4} - 2b - s)^2 ds + \int_{\frac{1}{2}-2b}^{1} -(\frac{3}{4} - 2b - s)^2 ds = -\frac{1}{3}[(\frac{1}{4} + 2b)^3 + (\frac{1}{4} - 2b)^3] - \frac{1}{96}.$$

In general, if $t_{i-1}$, $t_i$ and $t_{i+1}$ are three equilibrium cutoffs, then we should have:

$$t_i + b - \frac{1}{2}(t_{i-1} + t_i) = \frac{1}{2}(t_{i+1} + t_i) - (t_i + b)$$

and hence

$$4b = t_{i-1} + t_{i+1} - 2t_i. \tag{3}$$

Equation 3 is a difference equation about $t_i$'s. The solution to this difference equation is $t_i = it_1 + 2i(i - 1)b$. Notice that the last cutoff $t_N = Nt_1 + 2N(N - 1)b = 1$, which implies that:

$$2N(N-1)b < 1 \Longrightarrow N < [1 + \sqrt{(1 + \frac{2}{b})}]/2.$$

The above discussions can be summarized by the following theorem:

**Theorem 1** *Denote $N^\star$ to be the largest integer less than $[1+\sqrt{(1+\frac{2}{b})}]/2$. For each integer $1 \leq N \leq N^\star$, there exists an equilibrium in which the number of induced actions is $N$. The equilibrium is described by a partition*

$$0 \leq t_0 < t_1 < \cdots < t_{N-1} < t_N = 1$$

*satisfying the difference equation $4b = t_{i-1} + t_{i+1} - 2t_i$.*

**Remark 2** *The above theorem crucially depends on the quadratic utility assumption. By this assumption, sender at different states has different ideal points. This makes the sender has incentives to reveal some information. However, if we assume that the sender's utility function is monotonic in $a$ (for example, $U^S = a - s$), then the unique equilibrium is the babbling equilibrium.*

## 1.4 Enhancing Communication

It is typically possible to improve decisions using more general communication protocols or adding more senders.

### 1.4.1 Different Communication Procedures

Krishna and Morgan (2004) describe a two-stage communication procedure to improve the cheap talk equilibria. They amend the original model of Crawford and Sobel to allow the expert and the decision maker to meet face-to-face in the first stage and engage in (simultaneous) communication. In the second stage, the expert is allowed to send a further message, possibly conditioning this on the outcome of the face-to-face meeting. Their construction of equilibria is as follows.

- In the face-to-face meeting, the expert's message reveals whether the state y is less than a given quantity $x$ or not and a second message $A_1$ chosen from some suitable set of messages $\mathcal{A}$. Formally, the expert's message is either of the form $\{Low, A_1\}$ or of the form $\{High, A_1\}$: The first component, $Low$ or $High$, conveys whether the state $\theta < x$ or not. The decision maker also sends a message $A_2 \in \mathcal{A}$.

- Subsequent play depends on the messages that were exchanged in the face-to-face meeting.

  - If the expert says Low, this is interpreted to mean that $\theta < x$. In the remaining game, a partition equilibrium in the interval $[0, x]$ is played regardless of what other messages are exchanged.

  - If the expert says High, this is interpreted to mean that $\theta \geq x$. The subsequent play then depends on whether the meeting is deemed to be a success. The success or failure of the meeting is determined by a "success function" $S : \mathcal{A} \times \mathcal{A} \to \{0, 1\}$. A meeting is a success if and only if $S(A_1, A_2) = 1$. Then in the subsequent play the expert further reveals whether $\theta \in [x, z]$ or $\theta \in [z, 1]$ for some suitably chosen $z$. If the meeting is deemed to be a failure, then there is no further information conveyedthat is, a babbling equilibrium in the interval $[x, 1]$ is played.

The message sets $\mathcal{A}$ and the success function $S$ are chosen so that the strategies described above constitute an equilibrium.

**Example 1** *Suppose that the expert's bias $b = 1/10$. During the face-to-face meeting, the expert reveals whether the state, $\theta$, is above or below, $x = 2/10$. If he reveals during the meeting that $\theta < 2/10$, the decision maker then plays a low action $y_L = 1/10$ that is optimal given the information.*

*But if the expert reveals during the meeting $\theta \geq 2/10$, then the informativeness of the subsequent report depends on whether the meeting was deemed to be a success or a failure. In the event of a failure, there is no further information contained in the subsequent report. The decision maker then plays the "pooling" action $y_P = 6/10$. In the event the meeting is a success, however, information in the written report results in a further division of the interval $[2/10, 1]$ into $[2/10, 4/10]$ and $[4/10, 1]$. In the first subinterval, the medium action $y_M = 3/10$ is taken and in the second subinterval the high action $y_H = 7/10$ is taken.*

*Notice that in state $2/10$ the expert prefers $y_L$ to $y_P$ and prefers $y_M$ to $y_L$. Thus, if there were no uncertainty about the outcome of the meetingfor instance, if all meetings were "successes", then it would be in the interests of the expert to say that $\theta \geq 2/10$ for states $\theta = 2/10 - \epsilon$. Similarly, if all meetings were failures, then for states $\theta = 2/10 + \epsilon$, it would be in the interests of the expert to say $\theta < 2/10$.*

*Clearly, there exists a probability $p$ such that in state $2/10$ the expert is indifferent between*

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ | $A_8$ | $A_9$ |
|---|---|---|---|---|---|---|---|---|---|
| $A_1$ | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| $A_2$ | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| $A_3$ | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| $A_4$ | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| $A_5$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| $A_6$ | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| $A_7$ | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| $A_8$ | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| $A_9$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |

Figure 3: A jointly controlled lottery

*$y_L$ and a $(p : 1 - p)$ lottery between $y_M$ and $y_P$. It is straightforward to verify that* $p = 5/9$*. A jointly controlled lottery can be used to guarantee that it is deemed to be a success with probability* $p = 5/9$ *(see Figure 3). It may be verified that this indeed constitutes an equilibrium. In particular, given the proposed play in the subgames, in every state* $\theta < 2/10$*, the expert prefers to send the message Low, and in every state* $\theta > 2/10$*, the expert prefers to send the message High. Moreover, given the strategies neither the expert nor the decision maker can affect the play in the subgame by strategically sending the various coordinating messages $A_i$ or $A_j$.*

*The equilibrium of the extended game constructed above conveys more information to the decision maker than any of the equilibria of the CS model. In face, it is ex ante Pareto superior. The ex ante expected payoff of the decision maker is now* $-36/1200$*, whereas that of the expert is* $-48/1200$*. In the most informative CS equilibrium, the ex ante expected payoff of the decision maker is* $-37/1200$*, whereas that of the expert is* $-49/1200$*. Moreover, Krishna and Morgan (2004) show that for almost all* $b < 1/8$*, there exists a perfect Bayesian equilibrium in the model with face-to-face communication which is Pareto superior to all*

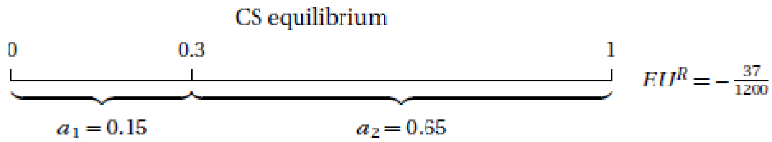


Figure 4: Cheap talk equilibrium with $b = \frac{1}{10}$

*equilibria in the CS model.*

The remarkable fact about the example is that this improvement in information transmission is achieved by adding a stage in which the uninformed decision maker also participates by injecting uncertainty into the resulting actions despite the fact that both parties are risk-averse. Similar ideas can be found in Blume, Board, and Kawamura (2007) as well.

### 1.4.2 Multiple Senders

It is natural to seek advice from more than one person. Having multiple senders allows the receiver to check facts. If the senders have access to identical information, the receiver expects identical messages. If the set of actions is sufficiently rich, then the receiver can respond to conflicting messages with an action that punishes both senders, inducing informative reporting in equilibrium.

Battaglini (2002) considers a model with one decision maker and two experts. The preferences are quadratic with biases $b_1 > 0$ and $b_2 < 0$. The state $\theta \in \Theta = [-W, W]$. The timing of the interaction is as follows: (a) at time 0 nature chooses $\theta$ according to $F(\theta)$ and each expert observes the true $\theta$; (b) at time 1 the experts are asked to report simultaneously or privately the state of nature $\theta$ to the decision-maker; (c) the decision-maker decides action $y$.

A strategy for the decision-maker is a function $y : \Theta \times \Theta \to Y$ that associates each couple of declarations of the experts to a decision in $Y$.

**Proposition 1** *$|b_1| + |b_2| > W$ is a necessary and sufficient condition for the nonexistence of a fully revealing equilibrium.*

If $|b_1| + |b_2| \leq W$, there is an entire class of equilibria that would be fully revealing. The following example is one of them. Senders report the truth in equilibrium. The decision-maker believes in the declaration if they are consistent; if $\theta_1 \leq \theta_2$, he believes that the state is $(\theta_1 + \theta_2)/2$; and if $\theta_1 > \theta_2$, he believes that the state is $W$ when $\theta_1 < 2|b_2| - W$ and $-W$ when $\theta_1 \geq 2|b_2| - W$.

## 1.5 Optimal Communication Mechanism

In this section, we will use the mechanism design approach to investigate the optimal communication mechanism for the receiver. The analysis follows Holmström (1984) and Melumad and Shibano (1991).

We will also consider a uniform-quadratic setting. State space $S = [0, 1]$, action space $A = [0, 1]$, message space $M = [0, 1]$. The prior is such that $s \sim U[0, 1]$.

The receiver can commit to a mechanism which is a mapping from the message space to the action space: $y : M \to A$. We do not allow any transfer from the receiver to the sender. This mechanism just assigns a deterministic action for each message sent by the sender.[2] By the revelation principle, we will focus on the direct mechanism such that the

[2] A deterministic mechanism means that $y(m)$ is a single point in $A$. Kovac and Mylovanov (2009) and Golstsman, Hörner, Pavlov, and Squintani (2009) also consider stochastic mechanisms, where $y(m)$ randomizes on $A$. However, it is shown by Golstsman, Hörner, Pavlov, and Squintani (2009) that the stochastic mechanism cannot do better than the deterministic mechanism in this setting.

sender truthfully reveals $s \in S$. This requires that $-(y(s) - s - b)^2 \geq -(y(s') - s - b)^2$ for any $s, s' \in S$.

**Lemma 1** $y(s)$ *is non-decreasing in* $s$.

**Proof.** Consider any $s \neq s'$. The IC condition implies that:

$$-(y(s) - s - b)^2 \geq -(y(s') - s - b)^2 \quad \text{and} \quad -(y(s') - s' - b)^2 \geq -(y(s) - s' - b)^2.$$

These two inequalities yield:

$$-(y(s) - s - b)^2 - (y(s') - s' - b)^2 \geq -(y(s') - s - b)^2 - (y(s) - s' - b)^2,$$

which is simplified as:

$$(y(s) - y(s'))(s - s') \geq 0.$$

Therefore, $y(s)$ is non-decreasing in $s$. ■

The following analysis is based on the assumption that $y(s)$ is continuous in $s$. Melumad and Shibano (1991) have shown that the optimal $y(s)$ must be continuous in $s$.

**Lemma 2** *Suppose* $y(s)$ *is continuous and strictly increasing in* $s$ *for* $s \in (s_1, s_2)$. *Then* $y(s) = s + b$ *for* $s \in (s_1, s_2)$.

**Proof.** Suppose on the contrary there exists $s$ such that $y(s) \neq s + b$. Without loss of generality, assume $y(s) > s + b$. Since $y(\cdot)$ is continuous and strictly increasing, there exists

$s' < s$ such that $s + b < y(s') < y(s)$. Then it is optimal for a sender under state $s$ to misreport $s'$. But this violates the IC condition! ■

The above lemma also implies that $y(\cdot)$ cannot be strictly increasing on two disjoint intervals. Any candidate incentive compatible communication mechanism $(y(s))_{s\in S}$ must have the following form: ($s_1$ could be 0 and $s_2$ could be 1)

$$y(s) = \begin{cases} s_1 + b & \text{if } s \leq s_1; \\ s + b & \text{if } s_1 < s < s_2; \\ s_2 + b & \text{if } s \geq s_2. \end{cases}$$

The receiver chooses $s_1 \geq 0$ and $s_2 \in [s_1, 1]$ to maximize:

$$W(s_1, s_2) = -\int_0^{s_1} (s_1 + b - s)^2 ds - \int_{s_1}^{s_2} b^2 ds - \int_{s_2}^{1} (s_2 + b - s)^2 ds.$$

It is shown that

$$\frac{\partial W}{\partial s_1} = -2\int_0^{s_1} (s_1 + b - s) ds < 0,$$

which implies that $s_1^\star = 0$. Similarly,

$$\frac{\partial W}{\partial s_2} = -2\int_{s_2}^{1} (s_2 + b - s) ds.$$

$\frac{\partial W}{\partial s_2} = 0$ at $s_2 = s_2^\star = 1 - 2b$ or $s_2 = 1$. However, the second derivative is negative at $s_2^\star$ but positive at 1. Therefore, $W$ is maximized at $s_2^\star = 1 - 2b$.

**Theorem 2** *The optimal communication mechanism takes the following form:*

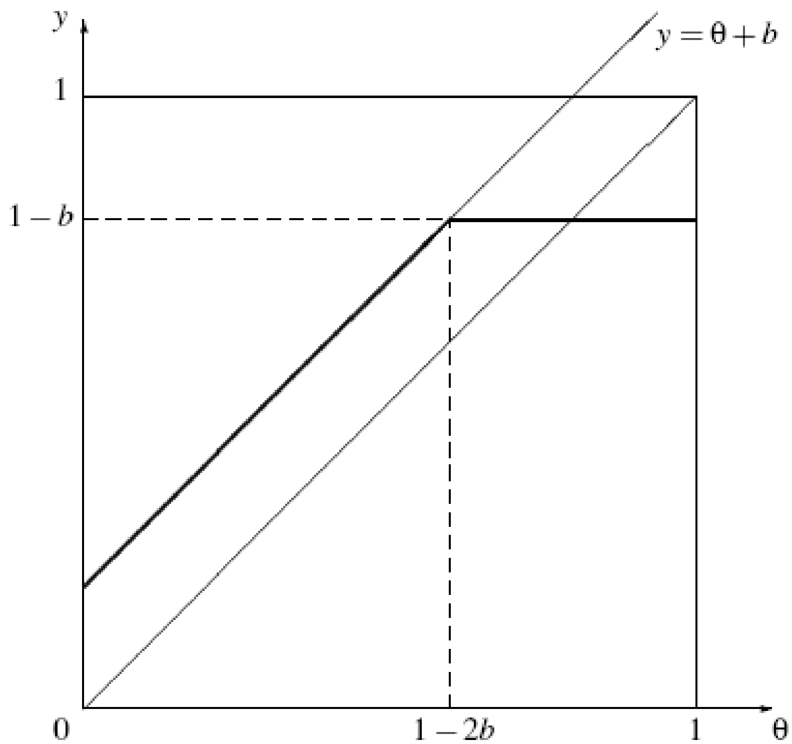


Figure 5: Optimal Communication Mechanism

$$y(s) = \begin{cases} s + b & \text{if } s < 1 - 2b; \\ 1 - b & \text{if } s \geq 1 - 2b. \end{cases}$$

The optimal communication mechanism can be implemented by delegation. The receiver delegates the action choice to the sender with a restriction on the set of actions. The sender is free to choose his mostly preferred action as long as $y \leq 1 - b$. Sometimes, delegation is subject to commitment problem: ex ante, the receiver cannot commit to let the sender choose his mostly preferred action ex post. In this situation, the optimal communication mechanism can also be implemented by arbitration. Under arbitration, the sender communicates privately or publicly with a neutral trustworthy arbitrator (e.g., the United Nations), who then chooses a final decision. We assume the arbitrator has the same preference as the receiver but has the commitment power to enforce a binding decision.

*Comparison to the cheap talk equilibria ($b = \frac{1}{10}$):*

When the sender's mostly preferred action is always chosen ($y(s) = s + b$), $EU^R = -b^2 = -\frac{1}{100} > -\frac{37}{1200}$. When the optimal communication is implemented, $EU^R = \frac{4}{3}b^3 - b^2 > -\frac{1}{100}$.

# 2 Disclosure Games: Verifiable Talk

In the cheap talk game, the sender is free to send any unverifiable signal. However, in the reality, information can be "verifiable" either because the receivers can directly check its accuracy or because there are institutions in place that effectively deter false claims by the senders. In other words, the message space $M$ depends on the state $s$. It turns out that some results in the cheap talk games do not hold any longer.

## 2.1 A Motivating Example

State space $S = \{0, 1\}$, message space $M = \{0, 1\}$, action space $A = [0, 1]$. The prior is such that each state happens with equal probability ($\frac{1}{2}$).

Payoffs are $U^S(a, s)$ and $U^R(a, s)$. In particular, we will use quadratic utility functions:

$$U^S(a, s) = -[a - (s + b)]^2 \quad \text{and} \quad U^R(a, s) = -[a - s]^2,$$

where $b > 0$ measures how nearly the S's interests coincide with the R's.

In the cheap talk game, there always exists a babbling equilibrium and if $b > \frac{1}{2}$, the "unique" equilibrium is the babbling equilibrium.

Disclosure game: We allow the sender to send a subset $m$ of $M$. The message $m$ has to be verifiable in the sense that it includes the true state $s$. For example, if the true state is 0, there are two possible messages to be sent: $\{0\}$ and $\{0,1\}$. If the true state is 1, there are also two possible messages to be sent: $\{1\}$ and $\{0,1\}$.

For S, a strategy $q(m = \{0,1\}|s)$ is the probability that S sends message $m = \{0,1\}$ when the true state is $s$.

For R, a strategy must specify an action $a(m) \in A$ for each message $m \subset M$. Also, we use $\beta(s|m)$ to denote the receiver's belief that the state is $s$ when he receives message $m$.

**Definition 3** *A triple $\{q, \beta, a\}$ is a sequential equilibrium if:*

1. *For every possible message $m \subset M$, $a(m)$ maximizes $\mathbb{E}[-(s-a)^2|\beta(\cdot|m)]$;*
2. *For every $s \in S$, if $m$ is in the support of $q(\cdot|s)$, then $m$ maximizes $-(s - a(m) - b)^2$;*
3. *Belief $\beta$ satisfies the structural consistency requirement: Supp$\beta(\cdot|m) \subset m$, and is updated by Bayes' rule.*

Notice that the babbling strategies (the sender sends $\{0,1\}$ irrespective of the state and the receiver always chooses $\frac{1}{2}$) is still a Nash equilibrium. But it is NOT a sequential equilibrium. Nash equilibrium only requires that for messages appeared on the equilibrium path, the receiver chooses the optimal action based on belief $\beta$. However, sequential equilibrium requires that for any possible message, the receiver is obliged to form a conjecture consistent with that message, and base his choice upon that conjecture. Since $m = \{1\}$ can be only sent at state 1, $\beta(1|m = \{1\}) = 1$ and hence $a(\{1\}) = 1$. As a result, the babbling strategies is not a sequential equilibrium because at state 1, the sender always would like to deviate to send message $\{1\}$. We say a strategy $q$ is full revealing if the supports at $s = 0$ and $s = 1$ have no intersection ($q(m = \{0, 1\}|s = 0)q(m = \{0, 1\}|s = 1) = 0$).

**Proposition 2** *For any $b > 0$, at every sequential equilibrium, the sender uses full-revealing strategy.*

**Proof.** In a sequential equilibrium, $\beta(1|m = \{1\}) = 1$ and therefore $a(\{1\}) = 1$ for sure. If $q(m = \{0, 1\}|s = 0)q(m = \{0, 1\}|s = 1) > 0$, $a(\{0, 1\}) < 1$ and the sender is indifferent between sending $\{1\}$ and $\{0, 1\}$ at state 1. This leads to a contradiction. ■

In particular, if $b > \frac{1}{2}$, the sender will always send $\{1\}$ at state 1 and the belief at $m = \{0, 1\}$ is *skeptical*: $\beta(s = 0|\{0, 1\}) = 1$.

## 2.2 Skepticism and Unraveling

The following analysis is based on Milgrom (1981) and Milgrom (2008). There are two players: an informed seller and an uninformed buyer. The seller has private information about the state of the world $\theta$, which belongs to a finite set $\{1, \cdots, N\}$. Higher values of $\theta$ represent better quality. The seller's only move in the game is to make a report about $\theta$ to the otherwise uninformed buyer, who then makes a purchase decision. The report has to be verifiable in the sense that $\theta \in m(\theta)$.

The buyer chooses the quantity of purchase $q$ based on the report. We assume that the seller's payoff is given by $v(q)$ where $v$ is strictly increasing in $q$ and the buyer's payoff is given by $u(\theta, q)$, where $u$ is strictly concave and differentiable in $q$. For example, we may assume that $v(q) = Pq$ and $u(\theta, q) = \theta u(q) - Pq$. There is a unique interior solution $q^\star(\theta)$ maximizing

$u(\theta, q)$ and $q^\star(\theta)$ is strictly increasing in $\theta$. This is satisfied when $u(\theta, q) = \theta u(q) - Pq$ if we impose the Inada condition on $u(\cdot)$.

The seller's reporting strategy is denoted as $\alpha(m|\theta) \in [0, 1]$ where $m \subset \{1, \cdots, N\}$ and the buyer's purchasing strategy is $q(m)$. The belief function is $\beta(\theta|m)$ for each $\theta \in m$. The equilibrium $(m, q, \beta)$ is a fully-revealing equilibrium if

$$\alpha(m|\theta) > 0 \Longrightarrow \alpha(m|\theta') = 0, \forall \theta' \neq \theta.$$

We will focus on sequential equilibrium where:

1. for every $m$ in the support of $\alpha(\cdot|\theta)$, $m$ maximizes $q(m)$;
2. for every $m \subset \{1, \cdots, N\}$,
$$q(m) \in \text{argmax}_q \mathbb{E}[u(\theta, q)|\beta(\cdot|m);$$
3. belief $\beta$ satisfies the structural consistency requirement: Supp$\beta(\cdot|m) \subset m$, and is updated by Bayes' rule whenever that applies.

Denote $L(m) = \min\{\theta|\theta \in m\}$. The belief function $\beta$ is skeptical if $\beta(L(m)|m) = 1$ for every non-empty $m$.

**Theorem 3** *Every sequential equilibrium must be fully revealing and the belief $\beta$ in every sequential equilibrium is skeptical.*

The argument used to prove the above statement is commonly called the unraveling argument. The usual presentation first shows that the highest quality sellers always make reports of quality that distinguish their products from all others, and then the remaining sellers face a similar game. The next highest quality sellers therefore report quality levels to distinguish themselves from lower-quality types, and the process repeats itself. For this argument to work, it must be common knowledge that a seller *can* distinguish its product from lower-quality products and sellers must *benefit* by doing so. The latter condition is guaranteed since $q^\star(\theta)$ is strictly increasing in $\theta$. Given the equilibrium is fully revealing, it is straightforward to see that the belief $\beta$ is skeptical. If not, there exists some $m$ such that $\mathbb{E}(\theta|m) > L(m)$. Then a seller with $\theta = L(m)$ has incentive to report $m$ but this equilibrium is not fully revealing.

## 2.3 Uncertainty Mutes Skepticism

The unraveling result changes when we introduce uncertainty about whether the seller is able to supply verifiable information. In the above example, if there is some probability that the seller has no information about $\theta$, then fully revelation cannot be an equilibrium. The intuition is the following: news that a product is very bad is strictly worse than no news and leads the buyer to purchase a smaller quantity. Consequently, in equilibrium, the seller always withholds very bad news and reports only relatively good news. The buyer still casts a skeptical eye on missing information, but the skepticism is muted because the buyer is unsure about what information the seller could have reported.

Suppose that $\theta$ in the above example is uniformly distributed on $[0, 1]$ and the probability that the seller is able to supply verifiable information is $\gamma$. Then in equilibrium, there exists $\bar{\theta} \in (0, 1)$ such that seller with $\theta > \bar{\theta}$ will fully reveal the true $\theta$ while seller with $\theta \leq \bar{\theta}$ will hide the information and just claim that the information is non-verifiable.

$\bar{\theta}$ is set such that seller with type $\bar{\theta}$ is just indifferent between revealing and not revealing information. This gives us that:

$$\frac{\gamma\bar{\theta}}{\gamma\bar{\theta}+(1-\gamma)}\frac{\bar{\theta}}{2}+\frac{1-\gamma}{\gamma\bar{\theta}+(1-\gamma)}\frac{1}{2}=\bar{\theta}$$

and hence

$$\bar{\theta}=\frac{\sqrt{1-\gamma}-(1-\gamma)}{\gamma}=\frac{1-\gamma}{\sqrt{1-\gamma}+(1-\gamma)}\in(0,1).$$

Obviously, $\bar{\theta}(\gamma)\in(0,1)$ with $\lim_{\gamma\to 0}\bar{\theta}(\gamma)=\frac{1}{2}$ and $\lim_{\gamma\to 1}\bar{\theta}(\gamma)=0$. $\bar{\theta}(\gamma)$ is decreasing in $\gamma$: the higher the probability of verifiability, the higher the probability of voluntary revelation as well.

## 2.4 Commitment by the Sender

In many cases, the sender can *commit* to an information disclosure rule. Examples include:

- Internet platform (for example, Taobao) can provide some information about the sellers to the buyers. The platform, however, may care not just about buyer welfare, but about its own profits. And the problem is to design a rule to rate the sellers.
- A bond rating agency chooses a rule about what information to disclose to investors about bond issuers, who also make payments to the agency for the rating.
- A school chooses what information to disclose to prospective employers about the ability of its students, who also pay tuition to the school.

As shown by Rayo and Segal (2010) and Kamenica and Gentzkow (2011), the profit-maximizing disclosure rule may be partially but not fully revealing when sender can make such a commitment.

### 2.4.1 A Motivating Example

Suppose the judge (Receiver) must choose one of two actions: to acquit or convict a defendant. There are two states of the world: the defendant is either guilty or innocent. The judge gets utility 1 for choosing the just action (convict when guilty and acquit when innocent) and utility 0 for choosing the unjust action (convict when innocent and acquit when guilty). The prosecutor (Sender) gets utility 1 if the judge convicts and utility 0 if the judge acquits, regardless of the state. The prosecutor and the judge share a prior belief $\Pr(guilty) = 0.3$.

The prosecutor conducts an investigation and is required by law to report its full outcome. We can think of the choice of the investigation as consisting of the decisions on whom to subpoena, what forensic tests to conduct, what questions to ask an expert witness, etc. We formalize an investigation as distributions $\pi(\cdot|guilty)$ and $\pi(\cdot|innocent)$ on some set of signal realizations. The prosecutor chooses $\pi$ and must honestly report the signal realization to the judge.

If the prosecutor chooses a fully informative investigation, one that leaves no uncertainty about the state, the judge convicts 30 percent of the time. The prosecutor can do better, however. For example, he can choose the following binary signal $\{i, g\}$ such that:

$$\pi(g|guilty) = 1 \quad \text{and} \quad \pi(g|innocent) = \frac{3}{7}.$$

This leads the judge to convict with probability 60 percent. Note that the judge knows 70 percent of defendants are innocent, yet she convicts 60 percent of them! She does so even though she is fully aware that the investigation was designed to maximize the probability of conviction.

### 2.4.2 A General Model

We develop a model to study the general problem of persuading a rational agent by controlling her informational environment. State space $S = \{0, 1\}$ and two actions $A = \{0, 1\}$. The prior is such that state $s = 0$ happens with probability $\mu_0$ and state $s = 1$ happens with probability $\mu_1 = 1 - \mu_0$. Under state $s$, the action $a = 1$ generates payoffs $u_s$ and $v_s$ to the sender and the receiver, respectively, The action $a = 0$ always generates payoff zero to the sender and always generates payoff $\bar{v}$ to the receiver.

There are two cases to consider

1. $\bar{v}$ is a constant and is common knowledge to both players (considered by Kamenica and Gentzkow (2011));
2. $\bar{v}$ is random and only known to the receivers when the sender designs the disclosure rule (considered by Rayo and Segal (2010)).

**Definition 4** *A information disclosure rule* $< \pi, \Omega >$ *consists of a finite set* $\Omega$ *of signals and a mapping* $\pi : S \to \Delta(\Omega)$ *that assigns to each state* $s$ *a probability distribution* $\pi(\cdot|s)$ *over* $\Omega$*.*

Given a disclosure rule, each signal realization $\omega$ leads to a posterior belief $\mu_\omega \in \Delta(S)$. Accordingly, the disclosure rule leads to a distribution over posterior beliefs $\tau \in \Delta(\Delta(S))$.

By Beyes rule, we have

$$\mu_\omega(s) = \frac{\pi(\omega|s)p_s}{\sum_{s'} \pi(\omega|s')p_{s'}}$$

and

$$\tau(\mu) = \sum_{\omega:\mu_\omega=\mu} \sum_{s'} \pi(\omega|s')p_{s'}.$$

A distribution of posteriors is Bayes plausible if the expected posterior probability equals the prior:

$$\sum_{Supp(\tau)} \mu\tau(\mu) = \mu_0.$$

Given $\mu_\omega$, the receiver will choose $a = 1$ if $\mu_\omega(s = 1)v_1 + \mu_\omega(s = 0)v_0 \geq \bar{v}$. Therefore, we can calculate the receiver's optimal choice $\hat{a}(\omega, \bar{v})$ and the sender's expected payoff $\hat{u}(\omega)$ for each signal $\omega$. The sender's expected payoff is

$$u(\pi, \Omega) = \sum_{\omega} \sum_{s} \hat{u}(\omega)\pi(\omega|s)p_s.$$

We can simplify the analysis further by noting that, without loss of generality, we can restrict our attention to a particular class of signals. Say that a disclosure rule is *straightforward* if $\Omega = A = S$ and the receiver's equilibrium action equals the signal realization. In other words, a straightforward disclosure rule produces a "recommended action" and the receiver always follows the recommendation.

**Lemma 3** *If there exists information disclosure rule with expected payoff* $u(\pi, \Omega)$*, then there* *must exist a straightforward information disclosure rule which generates the same expected payoff.*

### 2.4.3 Case 1: $\bar{v}$ is a constant

From the discussions above, we can focus on straightforward information disclosure rules. Denote $\mu$ to be the posterior that $s = 0$. Let $\bar{v}$ to be such that $\bar{v} \in (v_0, v_1)$. Then, there exists $\bar{\mu}$ such that $a = 1$ is chosen if and only if the posterior $\mu$ is smaller than $\bar{\mu}$. The sender's expected payoff under posterior $\mu$ is $\hat{v}(\mu) = \mu u_0 + (1 - \mu)u_1$ if $\mu \leq \bar{\mu}$ and is zero otherwise.

Let $\mu^1$ ($\mu^0$) be the posterior that $s = 0$ when $\omega = 1$ ($\omega = 0$). Then the optimal disclosure rule is to solve the following optimization problem:

$$u^\star = \max_{\tau,\mu} \tau_1 \hat{v}(\mu^1) + \tau_0 \hat{v}(\mu^0)$$

subject to

$$\mu^1 \tau_1 + \mu^0 \tau_0 = \mu_0 \quad \text{and} \quad \tau_1 + \tau_0 = 1.$$

Notice that the sender can also choose to not reveal any information. In this case, the payoff is just $\hat{v}(\mu_0)$. The problem is to characterize $u^\star$ and find conditions such that $u^\star > \hat{v}(\mu_0)$.

Given $\hat{v}(\mu)$, denote $V(\mu)$ be the concave closure of $u$, which is the smallest concave function that is everywhere weakly greater than $\hat{v}(\cdot)$. The following Figure 5 provides an illustration.of the construction of $V$. It turns out that $V(\mu_0)$ is the largest payoff the sender can achieve with any information disclosure rule when the prior is $\mu_0$.

**Proposition 3** *Given prior $\mu_0$, $u^\star$ must be $V(\mu_0)$ and revealing information is better than non-revealing if and only if $V(\mu_0) > \hat{v}(\mu_0)$.*

Intuition: for any point in $co(\hat{v}) = (\mu', z)$, there exists a distribution of posteriors such that $\mu^1\tau_1 + \mu^0\tau_0 = \mu'$ and $\tau_1\hat{v}(\mu^1) + \tau_0\hat{v}(\mu^0) = z$. Hence, $V(\mu_0)$ is the largest payoff the sender can achieve under prior $\mu_0$.

If $\hat{v}$ is concave, the sender does not benefit from persuasion for any prior. If $\hat{v}$ is convex and not concave, the sender benefits from persuasion for every prior. If $\hat{v}$ is (strictly) concave, no disclosure is (uniquely) optimal, and if $\hat{v}$ is (strictly) convex full disclosure is (uniquely) optimal.

In the motivating example, we have $u_0 = u_1 = 1$, $v_1 = 1$, $v_0 = -1$ and $\bar{v} = 0$. It is straightforward to verify that $\bar{\mu} = 0.5$ and revealing information is better than non-revealing if and only if $p_0 < 0.5$. The concave closure $V$ in this example is illustrated by Figure 6.

**Application: Lobbying.** Consider a setting where a lobbying group commissions a study with the goal of influencing a benevolent, but nonetheless rational, politician. The politician (Receiver) chooses a unidimensional policy $a \in [0, 1]$. The state $s \in [0, 1]$ is the socially optimal policy. The lobbyist (Sender) is employed by the interest group whose preferred action is $a^\star = \alpha s + (1-\alpha)s^\star$ with $\alpha \in [0, 1]$ and $s^\star > 1$. Politician's payoff $-(a-s)^2$ and lobbyist's payoff $-(a - a^\star)^2$.

It is straightforward to verify that

$$\hat{v}(\mu) = -(1-\alpha)^2(s^\star)^2 + 2(1-\alpha)^2 s^\star \mathbb{E}_\mu s - \alpha^2 \mathbb{E}_\mu s^2 + (2\alpha - 1)(\mathbb{E}_\mu s)^2.$$

$\hat{v}$ is linear in $\mu$ if $\alpha = 1/2$, strictly convex if $\alpha > 1/2$, and strictly concave when $\alpha < 1/2$.

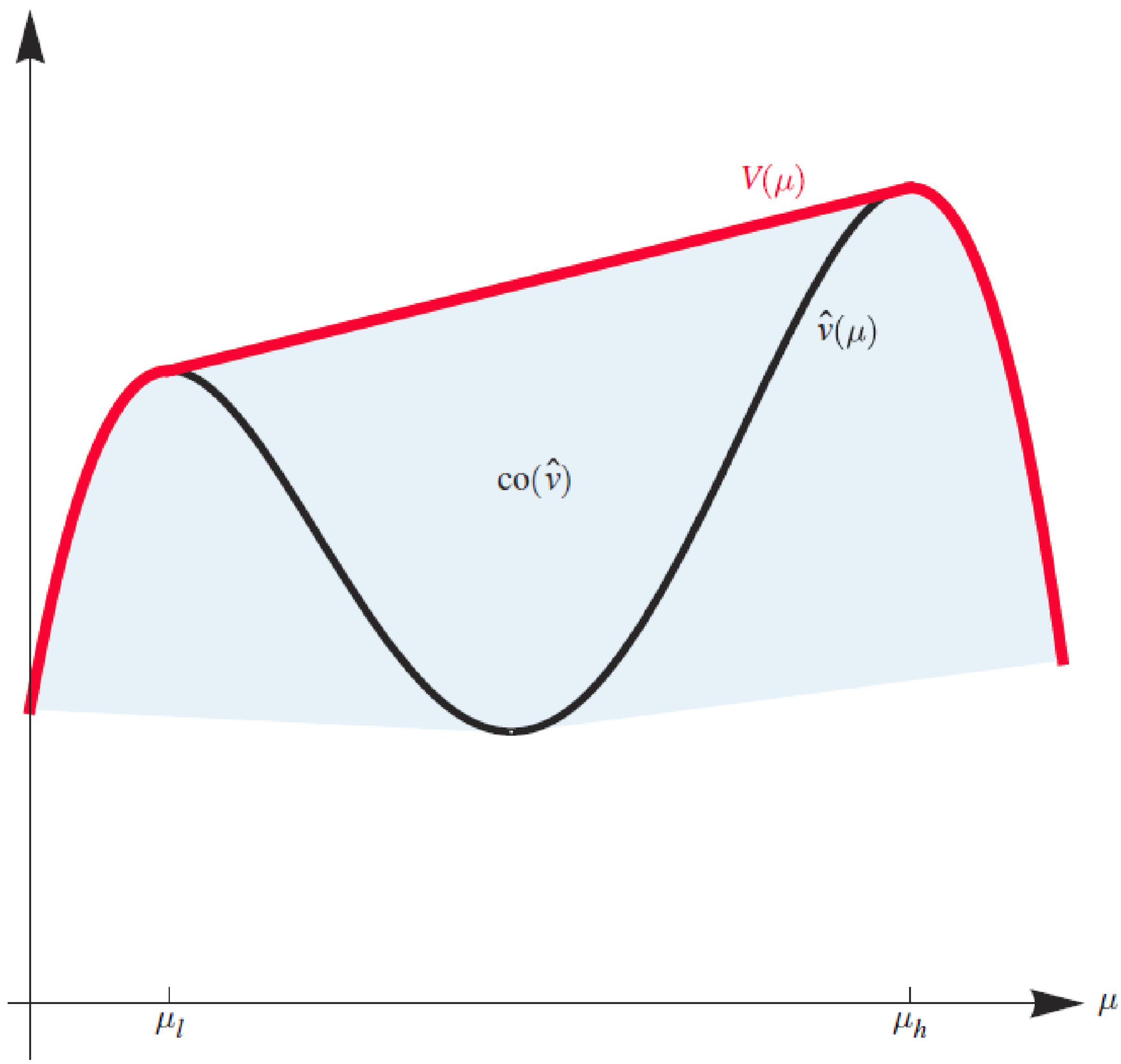


Figure 6: An Illustration of Concave Closure

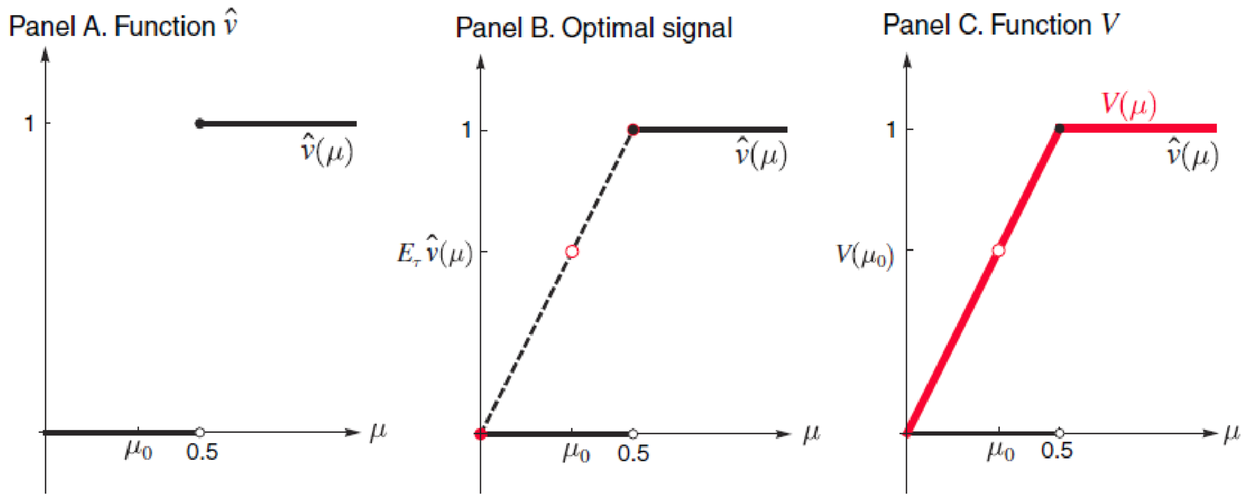


Figure 7: The Motivating Example

Therefore we have full disclosure if $\alpha > 1/2$ and no disclosure if $\alpha < 1/2$. Thus theory suggests that lobbyist either commissions a fully revealing study or no study at all.

### 2.4.4 Case 2: $\bar{v} \sim U[0, 1]$

Denote $\mu$ to be the posterior that $s = 1$. When $\bar{v} \sim U[0, 1]$, then under posterior $\mu$, the probability of choosing $a = 1$ is $\mu v_1 + (1 - \mu)v_0$ (we assume that $v_0, v_1 \in [0, 1]$ to make sure that this term is well-defined). Then we can write down $\hat{v}(\mu)$ as:

$$\hat{v}(\mu) = (\mu v_1 + (1 - \mu)v_0)(\mu u_1 + (1 - \mu)u_0).$$

**Definition 5** *Two states $i, j$ are ordered if either $(u_i, v_i) \geq (u_j, v_j)$ or $(u_i, v_i) \leq (u_j, v_j)$. The two states are unordered if otherwise. The two states are strictly ordered if they are ordered and not unordered; they are strictly unordered if they are unordered and not ordered.*

**Proposition 4** *Pooling two states yields (strictly) higher payoffs for the sender than sep-*

*arating them if the states are (strictly) unordered, and yields (strictly) lower payoffs if the states are (strictly) ordered.*

Intuition: The $\hat{v}$ function is convex if the states are ordered, and concave if the states are unordered.

## 2.5 Conclusion

When buyers are sophisticated, markets provide powerful incentives for sellers to supply useful and verifiable product information. Such incentives can sometimes go a long way toward alleviating problems of adverse selection. Yet sellers may still have an incentive to hide information. Regulation, especially in the form of creating liability for withholding material information, can help to mitigate the costs of nondisclosure. In practice, there are many other different problems which may require different solutions by regulator.

First, the sellers may test selectively, based on their unverifiable suspicions about which tests will make their products look good. In the pharmaceutical industry, the seller can use its unverifiable private information to make proper decisions about which tests to perform. What is needed in this setting is to hold the seller liable for failures to reveal promptly not only the verifiable information that the seller knew, but also the information that it should have known under the circumstances. Such a regulation can work only if it is eventually possible to establish what the seller should have known and when it should have known it.

Second, the consumers don't know enough about the relevant product even to ask the most relevant questions. Sometimes, the buyer is able to name some characteristics and asks the seller for information about a characteristic that the buyer can name. Then the seller can convey some potentially useful information. One possibility that an expert regulator could, by requiring product labeling for relevant characteristics, encourage more useful and informative disclosures. With too much consumer heterogeneity, however, the labeling solution fails, because the regulator does not know which characteristic needs to be reported. Reporting too much information in this situation leads to information overload, in which the buyer may fail to notice the most relevant information.

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