Learning and Efficiency with Search Frictions*

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Abstract

This paper studies the allocative efficiency in a Moscarini (2005)-type equilibrium search environment with learning. It is shown that the stationary equilibrium is efficient if and only if the Hosios condition holds no matter whether learning is about firm-specific human capital or about general human capital. However, the stationary equilibrium can never be efficient if there exists externality for being unemployed. In contrast, even with externality, the stationary equilibrium can be efficient under some modified Hosios condition if there is no uncertainty (standard Mortensen and Pissarides (1994)-type equilibrium search environment). The key intuition is that the equilibrium can be efficient if the firm-worker matching is formed and terminated efficiently but can never be efficient otherwise.

Keywords. Learning. Efficiency. Search Friction. Nash Bargaining. Firm Specific Human Capital. General Human Capital.

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1 Introduction

Ever since Jovanovic (1979), learning has been incorporated in the standard labor search model to explain job turnovers and tenure effects. Learning and job matching are integrated into a Mortensen and Pissarides (1994)-type equilibrium search environment nicely by Moscarini (2005).[1] Moscarini (2005) analyzes a situation where a firm-worker pair is learning about the firm-specific human capital of the worker through noisy Gaussian output. The unique stationary competitive search equilibrium can be explicitly solved, featuring endogenous market tightness and ergodic wage distribution. However, it is still an open question about the efficiency of such equilibrium.

From Hosios (1990), it is well known that in a Mortensen and Pissarides (1994)-type environment where wages are determined by Nash Bargaining, the competitive search equilibrium is efficient if and only if the Hosios condition is satisfied. Under this condition, the search externality can be internalized. This result can be generalized to endogenous search intensity, firm-specific productivity, and so on (see, e.g., Pissarides (2000), Mortensen and Wright (2002), Rogerson, Shimer, and Wright (2005)). However, in general, Hosios condition may not guarantee efficiency as shown by Acemoglu and Shimer (1999).

This paper first shows that in a Moscarini (2005) -type environment, the Hosios condition is sufficient and necessary to guarantee efficiency. The intuition is simple. In such an environment, Nash bargaining does not cause additional inefficiency on the learning part. As a consequence of the (private) efficiency of the Nash solution, the interests of the firm and the worker are aligned with the total surplus. This implies that learning is terminated efficiently under Nash bargaining. As a result, the Hosios condition is able to restore efficiency because it fully internalizes the search externality. This is different from Acemoglu and Shimer (1999), where the Hosios condition can never restore efficiency because Nash bargaining causes additional inefficiency due to the hold-up problem.

To better illustrate the above intuition, the paper further considers two extensions of the model. In the first extension, there is an "externality" for being unemployed such that the social value of being unemployed is different from the private value of being unemployed. The stationary competitive search equilibrium is no longer efficient under all parameter values. However, if we merely introduce such "externality" into a Mortensen and Pissarides (1994)-type environment, the

[1] Gonzalez and Shi (2010) discuss an alternative model where learning is about the worker's job-finding ability.

competitive search equilibrium can be efficient under a modified Hosios condition. The above intuition goes through as well since learning is always terminated inefficiently as the "externality" exists while the firm-worker pair still forms efficiently as long as the positive externality for being unemployed is not too large. In the second extension, the firm-worker pair is learning about the *general* instead of *firm-specific* human capital. It seems that some "externality" might exist since the information learned in a firm can be used by other firms. However, we show that learning is also terminated efficiently due to the nature of Nash bargaining. As a result, the stationary competitive search equilibrium is efficient as well if the Hosios condition holds. We conclude from these two examples that whether the firm-worker pair could be formed and terminated efficiently is crucial for restoring efficiency.

The paper is related to macroeconomic search models of the labor market, which use Nash bargaining to determine the wages (see, e.g., Hall (2005), Pries and Rogerson (2005), Shimer (2005), Hagedorn and Manovskii (2008), Rogerson and Shimer (2011)).[2] When calibrating such models, the usual strategy is to choose the bargaining weight in a way that guarantees the efficiency of the model (i.e., to satisfy the Hosios condition). Hagedorn and Manovskii (2008) question this identification strategy and show that in a model with tax, we could choose a different bargaining weight which reduces the inefficiency. Although our paper focuses on efficiency in the presence of learning, the general intuition seems to hold in more general environments. As long as the firm-worker matching is formed and terminated efficiently, the competitive search equilibrium can be efficient under some conditions (may not be the same as the Hosios (1990) condition). Otherwise, the competitive search equilibrium can never be efficient.

The remaining of the paper is organized as the following. Section 2 sets up the baseline model. The setup is general enough to include both the firm-specific human capital case and the general human capital case. Section 3 discusses the efficiency of the stationary equilibrium in the baseline model. In this section, we also consider an extension where there is externality for being unemployed and discuss the efficiency of the stationary equilibrium in this situation. Section 4 investigates the efficiency of the stationary equilibrium when the firm-worker pair is learning about the general human capital. The last section concludes the paper.

[2]Pries and Rogerson (2005) argue without formal proof that the Hosios condition applies to an environment of learning about firm-specific human capital. Their specification of learning is very special in the sense that it reveals either everything or nothing.

2 The Model Economy

Population of Firms and Workers. The economy is populated by a unit measure of workers and a sufficiently large measure of long-lived firms, ensuring free entry. Both firms and workers are *ex ante* homogeneous. One firm and one worker is matched together to produce a consumption good. Workers die with exogenous probability δ. New workers are born at the same rate.

Preferences. Workers and firms are risk-neutral and discount future payoffs at rate $r = 0$. The exogenous death rate hence plays the role of discounting.[3] Utility is perfectly transferable. A jobless worker enjoys a flow unemployment benefit b while vacant firms get zero flow returns. The firm must pay a flow sunk cost $k > 0$ to keep a vacancy open to applications from unemployed workers. A jobless worker contacts an open vacancy at finite Poisson arrival rate. The arrival rate is determined by a homogeneous of degree one matching function $m(u, v) = u^{\theta}v^{1-\theta}$, where u denotes the measure of unemployed workers and v denotes the measure of vacant firms. According to this matching function, the matching rate for an unemployed worker is $\lambda = q^{1-\theta}$ and for a vacant firm is $q^{-\theta}$, where $q = \frac{v}{u}$ denotes the market intensity.

Production. The cumulative output of a match of duration t, X_t, follows a Brownian motion with drift μ and known variance σ^2:

$$X_t = \mu t + \sigma Z_t \sim N(\mu t, \sigma^2 t).$$

The average productivity or "quality" of each match, μ, is ex ante uncertain for both the worker and the firm. μ has two possible values $\mu_H > \mu_L$. Without loss of generality, we normalize $\mu_H = 1$ and $\mu_L = 0$. A worker's past history of outputs is observable to all firms.

The common belief is that for a new worker, with probability p_0, $\mu = \mu_H$ and with complementary probability, $\mu = \mu_L$. To make learning non-trivial, we assume $0 < b < p_0$. Therefore, a new match should always be accepted, because it produces more than the joint value of inactivity b, and should be dissolved if it is common knowledge that $\mu = \mu_L$

Firm Specific vs. General Human Capital. In the paper, we will investigate two different cases. When human capital is match specific, a worker's average productivity in a certain firm is independent of the average productivities in other firms. Upon matching, the firm and the worker always

[3] It is important to assume away discounting such that the social planner's problem can be written as a static maximization problem.

share the common prior belief p_0 on μ, independent of their past history. When human capital is general, a worker's average productivity in a certain firm is the same as the average productivities in other firms. By observing a worker's past production history, all firms share a common posterior belief p on this worker's μ. But this belief p may not be p_0.

Belief Updating. Based on the realized output, both the firm and the worker will update the posterior belief that $\mu = \mu_H$. Standard results imply that the posterior belief p_t follows another Brownian motion:

$$dp_t = sp_t(1-p_t)d\bar{Z}_t,$$

where $s = 1/\sigma$ and $\bar{Z}_t$ is a standard Brownian motion.

Wage Determination. Firms and workers cannot commit to a wage contract. The spot wage is determined according to a generalized Nash bargaining rule, assigning a geometric weight β to the worker's surplus.

3 Firm Specific Human Capital

3.1 Stationary Equilibrium

We analyze the stationary equilibrium of this economy, where the workers and the firms have stationary value functions with posterior belief p as the unique state variable. Aggregate variables (including the wage distribution, the matching rates, etc) do not change over time. But a matching pair may dissolve due to the idiosyncratic death shock or endogenous separation decisions. The analysis in this subsection directly follows the one in Moscarini (2005).

Let $W(p)$ denote the total discounted value that a worker receives in equilibrium, when employed in a match with posterior belief p. Let U denote the worker's value of unemployment, independent of p because of the firm-specific nature of μ. Similarly, we use $J(p)$ to denote the value of the firm, and finally V denotes the value to the firm of holding an open vacancy. Free entry implies that $V = 0$.

Denote $\Sigma(p) = \frac{1}{2}s^2p^2(1-p)^2$. The value functions can be written as (notice that $r = 0$):

$$\delta U = b + \lambda(W(p_0) - U); \tag{1}$$

$$\delta W(p) = w(p) + \Sigma(p) W''(p); \tag{2}$$

and

$$\delta J(p) = p - w(p) + \Sigma(p) J''(p). \tag{3}$$

The generalized Nash bargaining solution selects a wage:

$$w(p) \in \text{argmax}_w [W(p) - U]^{\beta} [J(p)]^{1-\beta}.$$

The first order condition implies that

$$\beta J(p) = (1 - \beta)[W(p) - U], \tag{4}$$

which yields a wage formula:

$$w(p) = (1 - \beta) b + \beta [p + \lambda J(p_0)]. \tag{5}$$

The derivation of the wage formula is similar to the one in Moscarini (2005) and hence is omitted here. Plug equation 5 into equation 3 and we get a second-order differential equation about firm's value J:

$$\delta J(p) = (1 - \beta)(p - b) + \Sigma(p) J''(p) - \beta \lambda J(p_0). \tag{6}$$

The firm-worker pair will agree to separate when the posterior belief hits the same threshold $\underline{p}$. Value matching and smooth pasting conditions apply at $\underline{p}$:

$$J(\underline{p}) = 0, \quad J'(\underline{p}) = 0. \tag{7}$$

Also free entry implies that:

$$0 = -k + q^{-\theta} J(p_0) \Longrightarrow J(p_0) = k q^{\theta}. \tag{8}$$

As in Moscarini (2005), a stationary equilibrium is defined as a vector of functions (J, w) satisfying equations 5 and 6, and a vector of scalars $(\underline{p}^e, q^e)$ satisfying equations 7 and 8.[4]

Denote $\gamma = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2\delta}{s^2}}$. Equations 6 to 8 yield a unique pair of separation cutoff $\underline{p}^e$ and market intensity q^e in the stationary equilibrium:

Proposition 1 *The stationary equilibrium $(\underline{p}^e, q^e)$ is uniquely determined by the following system of equations:*

$$\underline{p} = \frac{(\gamma - 1)[(1-\beta)b + \beta qk]}{(1-\beta)(\gamma - b) - \beta qk} \tag{9}$$

$$kq^\theta = -\frac{[(1-\beta)(\underline{p} - b) - \beta qk]p_0^{1-\gamma}(1-p_0)^\gamma}{\delta \underline{p}^{1-\gamma}(1-\underline{p})^\gamma} + \frac{(1-\beta)(p_0 - b) - \beta qk}{\delta}. \tag{10}$$

The proof of Proposition 1 can be found in Moscarini (2005). This result implies that in the stationary equilibrium, the firm-worker pair will dissolve when the posterior belief reaches $\underline{p}^e$. Once the worker is fired, the worker can still find other jobs because the human capital is firm specific.

Denote $S(p) = W(p) + J(p)$ to be the total surplus of a firm-worker match with posterior belief p. From equation 4, we have: at the equilibrium cutoff $\underline{p}$, $S(p) = U$ and $S'(p) = 0$. This implies that the pair is dissolved efficiently. In other words, the generalized Nash bargaining does not lead to any inefficiency on the learning part.

3.2 Socially Optimal Allocation

We consider a planner's problem under stationarity, i.e., in the presence of an ergodic distribution. The planner chooses an allocation rule $(\underline{p}, u, v = uq)$ to maximize the aggregate steady state output, which consists of three parts: 1) the outputs produced by the firm-worker pairs; 2) the outputs produced by the unemployed workers; and 3) the costs of the vacant firms.

To evaluate the aggregate steady state output, the key is to compute the outputs produced by the firm-worker pairs, which are determined by the ergodic distribution of posterior beliefs about match quality. As shown by Moscarini (2005), the stationary and ergodic density $f(p)$ of posterior beliefs is determined by the allocation rule. From the Fokker-Planck (Kolmogorov forward) equation, the stationary and ergodic density $f(p)$ should satisfy the following differential equation:

[4]As shown in Moscarini (2005), given $\underline{p}^e$, we can uniquely pin down the ergodic distribution of posterior beliefs about match quality. Using that ergodic distribution, we can further compute the equilibrium measure of unemployed workers u^e and vacant firms v^e. Therefore, in the characterization of the stationary equilibrium, it is adequate to calculate the equilibrium separation cutoff $\underline{p}^e$.

$$0 = \frac{df(p)}{dt} = \frac{d^2}{dp^2}[\Sigma(p)f(p)] - \delta f(p). \tag{11}$$

The general solution to this differential equation is (see also Moscarini (2005)):[5]

$$f(p) = [C_1 p^{-1-\gamma}(1-p)^{\gamma-2} + C_2(1-p)^{-1-\gamma}p^{\gamma-2}]. \tag{12}$$

The integrability of $f(p)$ requires that $C_1 = 0$ if the infimum of supp$f$ is 0 and $C_2 = 0$ if 1 the supremum of supp$f$ is 1. The Fokker-Planck (Kolmogorov forward) equation is only valid for $p \neq p_0$. Since there is a flow in of new workers, for $p = p_0$ we should have a kink in the density function.

Suppose the social planner chooses $\underline{p}$ as the cutoff: the firm-worker pair will dissolve once the posterior reaches $\underline{p}$. Then, the ergodic density function is given by:

$$f(p) = [C_1 p^{-1-\gamma}(1-p)^{\gamma-2} + C_2(1-p)^{-1-\gamma}p^{\gamma-2}]\mathbb{I}(\underline{p} < p \leq p_0) + C_3 p^{-1-\gamma}(1-p)^{\gamma-2}\mathbb{I}(p > p_0). \tag{13}$$

The density functions are subject to the following boundary conditions (See also Moscarini (2005)).

First, $f(\underline{p}+) = 0$. Second, the density function is contiunous at p_0:

$$C_1 p_0^{-1-\gamma}(1-p_0)^{\gamma-2} + C_2(1-p_0)^{-1-\gamma}p_0^{\gamma-2} = C_3 p_0^{-1-\gamma}(1-p_0)^{\gamma-2}.$$

Since $r = 0$, the objective is to maximize the aggregate static output at the steady state.[6] Formally, the planner chooses $(\underline{p}^\star, u^\star, q^\star)$ to solve the problem:[7]

$$\max_{\underline{p},u,q} \int_{\underline{p}}^1 pf(p)dp + ub - uqk,$$

[5]Here the assumption that there is no heterogeneity in the prior p_0 substantially simplifies the solution to this differential equation. While there is no solution for a general distribution of priors, we have been able to solve the stationary distribution if the priors are drawn from a beta distribution. See Papageorgiou (2010) for an alternative model.

[6]If $r \neq 0$, the social planner's dynamic problem needs to add the density f as the state variable, which is very hard to solve.

[7]The objective function of the planner's problem seems only reflect the flow payoff of the economy. However, the planner's optimization also takes care of the inefficient delay of trade (match formation) due to the presence of the search frictions, which is reflected in the ergodic distribution of $f(p)$ and u.

$$\text{s.t. } uq^{1-\theta} = \delta(1-u) + \Sigma(\underline{p})f'(\underline{p}+).$$

The constraint comes from the fact that the total flows in and out of employment must balance. By some algebra, we can further simplify the planner's problem as the following:

Claim 1 *The planner's problem can be reformulated as:*

$$\max_{\underline{p},u,q} \frac{[(\frac{p_0}{1-p_0})^\gamma - (\frac{\underline{p}}{1-\underline{p}})^\gamma](1-u)}{(\frac{p_0}{1-p_0})^\gamma \frac{1}{p_0} - (\frac{\underline{p}}{1-\underline{p}})^\gamma \frac{1}{\underline{p}}} + ub - kuq,$$

$$s.t.\ uq^{1-\theta} = \delta(1-u)\frac{(\frac{p_0}{1-p_0})^\gamma \frac{1}{p_0}}{(\frac{p_0}{1-p_0})^\gamma \frac{1}{p_0} - (\frac{\underline{p}}{1-\underline{p}})^\gamma \frac{1}{\underline{p}}}.$$

Taking first order conditions with respect to the planner's problem yields:

Proposition 2 *The socially optimal allocation $(\underline{p}^\star, q^\star)$ is uniquely determined by the following system of equations:*

$$\underline{p} = \frac{(\gamma-1)[(1-\theta)b + \theta qk]}{(1-\theta)(\gamma-b) - \theta qk} \tag{14}$$

$$kq^\theta = -\frac{[(1-\theta)(\underline{p}-b) - \theta qk]p_0^{1-\gamma}(1-p_0)^\gamma}{\delta \underline{p}^{1-\gamma}(1-\underline{p})^\gamma} + \frac{(1-\theta)(p_0-b)-\theta qk}{\delta}. \tag{15}$$

Theorem 1 *The stationary equilibrium is efficient if and only if $\beta = \theta$.*

The above theorem implies that bargaining equilibrium achieves efficient search intensity and entry under the Hosios condition. Due to Nash bargaining, a firm-worker will always dissolve efficiently as shown in Section 3.1. Since learning does not cause any additional inefficiency, we just need to set $\beta = \theta$ to guarantee efficient entry.

3.3 Extension: Externality for Being Unemployed

In this section, we will discuss an extension of the baseline model where there is externality for being unemployed. As a result, the social value for being unemployed is $b + \kappa$. The externality may arise because of crime and social problems caused by unemployment or because the unemployment insurance benefit b is not included in the social welfare function. Although these are examples of negative externality, we allow κ to be either positive or negative in the model. The stationary equilibrium is still characterized by Proposition 1 while the socially optimal allocation changes to $(\underline{p}_e^\star, q_e^\star)$ where $(\underline{p}_e^\star, q_e^\star)$ satisfy:

$$\underline{p} = \frac{(\gamma-1)[(1-\theta)(b+\kappa)+\theta qk]}{(1-\theta)(\gamma-b-\kappa)-\theta qk} \tag{16}$$

and

$$kq^\theta = -\frac{[(1-\theta)(\underline{p}-b-\kappa)-\theta qk]p_0^{1-\gamma}(1-p_0)^\gamma}{\delta \underline{p}^{1-\gamma}(1-\underline{p})^\gamma} + \frac{(1-\theta)(p_0-b-\kappa)-\theta qk}{\delta}. \tag{17}$$

Proposition 3 *If there is externality for being unemployed, the stationary equilibrium can never be socially optimal.*

We may consider an extreme case where $p_0 = 1$. In this situation, there is no uncertainty and hence no learning as well. Standard technique (see Rogerson, Shimer, and Wright (2005)) implies that the stationary competitive search equilibrium is efficient if:

$$\frac{\delta + \beta q^{1-\theta}}{(1-\beta)q^{-\theta}} = \frac{\delta + \theta q^{1-\theta}}{(1-\theta)q^{-\theta}} + \frac{\kappa}{k}.$$

To achieve efficiency, the social planner simply needs to let $\beta(\theta)$ satisfy:

$$\frac{\delta + \beta q(\theta)^{1-\theta}}{(1-\beta)q(\theta)^{-\theta}} = \frac{\delta + \theta q(\theta)^{1-\theta}}{(1-\theta)q(\theta)^{-\theta}} + \frac{\kappa}{k},$$

where $q(\theta)$ solves:[8]

$$\frac{1-b-\kappa}{k} = \frac{\delta + \theta q^{1-\theta}}{(1-\theta)q^{-\theta}}.$$

As long as κ is not large enough, $\beta(\theta)$ can be picked such that $\beta > \theta$ if $\kappa > 0$ and vice versa. When there exists positive (negative) externality, the bargaining power of the firm will decrease (increase) such that more workers will become unemployed (employed).

In summary, if we introduce an externality for being unemployed, the stationary equilibrium can be efficient under a modified Hosios condition when there is no uncertainty. However, with learning, the stationary equilibrium can never be efficient. These results are quite intuitive. Without uncertainty, it is socially optimal to always let matched firm and worker form a pair, and never dissolve it as long as the externality is not large enough. Since this is a binary decision, a firm-worker pair will form and dissolve *efficiently* in equilibrium even if there is externality. Therefore, we only

[8]With multiple solutions, we pick the one with higher social welfare.

need to modify the Hosios condition to adjust for the externality. However, with learning, the socially optimal decision on dissolving a firm-worker pair is more complicated because it involves a continuous decision on when to dissolve the pair. Due to the existence of the externality, a firm-worker pair will always dissolve *inefficiently* because the externality κ is not considered in the individual problems. As a result, efficiency of the stationary equilibrium cannot be restored no matter how we modify the condition.

Remark 1 *Another way to introduce the wedge between private value and social value is by introducing a tax. It is straightforward to show in this situation, the stationary equilibrium with learning will always be inefficient while the stationary equilibrium without learning can be efficient under certain condition.*

4 General Human Capital

4.1 Stationary Equilibrium

Similar to the previous analysis, let $W(p)$ denote the total discounted value that a worker receives in equilibrium, when employed in a match with posterior belief p. Let $U(p)$ denote value of unemployment for a worker with posterior belief p. We use $J(p)$ to denote the value of the firm, and finally J denotes the value to the firm of holding an open vacancy. Free entry implies that $J = 0$.

The value functions can be written as:

$$\delta U(p) = b + \lambda(W(p) - U(p)); \tag{18}$$

$$\delta W(p) = w(p) + \Sigma(p)W''(p); \tag{19}$$

and

$$\delta J(p) = p - w(p) + \Sigma(p)J''(p). \tag{20}$$

Equations 19 and 20 are the same as the value functions with firm-specific human capital. However, equation 18 becomes different due to the fact that the human capital is general. On the other hand, on the equilibrium path, a firm-worker will not dissolve if the posterior belief p is higher than certain cutoff. Therefore, equation 19 describes the value off the equilibrium path.

Let u denote the measure of *new* workers and v denote the measure of vacant firms. Since the job finding rate of a type p worker off the equilibrium path is the same as the job finding rate of the new workers, the match function implies that

$$\lambda = q^{1-\theta} = (\frac{v}{u})^{1-\theta}.$$

Meanwhile, the generalized Nash bargaining solution selects wage $w(p)$ such that:

$$\beta J(p) = (1-\beta)[W(p) - U(p)].$$

Taking the second derivative with respect to p yields:

$$\beta J''(p) = (1-\beta)(W''(p) - U''(p)).$$

and

$$w(p) = \beta p + (1-\beta)(\delta U(p) - \Sigma(p)U''(p)).$$

Notice

$$\delta U(p) = b + \lambda[W(p) - U(p)]$$

implies that:

$$\delta U''(p) = \lambda[W''(p) - U''(p)] = \frac{\beta}{1-\beta}\frac{\lambda}{\delta}J''(p).$$

Plug the wage expression into the firm's value function and we get:

$$(\delta + \lambda\beta)J(p) = (1-\beta)(p-b) + (1+\beta\frac{\lambda}{\delta})\Sigma(p)J''(p).$$

This is a second order differential equation about $J(p)$ with boundary conditions: $J(\underline{p}) = 0$ and $J'(\underline{p}) = 0$. Therefore, denote $\gamma = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2\delta}{s^2}}$ and we can characterize the stationary equilibrium by the following proposition:

Proposition 4 *The stationary equilibrium* $(\underline{p}^e, q^e)$*is uniquely determined by the following system of equations:*

$$\underline{p} = \frac{b(\gamma - 1)}{\gamma - b} \tag{21}$$

$$kq^{\theta} = -\frac{(1-\beta)(\underline{p}-b)p_0^{1-\gamma}(1-p_0)^{\gamma}}{(\delta+\beta q^{1-\theta})\underline{p}^{1-\gamma}(1-\underline{p})^{\gamma}} + \frac{(1-\beta)(p_0-b)}{\delta+\beta q^{1-\theta}}. \tag{22}$$

Different from the firm specific human capital case, once the worker is fired, the worker cannot find other jobs because the human capital is general. However, the pair is dissolved efficiently in the sense that $S(\underline{p}^e) = U(p)$ and $S'(\underline{p}^e) = U'(\underline{p}^e) = 0$, where $S(p) = W(p) + J(p)$ is the total surplus of a firm-worker match with posterior belief p.

Remark 2 *Instead of assuming the unemployment benefit is b, we may assume that the unemployment benefit is some function $\phi(p)$ of the common belief p. In this situation, it is also true that the firm-worker pair is dissolved efficiently. The nature of Nash bargaining is the key reason for the result.*

4.2 Socially Optimal Allocation

Similarly, the social planner chooses allocation rule $(\underline{p}, u, v)$ to maximize the aggregate steady state output. Once $\underline{p} < p_0$ is picked, the ergodic density can be written as:

$$f(p) = [C_1 p^{-1-\gamma}(1-p)^{\gamma-2} + C_2(1-p)^{-1-\gamma}p^{\gamma-2}]\mathbb{I}(\underline{p} < p \leq p_0) + C_3 p^{-1-\gamma}(1-p)^{\gamma-2}\mathbb{I}(p > p_0), \tag{23}$$

with boundary conditions $f(\underline{p}+) = 0$ and

$$C_1 p_0^{-1-\gamma}(1-p_0)^{\gamma-2} + C_2(1-p_0)^{-1-\gamma}p_0^{\gamma-2} = C_3 p_0^{-1-\gamma}(1-p_0)^{\gamma-2}.$$

Since human capital is general, there are workers with posterior belief $\underline{p}$ in the economy, who cannot get any job. Denote the measure of those workers to be x. The total flows in and out of x must balance, which implies that:

$$\delta x = \Sigma(\underline{p}) f'(\underline{p}+). \tag{24}$$

On the other hand, the total measure of workers is one:

$$\int_{\underline{p}}^{1} f(p)dp + x + u = 1. \tag{25}$$

Equations 24 and 25 together imply that

$$x = \frac{(\frac{\underline{p}}{1-\underline{p}})^{\gamma}\frac{1}{\underline{p}}(1-u)}{(\frac{p_0}{1-p_0})^{\gamma}\frac{1}{p_0}}.$$

As a result, we can simplify the planner's problem as the following:

In this economy, there are measure u of unemployed workers with belief p_0, measure x of unemployed workers with belief $\underline{p}$ and measure $1-u-x$ of employed workers with ergodic distribution of beliefs $f(p)$. Similar to Proposition 2, we can reformulate the social planner's problem as:

$$\max_{\underline{p},u,q} \frac{[(\frac{p_0}{1-p_0})^{\gamma} - (\frac{\underline{p}}{1-\underline{p}})^{\gamma}](1-u)}{(\frac{p_0}{1-p_0})^{\gamma}\frac{1}{p_0}} + ub + \frac{b(\frac{\underline{p}}{1-\underline{p}})^{\gamma}(1-u)}{(\frac{p_0}{1-p_0})^{\gamma}\frac{1}{p_0}} - kuq,$$

$$\text{s.t. } uq^{1-\theta} = \delta(1-u).$$

Taking the first order conditions yields:

Theorem 2 *The stationary equilibrium is efficient if and only if* $\beta = \theta$.

The above theorem is proved by just taking first order conditions and hence is omitted. The intuition in the previous section is justified again: since the firm-worker pair is dissolved efficiently in the general human capital case, the Hosios condition guarantees that the stationary equilibrium is efficient.

5 Conclusion

This paper studies the allocative efficiency in a labor market with the following features: 1) the unemployed workers and the vacant firms meet according to a constant return matching function; 2) the wages are determined by generalized Nash bargaining; and 3) the firm and the worker are learning about the quality of the match over time. It turns out that the stationary competitive search equilibrium in such an environment is efficient no matter whether learning is about firm specific human capital or about general human capital. However, if there is a wedge between private value and social value, the stationary competitive search equilibrium can never be efficient. Whether efficiency can be achieved or not depends crucially on whether the firm-worker matching is formed and terminated efficiently or not. In Acemoglu and Shimer (1999), the firm-worker matching is formed inefficiently because of the hold-up problem while with externality, the firm-worker matching is terminated inefficiently. As a result, efficiency can never be achieved in these two cases.

Although the main results are derived in the presence of Bayesian learning, the above intuition goes through in other similar situations.

First, we may consider situations where the belief updating process is even more complicated than Brownian motion (Lévy process for example) or there is stochastic human capital accumulation instead of Bayesian learning (the belief updating does not follow a martingale process). It may be difficult to solve the ergodic distributions explicitly in these situations. But the intuition tells us that the stationary equilibrium should be efficient under the Hosios condition as well.

Second, when computing the welfare loss of various labor market policies, the usual calibration strategy is to choose the bargaining power to satisfy the Hosios condition. Obvious, the Hosios condition ceases to hold with labor market distortions and hence the welfare loss partly comes from the choice of bargaining power. Another possible calibration strategy is to choose bargaining power to maximize the social welfare given the policy. The intuition sheds light on the welfare comparison of various labor market policies using this calibration strategy. In particular, if the policy does not cause any inefficiency in the forming and termination of the firm-worker pair, efficiency can be restored under a modified Hosios condition.

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Appendix

A.1 Proof of Claim 1

Proof. From the boundary conditions, we can express C_1 and C_3 as functions of C_2:

$$C_3 = C_1 + C_2(\frac{p_0}{1-p_0})^{2\gamma-1}$$

and

$$C_1 = -C_2(\frac{\underline{p}}{1-\underline{p}})^{2\gamma-1}.$$

Finally, since $\int_{\underline{p}}^1 f(p)dp = 1-u$, we can solve that

$$C_2 = \frac{\delta(1-u)}{\sqrt{\frac{1}{4}+\frac{2\delta}{s^2}}s^2[(\frac{p_0}{1-p_0})^\gamma \frac{1}{p_0} - (\frac{\underline{p}}{1-\underline{p}})^\gamma \frac{1}{\underline{p}}]}.$$

Meanwhile, we have

$$\int_{\underline{p}}^1 pf(p)dp = C_2 \frac{s^2}{\delta}\sqrt{\frac{1}{4}+\frac{2\delta}{s^2}}[(\frac{p_0}{1-p_0})^\gamma - (\frac{\underline{p}}{1-\underline{p}})^\gamma].$$

Therefore, the objective function can be rewritten as

$$\frac{[(\frac{p_0}{1-p_0})^\gamma - (\frac{\underline{p}}{1-\underline{p}})^\gamma](1-u)}{(\frac{p_0}{1-p_0})^\gamma \frac{1}{p_0} - (\frac{\underline{p}}{1-\underline{p}})^\gamma \frac{1}{\underline{p}}} + ub - kuq.$$

The constraint $uq^{1-\theta} = \delta(1-u) + \Sigma(\underline{p})f'(\underline{p}+)$ can be rewritten as:

$$uq^{1-\theta} = \delta(1-u) + \frac{1}{2}s^2 C_2(2\gamma-1)p^{\gamma-1}(1-p)^{-\gamma}.$$

Since

$$C_2 = \frac{\delta(1-u)}{\sqrt{\frac{1}{4}+\frac{2\delta}{s^2}}s^2[(\frac{p_0}{1-p_0})^\gamma \frac{1}{p_0} - (\frac{\underline{p}}{1-\underline{p}})^\gamma \frac{1}{\underline{p}}]},$$

the above expression can be simplified as:

$$uq^{1-\theta} = \delta(1-u)\frac{(\frac{p_0}{1-p_0})^\gamma \frac{1}{p_0}}{(\frac{p_0}{1-p_0})^\gamma \frac{1}{p_0} - (\frac{\underline{p}}{1-\underline{p}})^\gamma \frac{1}{\underline{p}}}.$$

■

A.2 Proof of Proposition 2

Proof. Denote the Langrangian multiplier to be λ. By taking first order condtions with respect to u, q and $\underline{p}$ respectively, we can get:

$$\frac{[(\frac{p_0}{1-p_0})^\gamma - (\frac{\underline{p}}{1-\underline{p}})^\gamma]}{(\frac{p_0}{1-p_0})^\gamma \frac{1}{p_0} - (\frac{\underline{p}}{1-\underline{p}})^\gamma \frac{1}{\underline{p}}} - b + kq = \lambda[q^{1-\theta} + \delta \frac{(\frac{p_0}{1-p_0})^\gamma \frac{1}{p_0}}{(\frac{p_0}{1-p_0})^\gamma \frac{1}{p_0} - (\frac{\underline{p}}{1-\underline{p}})^\gamma \frac{1}{\underline{p}}}], \tag{26}$$

$$k = \lambda(1-\theta)q^{-\theta}, \tag{27}$$

and

$$\delta\lambda(\frac{p_0}{1-p_0})^\gamma \frac{1}{p_0} = [(\frac{p_0}{1-p_0})^\gamma - (\frac{\underline{p}}{1-\underline{p}})^\gamma] - \frac{\gamma\underline{p}}{\gamma-1+\underline{p}}[(\frac{p_0}{1-p_0})^\gamma \frac{1}{p_0} - (\frac{\underline{p}}{1-\underline{p}})^\gamma \frac{1}{\underline{p}}]. \tag{28}$$

Plug equation 28 into equation 26 yields:

$$b - kq + \lambda q^{1-\theta} = \frac{\gamma\underline{p}}{\gamma-1+\underline{p}}.$$

Equation 27 implies that $\lambda = \frac{kq^\theta}{1-\theta}$ and hence,

$$b + \frac{\theta qk}{1-\theta} = \frac{\gamma\underline{p}}{\gamma-1+\underline{p}},$$

which implies that:

$$\underline{p} = \frac{(\gamma-1)[(1-\theta)b + \theta qk]}{(1-\theta)(\gamma-b) - \theta qk}.$$

This is equation 14. Plug equation 14 into equation 28 and we can get 15 by simplification. ■

A.2 Proof of Proposition 3

Proof. Suppose there exist parameter values such that the stationary equilibrium is socially optimal. Then, equations 9 and 16 imply that:

$$\frac{(\gamma-1)[(1-\theta)(b+\kappa)+\theta qk]}{(1-\theta)(\gamma-b-\kappa)-\theta qk}=\frac{(\gamma-1)[(1-\beta)b+\beta qk]}{(1-\beta)(\gamma-b)-\beta qk},$$

which can be simplified as:

$$\frac{\kappa}{\beta-\theta}=\frac{qk}{(1-\beta)(1-\theta)}.$$

The sign of $\beta-\theta$ is the same as the sign of κ.

On the other hand, equations 9 and 16 imply that:

$$\begin{aligned}-\frac{[(1-\theta)(\underline{p}-b-\kappa)-\theta qk]p_0^{1-\gamma}(1-p_0)^\gamma}{\delta\underline{p}^{1-\gamma}(1-\underline{p})^\gamma}+\frac{(1-\theta)(p_0-b-\kappa)-\theta qk}{\delta}\\ =-\frac{[(1-\beta)(\underline{p}-b)-\beta qk]p_0^{1-\gamma}(1-p_0)^\gamma}{\delta\underline{p}^{1-\gamma}(1-\underline{p})^\gamma}+\frac{(1-\beta)(p_0-b)-\beta qk}{\delta},\end{aligned}$$

which can be simplified as:

$$[p_0-b+qk-\frac{(1-\theta)\kappa}{\beta-\theta}]\underline{p}^{1-\gamma}(1-\underline{p})^\gamma=[\underline{p}-b+qk-\frac{(1-\theta)\kappa}{\beta-\theta}]p_0^{1-\gamma}(1-p_0)^\gamma.$$

Plug $\frac{\kappa}{\beta-\theta}=\frac{qk}{(1-\beta)(1-\theta)}$ into the above equation and we can obtain:

$$[(1-\beta)(p_0-b)-\beta qk]\underline{p}^{1-\gamma}(1-\underline{p})^\gamma=[(1-\beta)(\underline{p}-b)-\beta qk]p_0^{1-\gamma}(1-p_0)^\gamma,$$

which implies that $q=0$ from equation 10. But this is impossible! ■