The Technological Origins of the Decline in Labor Market Dynamism*

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Abstract

In the last decades, there has been a marked decline in the job flows to and from unemployment and between employment. We ask whether and how technological change can account for his secular decline in labor market dynamism. We propose a theory that focuses on the determinants of technology broadly defined: 1. the complementarity between worker skill and firm productivity; and 2. the volatility in productivity shocks; and 3. search frictions. We derive job flows in a sorting model with search frictions and endogenous search effort both on and off the job, as well as shocks that lead to mismatch. We quantify our model using the US data and find an increase in the complementarity between labor and technology, a decline in the frequency and volatility of productivity shocks, and a decline in the match efficiency as well as an increase in the search costs. The changing nature of these features of the technology contribute to the secular decline in labor market dynamism.

Keywords. Sorting. Declining Labor Market Dynamism. Job Flows. Labor Reallocation. Complementarities. Technological Change.

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1 Introduction

The decline in labor market dynamism in the last decades is without doubt the most striking development in the US labor market. The transition of workers between unemployment and employment, and between jobs now is substantially lower than it was a few decades ago. The Current Population Survey (CPS) that is the basis for the calculation of the economy's unemployment rate shows that the job to job transitions in 1997 were 50% higher than they are in 2016, while the transitions between unemployment and jobs in both directions were each about 25% higher.[1] This decline in labor market dynamism has important implications for welfare and has been linked to the decline in startups of new businesses, the decline in mobility up the job ladder, and the decrease in migration rates.

In this paper, we investigate the role of technological change in explaining the decline in labor market dynamism. In particular, we focus on the evolution of 1. complementarity between worker ability and job productivity, 2. volatility in the productivity process, and 3. adjustment costs (directed search frictions). To that end, we build a theory of the unemployment due to labor market frictions that builds on Garibaldi, Moen, and Sommervoll (2016) and that incorporates sorting and on-the-job search, and where the transitions are determined through individual choices: through search intensity or the decision to terminate a match. The model has three key features. First, there is two-sided heterogeneity and sorting between workers and firms. Second, stochastic productivity generates mismatch and endogenous separations (EE and EU). Third, search intensity of workers determines UE and EE flows. This setup results in stochastic sorting with Beckerian assortative matching (Becker (1973)) where the match surplus is stochastic (see Chade and Eeckhout (2017)) and where separations are endogenous.

Because of sorting and technology shocks, mismatched workers determine the rate at which they switch jobs. They search more intensely when the extent of mismatch is larger, or they terminate the match altogether if the mismatch between their ability and the job is too big. Therefore both sorting and the volatility of the shocks play a role. The larger the complementarity between job productivity and worker ability, the higher the opportunity cost of mismatch. And the higher the volatility of shocks, the more likely a worker is mismatched. Volatility of shocks and complementarity thus affect the value of being mismatched and as a result, both determine the worker's response in search intensity or in their separation decision. But also the search technology plays a role. The match efficiency, the rate at which workers find a job, determines how intensely workers search, and so does the cos of job search.

Our main empirical findings are as follows. We find: 1. a modest increase in complementarities be-

[1] See Figure 3 below. While different data sources indicate some variation in the magnitudes of the change over time, all agree that there is a marked decline (see for example Moscarini and Postel-Vinay (2017) who analyze data from the Survey of Income and Program Participation (SIPP) instead of CPS. Moreover, this pattern of declining job flows is robust across geography (States), industry, firm size, firm age, worker demographics,...

tween labor and technology (30%); 2. The frequency and variance of the shock process declines sharply; 3. the match efficiency declines across all skill types. This is consistent with Hall and Schulhofer-Wohl (2018) who find a decline in the match efficiency from the 2001 to 2013. They also find that the composition of the unemployed shifted dramatically towards groups with lower matching efficiency. 4. We find that the cost of search has increased by 47%.[2] Quantitatively, each of these components contribute to the secular decline in labor market dynamism. In addition, we find that UE rate across wages is flat. This fact was first documented by Mueller (2017), namely that the average and cyclicality of job findings tend to be very similar across wage groups. This fact is a challenge to match for models with ex ante worker heterogeneity, and our model does well matching this fact.

Related Literature. Since the phenomenon of declining labor market dynamism was first documented, several potential explanations have been put forward. Those are complementary to the one we investigate in the current paper, namely the role of technological change. The most notable of the alternative explanations are those based on compositional changes.

The foremost of those demographic change is the impact from an aging population – not least due to the baby boom generation moving from young workers in the 1980s to older workers now. Since older workers have lower labor market dynamism than younger workers (see for example Jovanovic (1979)), a higher proportion of older workers implies that the average labor dynamism is lower (see for example Karahan, Pugsley, and Şahin (2024); Dent, Karahan, Pugsley, and Şahin (2016); Fallick, Fleischman, and Pingle (2010); Engbom (2017); Hyatt and Spletzer (2013); Molloy, Smith, Trezzi, and Wozniak (2016); Fallick and Foote (2022); Crump, Gianonni, Eusepi, and Sahin (2019)). While a simple decomposition analysis reveals seemingly small effects of aging (see for example Fallick et al. (2010)), more recent work shows that indirect and general equilibrium effects are important and sizable (see Crump, Gianonni, Eusepi, and Sahin (2019); Engbom (2017); Fallick and Foote (2022)). Relevant for our exercise is that this work shows that the age composition affects turbulence and the frequency of the shocks, which in our exercise we take as exogenous while keeping the age composition fixed.

Likewise, a shift in the skill composition towards a more educated work force also leads to a decrease in average labor market dynamism because the high skilled switch jobs less often (EE) and they separate jobs at a lower rate (EU) than the low skilled (though job finding rates (UE) are similar across skill groups). The composition shift of a structurally changing economy with an increasing share of services and a decreasing share of manufacturing (or the rise of the retail sector) could also affect the average job flows. Another force that can affect the flow is the decline in union coverage. However, this would have

[2] This is consistent with the findings of Molloy et al. (2016). They suggest that a decline in social trust, which may has increased the cost to job searchers or makes firms and workers more risk averse, could be behind the rising search cost and the resulting decline in labor market dynamism.

lead to a rise in dynamism as union jobs tend to have longer duration.

Dent, Karahan, Pugsley, and Şahin (2016) and Decker, Haltiwanger, Jarmin, and Miranda (2014, 2016) also document a decline in entrepreneurship with fewer new startup firms, and link this to the decline in labor market dynamism. This work also points to policy changes such as the rise in the share of workers covered by licensing agreements that lead to a decrease in worker competition and hence dynamism. Similarly with the decline of employment at will. Yet another explanation that can rationalize the decline in entrepreneurship and startups is the rise in market power by dominant firms (see De Loecker et al. (2020, 2021)). In the presence of market power, passthrough of shocks is incomplete, which leads to lower entry of new firms and to lower reallocation rates of workers across firms.

Our model can be interpreted as a framework that allows for the measurement of complementarities in production. Abowd et al. (1999) measure complementarities using fixed effects wage regressions. However, in a labor market with search frictions and sorting, wages are not monotonic in firm productivities which renders the fixed effect regression in Abowd et al. (1999) biased. Even though output is higher in more productive firms, a low ability worker receives a lower wage than they would get in a less productive firm because the outside option of the high productivity firm is higher due to complementarities. Eeckhout and Kircher (2011) show that identification of the magnitude of the complementarity is possible with wage data only, typically information on job transitions is needed to also identify the sign of sorting (Bonhomme, Lamadon, and Manresa (2015); Lopes de Melo (2018); Borovičková and Shimer (2017); Hagedorn, Law, and Manovskii (2017)).[3] Not only are wages non-monotonic in the presence of search, which complicates the identification, wages are typically indeterminate.[4] Equilibrium pins down the utility – be it through matching outside offers in the presence of random search, or through firms posting discounted utility streams – but not the wage. Hence there are many wage schedules that are consistent with the same equilibrium allocation, even if that allocation is unique. Relying therefore on wages that are indeterminate is problematic for the identification of complementarities when there is sorting. To that end, we rely on a model that is exclusively identified from job flows and not wages.

2 The Model

We develop a directed search model with two-sided heterogeneity, stochastic types, on-the-job search and endogenous search intensity. We need all of the ingredients to generate stochastic sorting, endoge-

[3]Measuring the complementarity is important because it may be related to other secular trends, such as the increase in between-firm inequality and outsourcing (see Song et al. (2019)).

[4]Especially in search models with on-the-job search, whether search is random as in Postel-Vinay and Robin (2002) and Lise and Robin (2017) or directed as in ours. Search intensity can also give rise to cyclical fluctuations due to coordination frictions (Eeckhout and Lindenlaub (2019)). Though our focus is on the secular trend in job flows, there is ample cyclical variation, see Carillo-Tudela and Visschers (2023); Chodorow-Reich and Wieland (2020).

nous UE, EE, UE flows and a distribution of match qualities.

Agents and Technology. Time is continuous, $t \in \mathbb{R}_+$. There is a measure one of risk neutral workers, each with a type $x \in \mathcal{X}$. A worker can be in three possible states, either she is unemployed searching for a job, or she is employed not searching, or she is employed searching on the job. As a result, the space of the individual state ξ can be defined as the set $\Xi = \mathcal{Y} \cup \{-1\}$. Here we abuse the notation by allowing $\xi = -1$ if the worker is unemployed, and if the worker is employed, $\xi \in \mathcal{Y}$ is the type of the firm currently matched with the worker. The flow utility from being unemployed is $b(x)$ and the flow utility from being employed is the wage w. Workers who decide to search, whether while unemployed or employed, choose the intensity λ at a cost $c_\lambda(\lambda)$.

There is a large measure of potential jobs (firms), all of which are risk neutral and ex ante identical. Firms can pay a flow cost $k > 0$ to open a vacancy. After opening the vacancy, firms can freely choose type y with operational cost $c_y(y)$. In other words, a firm of type y that is matched with a worker of type x produces output $f(x,y) - c_y(y)$, where $f(x,y)$ is the production function. If firms stay inactive, their payoff is zero.

An important component of the technology are the shocks to worker and firm types. Given time is continuous, we model the arrival of a shock by the Poisson rate γ, in which case there is a new realization of the pair x', y'. We allow this shock process to be as general as possible. The shock is drawn from the distribution $G(x', y'|x, y)$ with density $g(x', y'|x, y)$. We allow for a very general distribution that allows for drift ($\mathbb{E}_{x'} G(x', y'|x, y) \geq x$) to capture human capital accumulation as well as dependence of the process on x.

Market Frictions. Search is directed. Firms post a wage contract consisting an initial promised utility to the worker and all possible continuation payoffs upon every contingency, observed by all workers. Workers then choose which firm (and wage contract) to direct their application to. Due to the nature of directed search, the market is segmented by types x and y, and workers' states ξ, that is, there is a particular market tightness in each submarket. In such a submarket, denote the density of unemployed workers of type x by $u(x)$ and the density of vacancies of type y by $v(y)$. We define the standard market tightness by $\tilde{\theta} = \frac{v(y)}{u(x)}$. Denote by Λ the (symmetric) search intensity of all other workers in the submarket. Because of the endogenous search intensity, what matters for the matching technology is not so much the number of searchers, but rather the efficiency units of searchers $\Lambda u(x)$. Then we write the argument of the matching function as $\frac{v(y)}{\Lambda u(x)} = \frac{\tilde{\theta}}{\Lambda}$.[5]

[5] As is common in directed search, we assume that workers who direct their search to a job y obtain that job with certainty. We could also allow that a worker who applies for a job y obtains a job y' in the neighborhood of y. We leave this possibility

The stochastic nature of the match formation is modeled by means of a standard matching function m where the matching rate for a worker with search intensity λ is given by $\varphi\lambda m\left(\frac{\tilde{\theta}}{\Lambda}\right)$, where m is increasing, concave and has constant returns to scale. φ is a parameter measuring the matching efficiency, and we allow φ to be different across employed (φ_e) and unemployed (φ_u) workers. The individual search effort affects the own matching probability through λ, but not the effective market tightness $\frac{\tilde{\theta}}{\Lambda}$ which depends on the aggregate search intensity Λ. In a symmetric equilibrium, we will have $\lambda = \Lambda$. Moreover, the standard consistency argument implies that the matching rate for a firm is given by $q\left(\frac{\tilde{\theta}}{\Lambda}\right) = \varphi m\left(\frac{\tilde{\theta}}{\Lambda}\right)\frac{\Lambda}{\tilde{\theta}}$.

Payoffs. We assume that the firms and the workers have the same discount rate ρ. And each worker exits the market exogenously at rate δ such that the stationary distribution of the workers' types is not degenerate. Upon exit, the old worker will be replaced by a new one whose type is drawn from density $f_0(x)$. Therefore, the effective discount rate is $r = \rho + \delta$.

Utilities are perfectly transferable. Define $E(x, y)$ to be the promised value received by a type x worker from working with a type y firm, and $J(x, y)$ to be the expected profits received by a type y firm from working with a type x worker. Then the total expected value created by this (x, y) pair is denoted by $S(x, y) = E(x, y) + J(x, y)$. We further assume that the firm decides whether to fire the worker or not, and if an employed worker decides to leave the firm, the worker has to pay a penalty $P(x, y)$ to the firm. P is allowed to be negative, in which case we can interpret it as severance pay.

Value Functions. Due to the stochastic nature of types, there is mismatch and the allocation of matches is potentially the entire domain $\mathcal{X} \times \mathcal{Y}$. However, there are two important subsets in the domain. The first is the allocation that is chosen when a new match is formed, either out of unemployment or from on-the-job search. This allocation is denoted by $y = \mu(x, \xi)$. In principle workers with type x and individual state ξ will choose to search for type $\mu(x, \xi)$ firms. The second subset is the matching set as a subset of the entire domain $\mathcal{X} \times \mathcal{Y}$. The matching set for type x workers is denoted by $\mathcal{M}(x)$, which consists of the set of firm types that are willing to employ type x workers. Moreover, we have to determine the market tightness $\tilde{\theta}(x, y, \xi)$, the individual search intensity $\lambda(x, y, \xi)$ and the aggregate search intensity $\Lambda(x, y, \xi)$ in each submarket.

We first consider the worker's optimization problem. For an unemployed worker, the worker chooses which submarket to search and with which search intensity. The characteristics of a submarket $(y', \tilde{\theta}, \Lambda)$ include firm type y', market tightness $\tilde{\theta}$ and aggregate search intensity Λ. The optimization problem can be written as:

for future work.

$$rU(x) = b(x) + \max_{\tilde{y},\tilde{\theta},\Lambda,\lambda} \left\{ \varphi_u \lambda m \left(\frac{\tilde{\theta}}{\Lambda} \right) [E(x,\tilde{y}) - U(x)] - c_\lambda(\lambda) \right\}. \tag{1}$$

Similarly, an employed worker's problem can be written as (given the promised value $E(x,y)$):

$$\begin{aligned} rE(x,y) = w(x,y) + \gamma \int \left[\eta E(x',y') + (1-\eta)U(x') - E(x,y) \right] dG(x',y'|x,y) \\ + \max_{\tilde{y},\tilde{\theta},\Lambda,\lambda} \left\{ \varphi_e \lambda m \left(\frac{\tilde{\theta}}{\Lambda} \right) [E(x,\tilde{y}) - E(x,y)] - c_\lambda(\lambda) \right\}, \end{aligned} \tag{2}$$

where $\eta \in \{0,1\}$ denotes the firm's separation decision: the worker is fired if $\eta = 0$ and retained otherwise.

For a matched firm, the firm's objective is to maximize its expected value subject to the following promise-keeping constraint. When a firm-worker pair x, y is initially formed, the firm promises the worker a promised utility of $\bar{E}(x,y)$. Thus, the promise-keeping constraint requires that the worker's continuation value upon matching with the firm, $E_0(x,y)$, has to satisfy $E_0(x,y) \geq \bar{E}(x,y)$. The firm's problem is to design an optimal contract subject to the promise-keeping constraint to maximize its continuation value upon matching with the worker. Formally, the firm's problem can be written as:

$$\begin{aligned} rJ_0(x,y) = \max_{w,\eta} f(x,y) - c_y(y) - w(x,y) + \gamma \int \left[\eta J(x',y') - J(x,y) \right] dG(x',y'|x,y) \\ - \varphi_e \lambda^\star m \left(\frac{\tilde{\theta}^\star}{\Lambda^\star} \right) J(x,y), \end{aligned} \tag{3}$$

subject to the promise-keeping constraint $E_0(x,y) \geq \bar{E}(x,y)$, and $\lambda^\star$, $\tilde{\theta}^\star$ and $\Lambda^\star$ are optimal solutions to (2).

Finally, the value of opening a vacancy is the expected profits of an entering firm. In particular, if $\mu(x,\xi) = y$, then this value becomes

$$V(y) = -k + q \left(\frac{\tilde{\theta}}{\Lambda} \right) J(x,y). \tag{4}$$

Equilibrium. We consider the block-recursive equilibrium (BRE) in our modeled economy. As shown by Menzio and Shi (2011), all equilibria are block recursive, and hence it is without generality in focusing on the BRE.

Definition 1 *A block-recursive equilibrium consists of a market tightness function $\tilde{\theta} : \mathcal{X} \times \mathcal{Y} \times \Xi \to \mathbb{R}_+$, an aggregate intensity function $\Lambda : \mathcal{X} \times \mathcal{Y} \times \Xi \to \mathbb{R}_+$, an individual intensity function $\lambda : \mathcal{X} \times \mathcal{Y} \times \Xi \to \mathbb{R}_+$, an allocation function $\mu : \mathcal{X} \times \Xi \to \mathcal{Y}$, value functions $U : \mathcal{X} \to \mathbb{R}$, $E : \mathcal{X} \times \mathcal{Y} \to \mathbb{R}$, $J : \mathcal{X} \times \mathcal{Y} \to \mathbb{R}$, $V : \mathcal{Y} \to \mathbb{R}$, and policy functions $(w, \eta) : \mathcal{X} \times \mathcal{Y} \to \mathbb{R} \times \{0, 1\}$. These functions satisfy the following conditions: (i) the value functions are given by Equations (1)-(4); (ii) the associated policy functions are optimal solutions to Equations (1)-(4); (iii) $V(y) = 0$ for all y and $\lambda = \Lambda$.*

Conditions $(i) - (ii)$ ensure that in a BRE, the strategies of each agent are optimal given the strategies of the other agents. Condition (iii) is the free entry condition: every entering firm receives a zero expected profit. The nice property of a BRE is that the agent's value and policy functions do not directly depend on the aggregate distribution of the individual states. This makes it much simpler to solve a BRE.

Planner's Solution. From the planner's point of view, $S(x, y) = J(x, y) + E(x, y)$ measures the surplus of a vacant job at firm y that is filled by a worker x:

$$rS(x,y) = f(x,y) - c_y(y) + \gamma \int \left[\max\{S(x',y'), U(x')\} - S(x,y)\right] dG(x',y'|x,y) + \max_{\lambda,\theta,\tilde{y}} \left\{\varphi_e \lambda m(\theta) \left[S(x,\tilde{y}) - S(x,y)\right] - c_\lambda(\lambda) - k\lambda\theta\right\} \tag{5}$$

where the flow of output is $f(x, y) - c_y(y)$, and at rate γ a shock arrives that generates a new set of types x', y', drawn from $G(x', y'|x, y)$. This involves an optimization decision, where the match is sustained whenever $S(x', y') \geq U(x')$ and is destroyed otherwise to move into unemployment. During the match, the worker also continuously searches on the job. She chooses a search intensity λ at cost c_λ as well as a submarket with effective tightness θ where to search at cost $k\lambda\theta$.[6] Obviously, the choice of $\tilde{y}$ is always to maximize $S(x, \tilde{y})$ and hence the worker always searches for her best match. When a new match is formed, the search intensity and the effective tightness are both zero as the firm-worker pair has achieved its best match, i.e., $S(x, y) \geq S(x, y')$ for all y'.

Similarly, $U(x)$ measures the surplus of an unemployed worker x:

$$rU(x) = b(x) + \max_{\lambda,\theta,\tilde{y}} \left\{\varphi_u \lambda m(\theta) \left[S(x,\tilde{y}) - U(x)\right] - c_\lambda(\lambda) - k\lambda\theta\right\}. \tag{6}$$

[6]Notice that if λ is exogenously given to be one, this model is equivalent to the standard on-the-job directed search models (e.g., Menzio and Shi (2011) and Li and Weng (2017)).

The planner's values (5) and (6) are the same as the decentralized equilibrium values in equations (10) and (11)

Steady State Distributions. The BRE is fully solved independent of the distribution of individual states. Now, given the BRE, we can further characterize the steady state distribution of individual states. Let $u(x)$ denote the density of unemployed workers with type x, $\psi(x)$ the density of perfectly matched workers with type x, and $\phi(x,y)$ the density of mismatched worker-firm pair (x,y). Then the stationary densities should satisfy:

$$(\gamma+\delta)\psi(x) = \varphi_u\lambda_u m\left(\theta_u\right) u(x) + \int \phi(x,y)\varphi_e\lambda_y m\left(\theta_y\right) dy \tag{7}$$

$$\begin{aligned}(\varphi_u\lambda_u m\left(\theta_u\right)+\delta)u(x) =& \gamma \int \int_{y\notin\mathcal{M}(x)} g(x,y|\tilde{x},\tilde{y})dxdy\phi(\tilde{x},\tilde{y})d\tilde{x}d\tilde{y} \\ &+ \gamma \int \int_{y\notin\mathcal{M}(x)} g(x,y|\tilde{x},\mu(\tilde{x}))dxdy\psi(\tilde{x})d\tilde{x} + \delta f_0(x)\end{aligned} \tag{8}$$

$$\left(\gamma+\delta+\varphi_e\lambda_y m\left(\theta_y\right)\right)\phi(x,y) = \gamma \int g(x,y|\tilde{x},\tilde{y})\phi(\tilde{x},\tilde{y})d\tilde{x}d\tilde{y} + \gamma \int g(x,y|\tilde{x},\mu(\tilde{x}))v(\tilde{x})d\tilde{x}. \tag{9}$$

Equation (7) implies that the flow-out of $\psi(x)$ (due to productivity shock and death) must be the same as the flow-in (coming from unemployment search and on-the-job search). Equation (8) implies that the flow-out of $u(x)$ (due to search and death) must be the same as the flow-in (coming from voluntary separation of the mis-matched workers). Finally, Equation (8) implies that the flow-out of $\phi(x,y)$ (due to search, productivity shock, and death) must be the same as the flow-in (coming from productivity shock of the other workers).

Illustration of the Mechanism. Figure 1 illustrates the model mechanism. Workers of type x are depicted on on the horizontal axis, firms/jobs y are on the vertical axis. Newly matched workers, whether it is out of unemployment or from on-the-job search, direct their search to a job y that corresponds to the perfectly sorted allocation μ (as in Eeckhout and Kircher (2010)). Now once they are matched, shocks to their type leads to mismatch, away from μ. Mismatch in turn induces search by the worker who trades off the cost of search against the higher wage when better matched. The search intensity λ solves the optimal choice for this trade off and is plotted to the left of the figure. Search intensity is zero at the optimal match μ and increases the further away from μ. If a shock to the type leads to mismatch outside the acceptance region (below $\underline{y}$ or above $\overline{y}$) then worker and firm dissolve the match and the worker becomes unemployed. The search intensity for all unemployed workers of a given type x is the same.

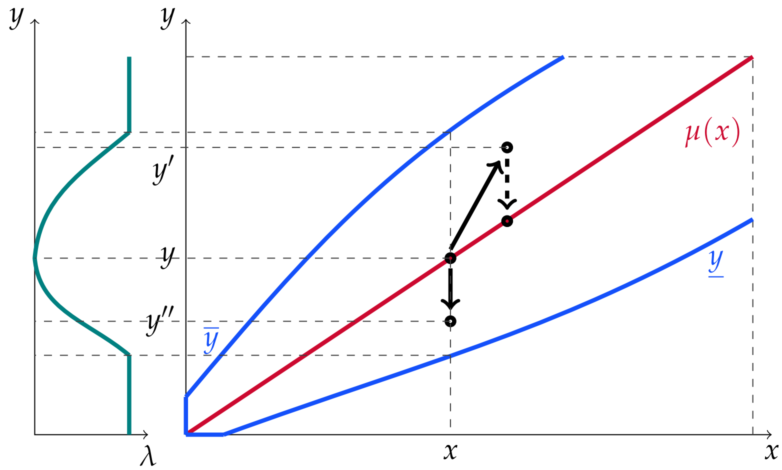


Figure 1: Illustration of the Model Mechanism

3 Solution and Results

General Results. We first rewrite the Bellman equations and show existence, then we solve them. Recall that we let $S(x,y)$ denote the total expected value created by a (x,y) pair: $S(x,y) = E(x,y) + J(x,y)$. The next theorem shows that we can characterize the BRE only based on values $S(x,y)$ and $U(x)$.

Proposition 1 *In any BRE, there exist values $U(x)$ and $S(x,y)$ satisfying*

$$rU(x) = b(x) + \max_{\lambda,\theta,\tilde{y}} \left\{ \varphi_u \lambda m(\theta) \left[S(x,\tilde{y}) - U(x) \right] - c_\lambda(\lambda) - k\lambda\theta \right\}, \tag{10}$$

and

$$\begin{aligned} rS(x,y) = f(x,y) - c_y(y) + \gamma \int \left[\max\{S(x',y'), U(x')\} - S(x,y) \right] dG(x',y'|x,y) \\ + \max_{\lambda,\theta,\tilde{y}} \left\{ \varphi_e \lambda m(\theta) \left[S(x,\tilde{y}) - S(x,y) \right] - c_\lambda(\lambda) - k\lambda\theta \right\}, \end{aligned} \tag{11}$$

and where the policy functions λ, θ and μ are optimal solutions to equations (10) and (11), $\Lambda = \lambda$ and $\tilde{\theta} = \theta\lambda$.

Proof. In Appendix. ■

As in Menzio and Shi (2011), the BRE is constrained socially efficient. It is not fully efficient because the workers do not internalize the externality caused by their search intensities. In other words, each worker chooses the search intensity λ not taking into account that $\Lambda = \lambda$.

We now solve for the Bellman equations. Consider a worker who is searching on the job. Equation (5) implies that the planner chooses $\lambda, \theta, \tilde{y}$ to maximize the surplus $S(x, y)$. The FOCs for λ, θ and $\tilde{y}$ satisfy:

$$
\begin{aligned}
\lambda &: \ \varphi_e m(\theta)\left[S(x,\tilde{y}) - S(x,y)\right] = c'_\lambda + k\theta \\
\tilde{\theta} &: \ \varphi_e m'(\theta)\left[S(x,\tilde{y}) - S(x,y)\right] = k \\
\tilde{y} &: \ \varphi_e \lambda m(\theta)\frac{\partial S(x,\tilde{y})}{\partial \tilde{y}} = 0.
\end{aligned}
$$

Similarly, the FOCs for an unemployed worker satisfy:

$$
\begin{aligned}
\lambda &: \ \varphi_u m(\theta)\left[S(x,\tilde{y}) - U(x)\right] = c'_\lambda + k\theta \\
\tilde{\theta} &: \ \varphi_u m'(\theta)\left[S(x,\tilde{y}) - U(x)\right] = k \\
\tilde{y} &: \ \varphi_u \lambda m(\theta)\frac{\partial S(x,\tilde{y})}{\partial \tilde{y}} = 0.
\end{aligned}
$$

Observe that for both the employed and unemployed workers, the solution for λ and $\tilde{\theta}$ only depends on the difference $S(x,\tilde{y}) - S(x,y)$ or $S(x,\tilde{y}) - U(x)$. Denote this difference to be Δ, and we therefore express the solution S to the optimal directed search problem (choosing search intensity and effective market tightness) as a function $S_j(\Delta) = \max_{\lambda,\theta}\left\{\varphi_j \lambda m(\theta)\Delta - c_\lambda(\lambda) - k\lambda\theta\right\}$ for $j = e, u$.

The Wage Contract. Firms offer a continuation value of the surplus that is precisely pinned down in equilibrium. However, the path of a worker's individual continuation values as well as the worker's wage path are not uniquely determined. In fact, there is a continuum of wage contracts that are consistent in equilibrium. Each of those wage contracts offers the same continuation utility when the firm-worker pair is initially formed, and generates zero continuation value to the firm after the first shock arrives.

One example of the wage contract is the following incubation contract. After a $(x, \mu(x))$ match is initially formed, the firm pays a constant wage

$$
w(x,\mu(x)) = f(x,\mu(x)) - c_y(\mu(x)) - (r+\gamma)\frac{k}{q},
$$

as long as no shock arrives where q is the equilibrium probability for the firm to be matched with a worker. This wage guarantees the free entry condition: $J(x,\mu(x)) = \frac{k}{q}$. After the arrival of a shock, the firm's continuation value becomes $J(x', y') = 0$ and hence $w(x', y') = f(x', y') - c_y(y')$.

Specific Results. In what follows, we will derive analytical results on a shock technology that is independent of y.

Assumption 1 *The shocks are independent of* $y : G(x', y'|x)$.

This simplifies the analysis since the future surplus at the optimal choice of y is independent of y. It means that once the worker-firm pair (x', y) is mismatched and a new shock arrives, this new shock only depends on x', and not on y. One possible shock technology that satisfies the assumption is that at a mismatched pair (x', y), the firm adjust y to match the worker's type so that y switches to x'. We will focus on this case in the subsequent analysis, and the implicit assumption is that the firm's adjustment occurs with a lag, i.e., y is adjusted to x' when x' is shocked again to x'' (we can similarly analyze the no lag case that y switches to $\mathbb{E}x''$).[7] We can then further calculate $\frac{\partial S(x,\tilde{y})}{\partial \tilde{y}}$ from the envelope theorem. As long as the distribution $G(x', y'|x, y)$ is not contingent on y as we have assumed, $\frac{\partial S(x,\tilde{y})}{\partial \tilde{y}} = 0$ is equivalent to $f_y(x, \tilde{y}) - c'_y(\tilde{y}) = 0$. Therefore, the optimal choice $\tilde{y}$ for both the employed and the unemployed worker in $S(x, \tilde{y})$ will be exclusively determined by $f_x(x, y) = c'_y(y)$. The optimal choice $\tilde{y} = \mu(x, \xi)$ hence is independent of the individual state ξ (which is either the current firm y or unemployment), and only depends on the worker type x.

Since at every new match, the type x worker is matched with the same $\mu(x)$ firm, we can use $S^\star$ to denote the surplus at this ideal match: $S^\star(x) = S(x, \mu(x))$. Likewise, let $f^\star(x) = \max_y \left[f(x, y) - c_y(y)\right]$. Then we can write $S^\star$ as:

$$(r + \gamma)S^\star(x) = f^\star(x) + \gamma \int \max\{S(x', y'), U(x')\} dG(x', y'|x), \tag{12}$$

which is just like the general surplus except for the fact that there is no search on the job when matched to the ideal partner $\mu(x)$.

Denote $\Delta(x, y) = S^\star(x) - S(x, y)$, and we can then rewrite (5) and (12) as:

$$(r + \gamma)\Delta(x, y) = f^\star(x) - \left[f(x, y) - c_y(y)\right] - S_e(\Delta(x, y)). \tag{13}$$

Equation (13) is an equation in $\Delta(x, y) = S^\star(x) - S(x, y)$, and hence $S^\star(x) - S(x, y)$ can be solved directly from this equation. Similarly, Equation (6) implies that

$$rU(x) = b(x) + S_u \left[S^\star(x) - U(x)\right]. \tag{14}$$

[7] When Assumption 1 is not satisfied, we cannot obtain analytical results on the matching pattern, except at the other extreme where shocks only depend on y (see Appendix D.1). For the general case, we attempt to solve the model computationally.

$U(x)$ can be solved from the above equation for any given $S^\star(x)$. Therefore, we can express both $S(x,y)$ and $U(x)$ in terms of $S^\star(x)$. And to solve $S(x,y)$ and $U(x)$, it is equivalent to solve $S^\star(x)$ satisfying Equation (12). The next theorem shows that the solution is always unique.

Proposition 2 *Under Assumption 1, the BRE is unique: there exists a unique pair of $(S(x,y),U(x))$ satisfying Equations (10) and (11).*

Proof. In Appendix. ■

In the subsequent analysis, we will impose several usual assumptions on the functions $f(x,y)$ and $c_y(y)$.

Assumption 2 *(i) c_y is an increasing, convex function: $c_y' > 0$ and $c_y'' \geq 0$; (ii) f is increasing and concave in each element: $f_x > 0$, $f_y > 0$, $f_{xx} < 0$ and $f_{yy} < 0$.*

Proposition 3 *Under Assumptions 1 and 2, there is positive assortative matching ($\mu'(x) \geq 0$) if and only if $f(x,y)$ is supermodular.*

Proof. In Appendix. ■

Proposition 4 *Under Assumptions 1 and 2, there exist $\overline{y}(x) \geq \underline{y}(x) \geq 0$ such that $y \in \mathcal{M}(x)$ if and only if $y \in [\underline{y}(x), \overline{y}(x)]$.*

Proof. In Appendix. ■

In order to obtain some further analytical results, we now introduce some specific functional form assumptions that we will also maintain in the simulations below. First, we assume the matching function is Cobb-Douglas: $m(\theta) = \varphi\theta^\alpha$. As a result of this assumption, the search intensity λ and the market tightness $\frac{v}{u}$ solve the maximization problems in (10) and (11) such that they always move in the same direction: if search intensity λ increases (decreases) then market tightness θ also increases (decreases).

Second, we assume the production technology is given by:

$$f(x,y) - c_y(y) = \omega\left(x^\beta y^{1-\beta} - (1-\beta)y\right) + (1-\omega)\beta x \tag{15}$$

with degree of complementarity ω. The total complementarity is given by the cross-partial and varies by x, y. We will denote by $\Omega = \omega\beta(1-\beta)$ since $f_{xy}(x,y) = \omega\beta(1-\beta)x^{\beta-1}y^{-\beta}$. This technology implies 1. that the ideal, frictionless match for all x is the diagonal $\mu(x) = x$; and 2. that by virtue of the

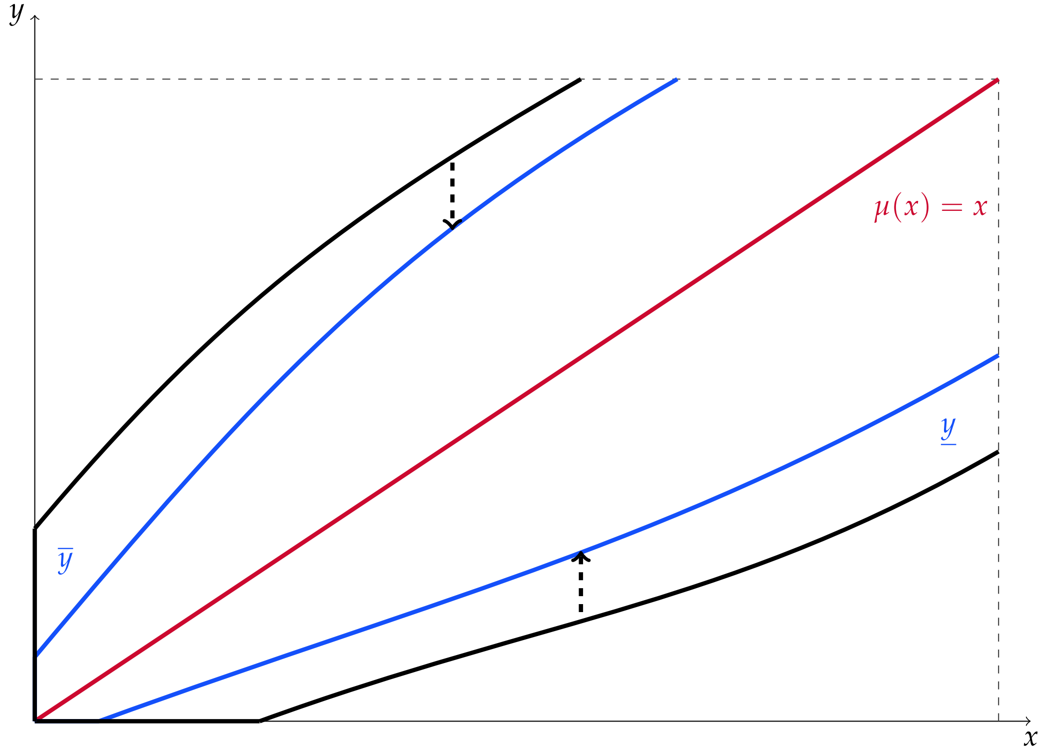


Figure 2: A higher ω leads to narrower boundaries, but does not change $\mu(x)$

term $-\omega(1-\beta)y$, an increase in the degree of complementarity ensures that the frictionless allocation is invariant of ω: $f^\star(x) = f(x,\mu(x)) - c_y(\mu(x)) = \beta x$. Therefore, as illustrated by Figure 2, ω controls the narrowness of the profitable match range, without moving the max or the argmax of the $f(x,y) - c_y(y)$ function.

Third, we model shocks only to the worker type x, not to the firm type y, and do this by means of the normal distribution with mean $(1-\kappa)x + \kappa$ and standard deviation σ, truncated below at $x' = 0$ and above at $x' = 1$:[8]

$$g(x'|x) \sim \mathcal{N}((1-\kappa)x + \kappa, \sigma), \forall x \in [0,1], \tag{16}$$

and where $\kappa \in [0,1]$ is a measure of the expected growth rate of x. To see this, the expected value of x' (the mean of the normal distribution) is determined by $(1-\kappa)x + \kappa$.[9] This is a weighted sum of x and 1, so if $\kappa = 0$ this is a martingale and the expected value of x' is equal to x; instead, if $\kappa = 1$ all workers immediately jump to the highest type $x = 1$. Then if wage growth for the high skilled is faster, we require that $\kappa_H > \kappa_L$. Note also that with $\kappa \neq 1$, then x is increasing at a decreasing rate x: higher types x have lower growth rates.

[8] Observe that this shock technology satisfies Assumption 1 since y' is assumed to be the same as x.

[9] Because the normal is truncated, there is no exact expression for the expected value. We do know that the mean is monotonic in $(1-\kappa)x + \kappa$.

Finally, The unemployment benefit is assumed to be a constant fraction of the perfect match x, bx where b is a positive constant. The cost of search is assumed quadratic: $c_\lambda = \frac{1}{2}c\lambda^2$.

Then the system of equations that determines equilibrium is (12), (13), and (14), where we substitute the technology for (15). Given this setup we obtain the following results on the effect of a change in the technology $(\omega, \gamma, \kappa, \sigma)$ on the equilibrium outcomes $\Delta(x, y)$ (the gap in the value between the mismatched and the perfectly matched worker), $S^\star(x)$ (the value of the perfectly matched worker), $\lambda_e(x, y), \lambda_u(x)$ (the search intensity of the employed and the unemployed.

Proposition 5 *Under Assumptions 1 and 2 and with the technology (15), the comparative statics results of the technology can be summarized as follows:*

	$\omega \uparrow$	$\gamma \uparrow$	$\kappa \uparrow$	$\sigma \uparrow$
$\Delta(x, y)$	$+$	$-$	$=$	$=$
$\lambda_e(x, y)$	$+$	$-$	$=$	$=$
$S^\star(x)$	$-$	$+/-$	$+/-$	$+/-$
$\lambda_u(x)$	$-$	$+/-$	$+/-$	$+/-$

Proof. In Appendix. ■

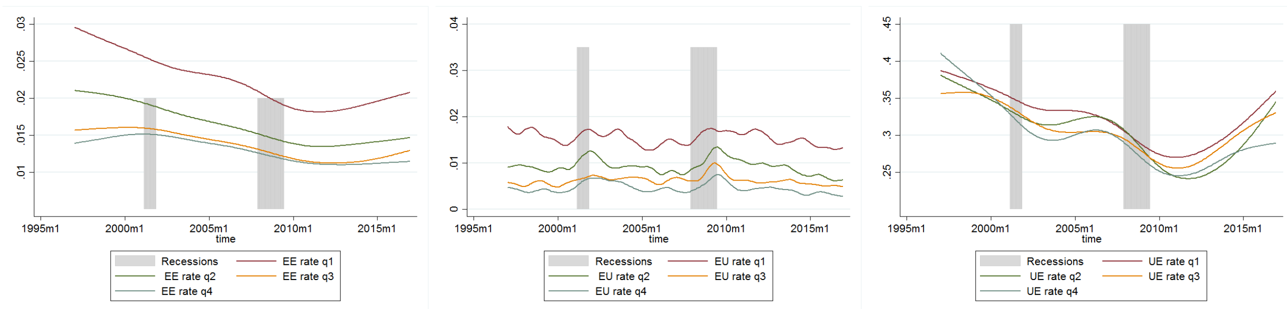
The effect of complementarity ω in the production technology is unambiguous: it increases the gap between the mismatch and the perfect match values (by the setup of the technology), and as a result, this increases search intensity on the job $\lambda_e(x, y)$; it decreases value of the perfect match since the mismatch in anticipation is of lower value, and this in turn decreases the search intensity of the unemployed.

The effect of the arrival rate γ is negative on the gap $\Delta(x, y)$ and therefore on $\lambda_e(x, y)$ and ambiguous on $S^\star(x)$ and therefore also ambiguous on $\lambda_u(x)$. Faster arrival of shocks implies more mismatch and hence lower $\Delta(x, y)$. That will lower $S^\star(x)$, but $S^\star(x)$ is also affected by the matching range between unemployment and remain matched, rendering the effect ambiguous. The ambiguity of effects in general stems from the fact that the matching range changes in equilibrium, which changes the continuation value.

The effect of the mean old the variance of the shock does not change the mismatch value gap $\Delta(x, y)$ (and hence search intensity on the job), where again the effect on $S^\star(x)$ (and $\lambda_u(x)$) is ambiguous because of the change in the matching range.

In general, we cannot make any unambiguous predictions about the effect of technological change on the flow rates (UE, EE, EU). Those are averages integrated over all accepted matches. The reason, again, is that the range of accepted matches changes with changes in the technology. The impact on flows is particularly big because the at the boundaries, search intensity is highest (search intensity increases the further away from the ideal match). Any change in the boundaries has therefore big effects on the flow rates.

4 Quantitative Exercise



(a) EE flow rate (b) EU flow rate (c) UE flow rate

Figure 3: Data Moments. Detrended using HP filter (smoothing parameter 1600 $\times 3^4$. Deseasonalized using the Census Bureau's X-13 ARIMA.

We now quantitatively analyze the implications of changes in the technology and how these changes affect the job market flows. Our objective is to solve the steady state equilibrium allocation numerically. To that end, we solve the values through value iteration and we derive the boundaries of matched and unemployed worker types. Then we derive the steady state distribution of matched and unmatched types. The algorithm is described in detail in Appendix C. We do this on a 50 point grid for the types x and y. We assume that y is uniform on $[\underline{y}, 1]$ with $\underline{y} = 0.1$. For $f_0(x)$, the distribution of newly born workers, we use the truncated normal by restricting a normal distribution with mean 0 and variance 0.05 to the interval of $[0, 1]$. At rate γ, a shock arrives and changes the worker type x according to a truncated normal on $[0, 1]$:

$$g(x'|x) \sim \mathcal{N}((1-\kappa)x + \kappa, \sigma), \forall x \in [0, 1].$$

We allow both γ and σ to be functions of x: $\gamma(x) = \gamma_0(1 - \gamma_1 x)$ and $\sigma(x) = \sigma_0(1 - \sigma_1 x)$. To simplify our analysis, we set $\kappa = 0$.

Given the above distribution assumptions, we can simulate our model for the 50 worker and job types, and match the 4 quartiles from the model to the data. With this numerical solution in hand for a set of parameters, we then estimate the model to match three moments for each of 4 quartiles in the data: the flow rates UE, EE, and EU.[10] The data moments we pick are those that match the trend in the series

[10]Implicitly, we also match the unemployment rate u, which is implied by UE, EE, and EU.

between January 1997 and September 2016, see Figure 3. Our unit of time is one month and we convert the monthly flow rate to instantaneous flow rates to match the continuous time model.[11]

For the quantitative exercise, we make additional assumptions on our model. First, we assume that $\phi_e = \phi_u = \phi(x) = \phi_0(1 - \phi_1 x)$. Second, we assume that $r = \delta$, and hence $\rho = 0$. Finally, we assume that $b(x)$ is determined by $b(x) = b_0 + b_1 x$. In total, the model has 14 parameters: ω, b_0, b_1, σ_0, σ_1, γ_0, γ_1, ϕ_0, ϕ_1, c, k, α, β and r. We set the last 3 of these parameters to be exogenously given and estimate the remaining 11. The exogenous parameter values are: $\alpha = 0.5$, $\beta = 0.7$ and $r = 0.005$.

The estimated parameters are listed in Table 2 below. We have 12 moments and 11 parameters to be estimated. We calculate the steady state of the model to match those moments in January 1997. We then calculate this again to match those moment in September 2016.

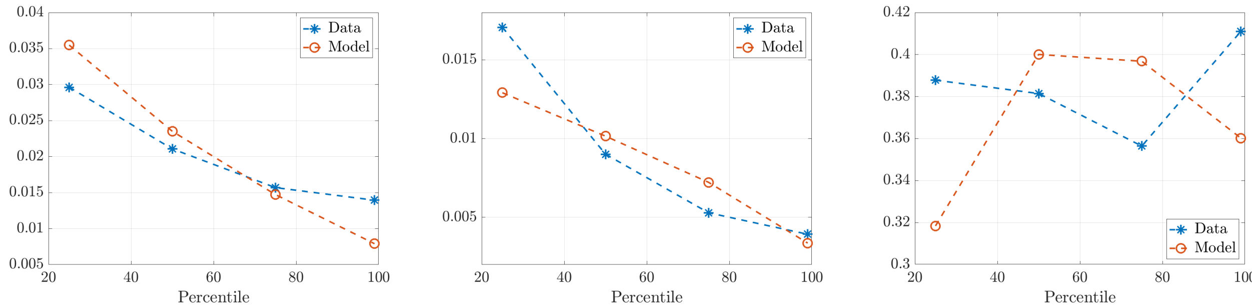
Simulation Results. In Table 1 and Figure 4 we report the data moments as well as those from the estimated Model. We systematically match UE, EE and u very well, but less so EU. Our model tends to underestimate the EU rate, and this deviation is increasing (from -10% in 1997 to -26% in 2016). This holds true for the two estimations of the sub economies by skill as well. The heterogeneity may be due to the the inability of our model to match EU because our underestimate for the high skill workers decreases while it decreases for the low skill workers.

Table 1: Data Targets and Model Estimates (in %)

	1997			2016			% Change 97-16	
	Data	Model	%Δ	Data	Model	%Δ	Data	Model
ue_{q1}	38.78	31.84	-17.90	35.97	27.05	-24.81	-7.25	-15.06
ue_{q2}	38.15	40.01	4.88	34.54	37.03	7.21	-9.46	-7.45
ue_{q3}	35.65	39.68	11.33	33.09	37.13	12.22	-7.18	-6.44
ue_{q4}	41.10	36.03	-12.36	28.96	34.36	18.66	-29.54	-4.61
ee_{q1}	2.96	3.55	19.78	2.08	1.55	-25.41	-29.77	-56.27
ee_{q2}	2.11	2.35	11.61	1.47	1.37	-6.83	-30.25	-41.78
ee_{q3}	1.57	1.47	-6.20	1.30	1.09	-15.75	-17.32	-25.74
ee_{q4}	1.39	0.79	-43.31	1.15	0.72	-37.42	-17.63	-9.07
eu_{q1}	1.71	1.29	-24.34	1.29	1.12	-13.54	-24.30	-13.50
eu_{q2}	0.90	1.02	12.70	0.61	0.54	-11.12	-32.75	-46.96
eu_{q3}	0.53	0.72	36.49	0.49	0.31	-36.48	-7.00	-56.72
eu_{q4}	0.39	0.34	-14.28	0.30	0.31	4.96	-24.19	-7.17

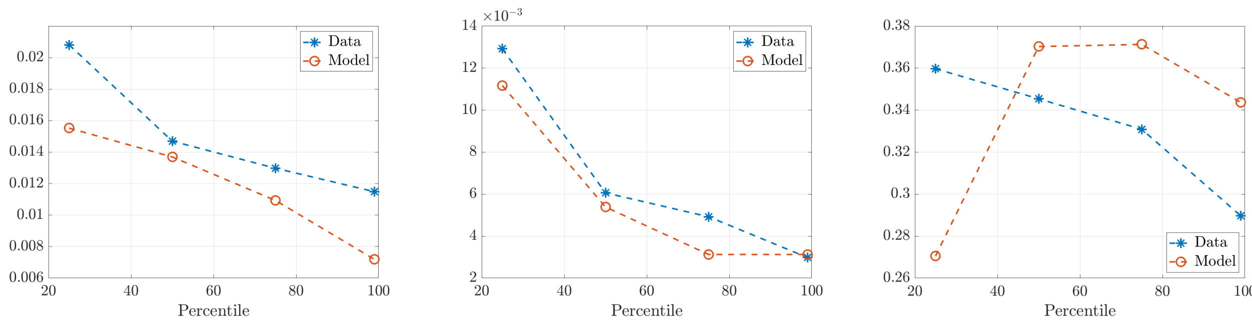
Note. For the data, we use the trend of the data over the entire period (1996-2016) to avoid cyclical and seasonal fluctuations.The 1996 data point corresponds to the trend in January 1996 and the 2016 data point corresponds to the trend in September 2016.

[11] We set $EU_{data} = 1 - e^{-EU}$.



(a) EE flow rate (b) EU flow rate (c) UE flow rate

Figure 4: Data-model Match 1997.



(a) EE flow rate (b) EU flow rate (c) UE flow rate

Figure 5: Data-model Match 2016.

We report the parameter estimates in Table 2. The parameters are grouped into three categories: complementarity, shocks and matching technology. The complementarity parameters in the Cobb-Douglas technology consist of ω, the multiplicative term, and β, the input share on worker skills. We find an increase in the complementarity through the rise in ω and the input share β.

Using the estimated parameters, in Figure 6 we plot the resulting model determinants and how they change with x: unemployment benefit $b(x)$, the shock arrival rate $\gamma(x)$, match efficiency $\varphi(x)$, and the shock variance $\sigma(x)$. Those plots show how the estimated determinants vary between 1997 and 2016 and across different worker types x.

The change in the estimated parameters that pertain to the shocks shows that shocks have become substantially less frequent (γ), and that the variance (σ) has gone down. Both are measures of the volatility. The parameters that pertain to the matching technology, entry and search costs and the exogenous separation rates show a decline in match efficiency (φ) and an increase in the search cost (c). The entry cost for firms (kk) has declined.

The UE rates drop because search frictions increase, but search frictions by themselves cannot explain the much larger relative drop in EE and EU rates compared to UE rates. A decline in the frequency of

Table 2: Parameter Estimates

		All		
		1997	2016	%Δ
ω	Complementarity	5.95	7.73	29.92
b_0	Unemployment benefit intercept	0.38	0.61	58.77
b_1	Unemployment benefit slope	0.11	0.14	29.30
σ_0	shock var. intercept	0.61	0.17	-72.05
σ_1	shock var. slope	0.63	0.27	-57.41
γ_0	shock freq. intercept	0.10	0.04	-56.76
γ_1	shock freq. slope	0.71	0.34	-51.90
φ_0	match eff. intercept	15.67	11.85	-24.40
φ_1	match eff. slope	0.33	0.25	-23.62
c	search cost	7.74	11.36	46.64
kk	entry cost	17.62	6.40	-63.66

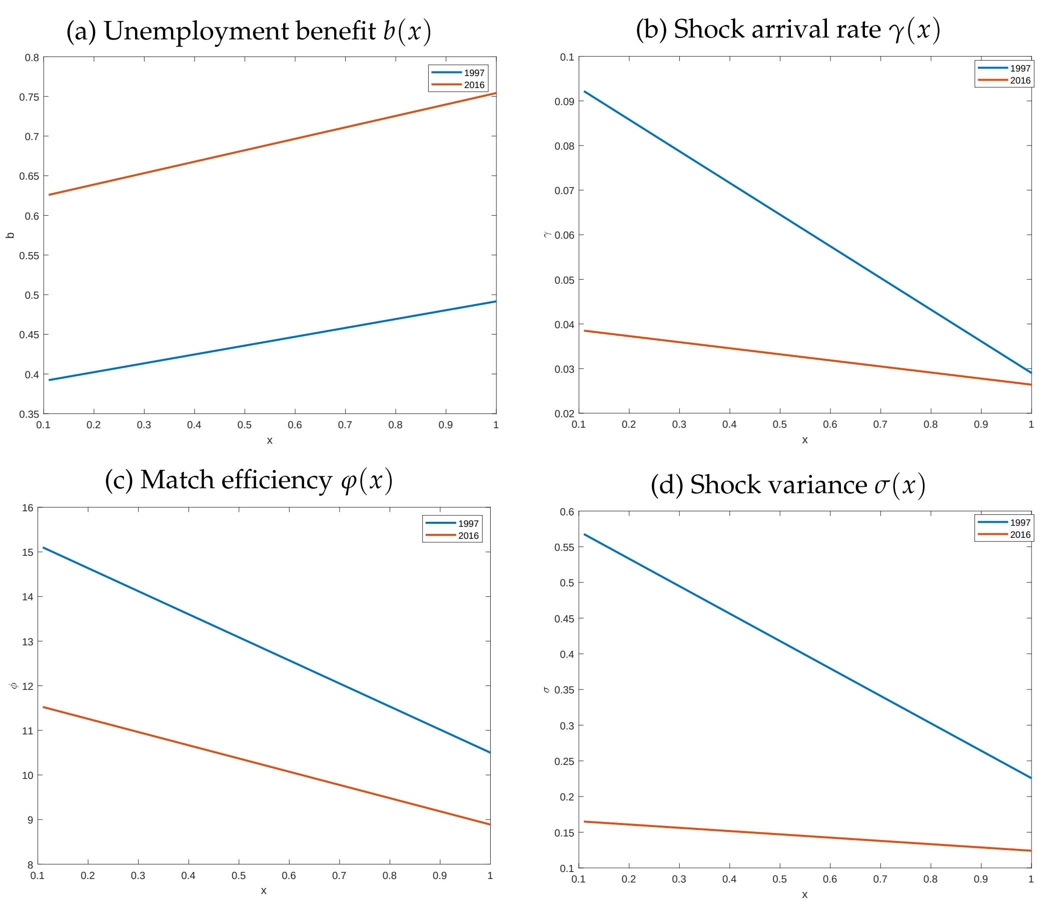


Figure 6: PARAMETER CHANGES AS A FUNCTION OF x: 1997 AND 2016

on-the-match shocks that decreases mismatch is not able to create slowdowns in EE and EU of the right magnitudes either because it slows down EU too much relative to EE. The slowdown in the shocks that create mismatch is partially offset by an increase in technological complementarity. The latter allows to offset the too-strong decline in EU while keeping more of the EE decline in place.

If we interpret shocks to type within a worker-firm match γ as mandating a change in a worker's activity, we should see a decline over time in workers changing work activity while at the same employer. This would provide suggestive evidence of a lower frequency at which these shocks occur, which is consistent with the estimated value for γ.[12]

To gain further insights into the contribution of each individual parameter to the three flow rates and unemployment, we calculate the elasticity. We start from the benchmark estimation and change one parameter at a time. We do this by inducing a 10% increase in that parameter, and we calculate the new steady state equilibrium. In this new steady state we obtain new flow and unemployment rates. We perform this exercise in both years, 1997 and 2017. Tables 3 and 4 report the elasticities obtained from that exercise.[13]

Table 3: Elasticities 1997

	σ_1	ω	γ_0	γ_1	φ_0	φ_1	c	kk	σ_0	β_0	β_1
ue_{q1}	-0.03	-0.85	-1.45	0.19	9.93	-2.20	-2.42	-4.82	-0.01	-5.58	-0.38
ue_{q2}	0.08	-0.27	-0.75	0.41	8.45	-5.15	-2.06	-4.09	-0.14	-4.16	-0.61
ue_{q3}	0.15	-0.15	-0.41	0.35	8.07	-7.51	-1.97	-3.92	-0.16	-4.10	-0.85
ue_{q4}	0.08	-0.06	-0.17	0.32	8.11	-10.99	-1.98	-3.91	-0.07	-4.04	-1.12
ee_{q1}	-0.66	5.19	3.48	-2.26	5.82	-1.19	-1.70	-3.27	1.05	-3.33	-0.77
ee_{q2}	-2.80	-1.25	4.75	-3.92	4.75	-2.74	-1.54	-2.55	2.27	-0.51	-0.05
ee_{q3}	-2.69	-0.71	4.49	-4.23	7.19	-2.98	-1.20	-2.77	2.32	-0.04	0.00
ee_{q4}	-2.21	0.20	2.95	-7.58	6.71	-6.76	-1.95	-4.03	-0.31	0.26	-0.00
eu_{q1}	-0.18	7.73	8.17	-1.27	1.57	-0.03	-0.02	-0.05	0.70	3.35	0.87
eu_{q2}	-1.36	11.00	9.11	-3.61	2.30	0.00	-0.00	-0.75	3.87	9.05	2.19
eu_{q3}	-5.06	4.73	9.42	-7.92	0.01	0.10	-0.05	-0.55	6.80	0.87	0.08
eu_{q4}	-7.05	9.52	10.14	-14.08	-0.06	0.28	-0.06	-0.03	9.38	2.70	0.16

[12] As pointed out by the referee, there is a question for employer stayers that inquires about changes in work activities in the monthly CPS, and based on personal calculation, the within firm worker activity mobility rate drops from about 1 in 1997 to about 0.7 in 2016.

[13] Read this table as follows. The elasticity of ω on EE of 5.19 means that a 10% increase in ω leads to 51.9% increase in the EE rate.

Table 4: Elasticities 2016

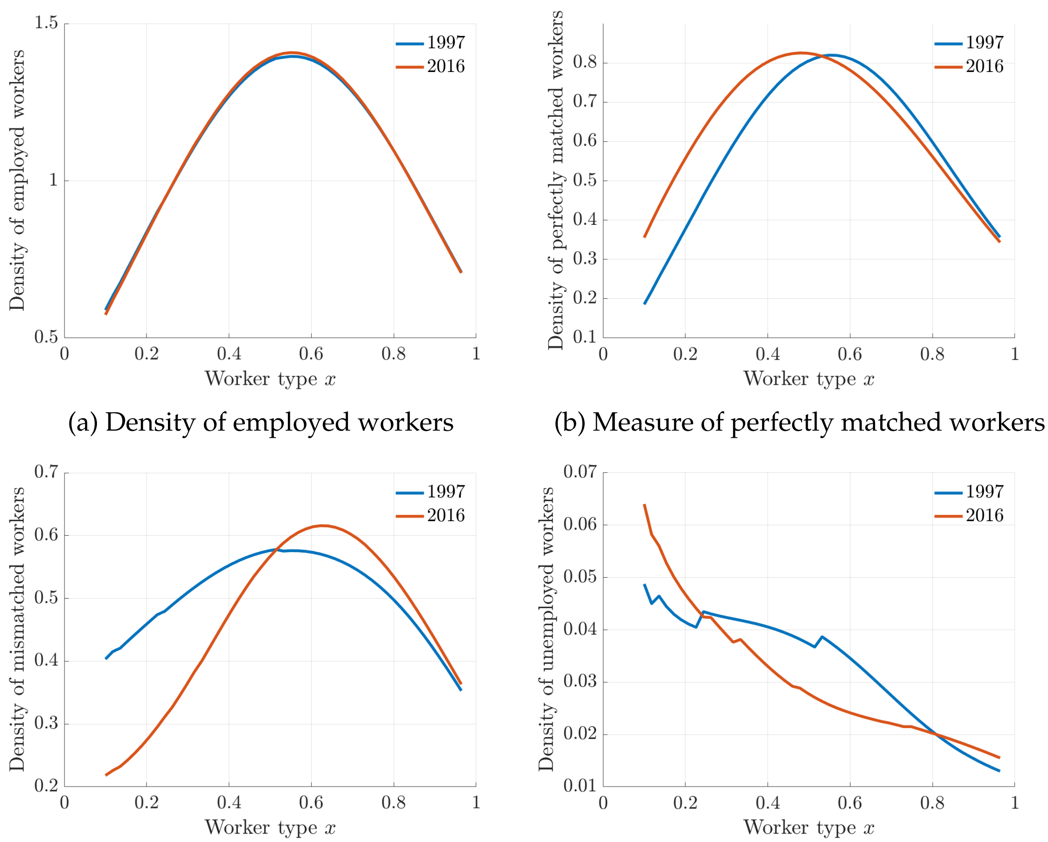
	σ_1	ω	γ_0	γ_1	φ_0	φ_1	c	kk	σ_0	β_0	β_1
ue_{q1}	-0.04	-0.31	-1.61	0.04	10.49	-1.40	-2.53	-5.04	0.23	-13.98	-0.79
ue_{q2}	0.06	-0.18	-0.49	0.09	8.44	-3.64	-2.05	-4.07	-0.37	-10.88	-1.43
ue_{q3}	0.05	-0.23	-0.27	0.08	8.19	-5.64	-1.98	-3.94	-0.24	-10.30	-2.08
ue_{q4}	0.05	-0.13	-0.17	0.08	8.26	-8.01	-1.99	-3.96	-0.17	-10.45	-3.02
ee_{q1}	-0.11	-1.47	5.84	-1.02	4.17	-0.68	-1.30	-2.25	-0.55	-12.88	-1.25
ee_{q2}	-0.84	1.69	5.05	-0.82	5.34	-1.46	-1.28	-2.64	3.69	-2.82	-0.45
ee_{q3}	-1.58	1.78	3.95	-1.83	6.65	-2.54	-1.67	-3.34	4.99	-1.96	-0.02
ee_{q4}	-2.41	5.14	2.64	-1.30	8.44	-4.26	-2.09	-4.17	6.63	0.59	-0.00
eu_{q1}	-0.51	9.84	8.76	-0.46	1.38	-0.02	-0.03	-0.64	6.78	13.28	1.27
eu_{q2}	-2.06	4.52	9.39	-1.40	0.58	-0.05	-0.04	-0.07	16.08	15.99	1.50
eu_{q3}	-0.96	11.15	9.83	-2.76	0.14	-0.02	-0.03	-0.04	8.03	13.94	1.68
eu_{q4}	1.99	0.69	9.57	-4.01	-0.01	0.00	-0.38	-0.38	-4.35	1.47	0.49

Because the variables induce changes in the flow rates that go in opposite directions, the decreases in the flow rates in this model must necessarily come from a combination of parameter changes. For similar findings, see for example Schaal (2017). The parameters that seem to have the digest impact on the flows are γ (the frequency of the shocks) and β (the input share of workers skills) with a positive elasticity on EE that is larger than one. This seems to indicate that an increase in β tends to increase the flows, as does an increase in γ.

Figure 7: Acceptance Region

We now proceed to illustrating the features of the allocation and properties of the equilibrium with a number of figures. First, Figure 7 shows the acceptance region in the model which has slightly narrowed between 1997 and 2016. We observe that due to higher search cost and lower shock frequency and variance, the acceptance region in 2016 is substantially smaller than in 1997. The decline in the acceptance region is a key driver of the decline in the flow rates.

In Figure 8 we report 4 plots that pertain to the distribution of workers and they illustrate how the endogenous distributions have changed over time. The density of employed worker types is unchanged. The density of perfectly matched worker types has shifted to the left and as a result, the density of mismatched worker types has shifted to the right. More high worker types are mismatched in 2016 than in 1997. Finally, the measure of unemployed workers is always lower for the high skilled workers than the low skilled workers, but there are more low skilled workers who are unemployed in 2016 than in 1997. Even if the average unemployment rate has not changed, the distribution has shifted towards more unemployed low skill workers.



(a) Density of employed workers

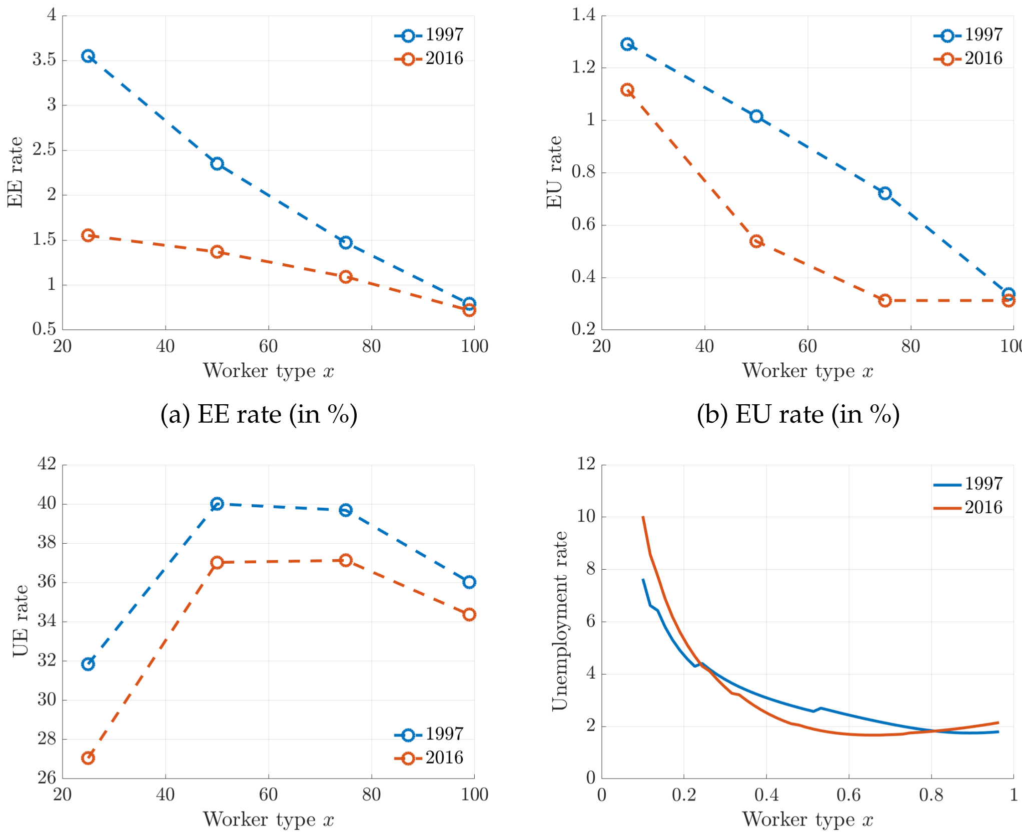
(b) Measure of perfectly matched workers

(c) Density of mismatch workers

(d) Measure of unemployed workers

Figure 8: Distribution of Workers.

The changing stationary distributions of types in Figure 8 have implications for the flow rates by worker skill x which we report in Figure 9. The UE rate has become flatter, decreased for the low skilled and slightly increased for the high skilled (Figure 9c). The EE rate that is increasing in skill has become somewhat flatter (Figure 9a), and the EU rate has uniformly shifted down for all skill levels (Figure 9b). The unemployment rate for the low skilled types x has increased from under 8% to 10% which reflects the net effect of the different flow rates.

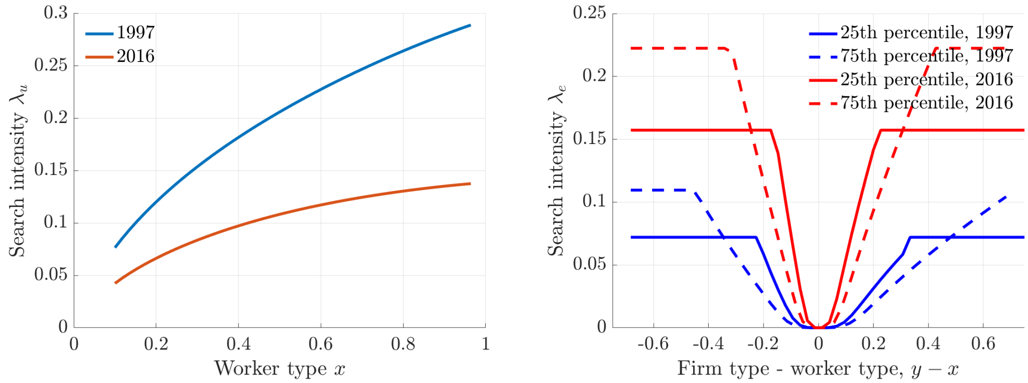


(a) EE rate (in %) (b) EU rate (in %)

(c) UE rate (in %) (d) Unemployment rate (in %)

Figure 9: Flow and Unemployment Rates.

Finally, Figure 10 illustrates the search intensity for different types. Figure 10a shows that the higher types search more intensely than the low types, but that search intensity has dropped to half in 2016 compared to 1997. The search intensity of the employed is depicted in Figure 10b. When perfectly matched ($x = \mu(x) = y$), there is no incentive to search and the search intensity is zero. The further mismatched and therefore the larger (smaller) $y - x$, the higher the search intensity. Eventually when unemployed, the search intensity is independent of the difference because the work is no longer matched to a y. Also here, the search intensity of the low skilled workers (measured here by the 25th) is lower than that of the high skilled workers (75th percentiles) and the search intensity has gone down between 1997 and 2016. This is consistent with the lower flow rates.



(a) Unemployed Search Intensity

(b) Employed Search Intensity

Figure 10: Search Intensity.

5 Concluding Comments

In this paper we have proposed a theory of the labor market with endogenous search intensity, sorting and directed on-the-job search. Matched workers are subjected to shocks and once mismatched, they start to search on-the-job to find a better allocation. Because search is directed, they can target the ideal job. The more mismatched the worker is, the higher the incentives to find the ideal job and the higher the search intensity. This leads to endogenous flow rates that vary by matched worker-job pairs.

The model generates predictions regarding the match surplus and the search intensity that are consistent with the data. We then quantify the model and estimate the model parameters to study the determinants of declining business dynamism as evidenced by declining labor market flow rates. We find that the decline in flows is driven by an increase in the complementarity between jobs and worker abilities, a decline in the variance and frequency of shocks, and an increase in the cost of search.

While the study of aggregate fluctuations is beyond the purpose of the current paper, our setup can be used to study the response of the economy to aggregate shocks. The Block Recursive nature of the equilibrium allows us to solve for the allocation without the need to keep track of the distribution. The analysis of aggregate fluctuations in this framework resembles that in Lise and Robin (2017) who model the economy by means of a random search model with sorting and on-the-job search. Our setup with directed search is simpler because it has the added advantage that the surplus of a match does not involve integrating over randomly arriving matches.

Finally, an open issue is to embed in the model non-technological determinants (such as demographic and structural change, skill composition, and market power) in conjunction with the existing technological determinants of the existing model. That would give a complete picture not only of the contribution of each determinant, but also of the interactions between them. This is a challenging venture, however, that we leave for future work.

Appendix A List of Variables

Table 5: List of Variables

x	type of the worker
y	type of the firm
$\tilde{\theta}$	market tightness
λ	search intensity
Λ	aggregate search intensity
δ	death rate
ρ	discount rate
$r = \delta + \rho$	effective discount rate
$f(x, y)$	output function
$c_y(y)$	operation cost
$c_\lambda(\lambda)$	search cost
k	entry cost
$m\left(\frac{\tilde{\theta}}{\Lambda}\right)$	matching function
$U(x)$	value of an unemployed worker
$E(x, y)$	value of an employed worker
$J(x, y)$	value of a matched firm
$S(x, y)$	value of a firm-worker pair, gross of $U(x)$; $S = E + J$
$V(y)$	value of vacancy
$G(x', y'\|x)$	transition distribution function
$\phi(x, y)$	density of mismatched workers
$u(x)$	density of unemployed workers
$\psi(x)$	density of perfectly matched workers
$v(y)$	density of vacant firms
$\nu(y)$	density of vacant firms to be matched with unemployed workers
$\Phi(x, y)$	density of vacant firms to be matched with mismatched workers

Appendix B Omitted Proofs

B.1 Proof of Proposition 1

Proof. First consider the unemployed worker's problem (1). The free entry condition $V(y) = 0$ implies that

$$J(x, \tilde{y}) = k \frac{\tilde{\theta}}{\varphi_u \Lambda} m^{-1} \left(\frac{\tilde{\theta}}{\Lambda} \right). \tag{17}$$

Plugging the above equation into equation (1) yields

$$rU(x) = b(x) + \max_{\lambda,\tilde{\theta},\Lambda,\tilde{y}} \left\{ \varphi_u \lambda m \left(\frac{\tilde{\theta}}{\Lambda} \right) [S(x,\tilde{y}) - U(x)] - c_\lambda(\lambda) - k\tilde{\theta}\frac{\lambda}{\Lambda} \right\}, \tag{18}$$

which implies equation (10) by letting $\theta = \frac{\tilde{\theta}}{\Lambda}$.

Second, for the firm's problem, notice that from equation (2), we can express $w(x,y)$ as

$$w(x,y) = rE(x,y) - \gamma \int [\eta E(x',y') + (1-\eta)U(x') - E(x,y)] \, dG(x',y'|x,y) \tag{19}$$

$$- \max_{\tilde{y},\tilde{\theta},\Lambda,\lambda} \left\{ \varphi_e \lambda m \left(\frac{\tilde{\theta}}{\Lambda} \right) [E(x,\tilde{y}) - E(x,y)] - c_\lambda(\lambda) \right\}. \tag{20}$$

We plug the above equation into equation (3) and collect terms by using $S = E + J$. This yields a rewriting of the firm's problem:

$$rS_0(x,y) = \max_{\eta,E,J} f(x,y) - c_y(y) + \gamma \int [\eta S(x',y') + (1-\eta)U(x') - S(x,y)] \, dG(x',y'|x,y)$$

$$+ \max_{\tilde{y},\tilde{\theta},\Lambda,\lambda} \left\{ \varphi_e \lambda m \left(\frac{\tilde{\theta}}{\Lambda} \right) [E(x,\tilde{y}) - E(x,y)] - c_\lambda(\lambda) \right\} - \varphi_e \lambda^\star m \left(\frac{\tilde{\theta}^\star}{\Lambda^\star} \right) J(x,y), \tag{21}$$

subject to the promise-keeping constraint $E_0(x,y) \geq \bar{E}(x,y)$.

We next will show that the optimal solution to the above problem satisfies equation (11). Obviously, $\eta S(x',y') + (1-\eta)U(x') = \max\{S(x',y'), U(x')\}$: when the firm makes its separation decision, it just compares S and U. Moreover, using the fact $E(x,y) = S(x,y) - J(x,y)$ and the free entry condition (17), we can rewrite $\varphi_e \lambda m \left(\frac{\tilde{\theta}}{\Lambda} \right) [E(x,\tilde{y}) - E(x,y)] - c_\lambda(\lambda)$ as

$$\varphi_e \lambda m \left(\frac{\tilde{\theta}}{\Lambda} \right) [E(x,\tilde{y}) - E(x,y)] - c_\lambda(\lambda)$$

$$= \varphi_e \lambda m \left(\frac{\tilde{\theta}}{\Lambda} \right) [S(x,\tilde{y}) - S(x,y)] - c_\lambda(\lambda) - k\lambda \frac{\tilde{\theta}}{\Lambda} + \varphi_e \lambda m \left(\frac{\tilde{\theta}}{\Lambda} \right) J(x,y). \tag{22}$$

Since $\lambda^\star$, $\tilde{\theta}^\star$ and $\Lambda^\star$ are the optimal solutions,

$$\max_{\tilde{y},\tilde{\theta},\Lambda,\lambda}\left\{\varphi_e\lambda m\left(\frac{\tilde{\theta}}{\Lambda}\right)[E(x,\tilde{y})-E(x,y)]-c_\lambda(\lambda)\right\}-\varphi_e\lambda^\star m\left(\frac{\tilde{\theta}^\star}{\Lambda^\star}\right)J(x,y)$$
$$\leq \max_{\tilde{y},\tilde{\theta},\Lambda,\lambda}\left\{\varphi_e\lambda m\left(\frac{\tilde{\theta}}{\Lambda}\right)[S(x,\tilde{y})-S(x,y)]-c_\lambda(\lambda)-k\lambda\frac{\tilde{\theta}}{\Lambda}\right\}. \tag{23}$$

Moreover, when $\lambda^\star m\left(\frac{\tilde{\theta}^\star}{\Lambda^\star}\right)>0$, the above equality is achieved only if $J(x,y)=0$; when when $\lambda^\star m\left(\frac{\tilde{\theta}^\star}{\Lambda^\star}\right)=0$, the above equality holds for any $J(x,y)\geq 0$. We also know from the free entry condition (17) that the firm's value has to be strictly positive upon matching with the worker.

Therefore, the optimal contract has to satisfy that when a new match is formed, the firm obtains a positive continuation value since the worker does search on the job; when the worker starts to search on the job, the firm obtains zero continuation value. In both cases, it is straightforward to verify that the value $S=E+J$ satisfies equation (11) by letting $\theta=\frac{\tilde{\theta}}{\Lambda}$.

The optimal contract can be implemented by many wage schemes. One wage scheme is designed as follows. When a $(x,\tilde{y})$ match is initially formed, the wage $w(x,\tilde{y})$ satisfies

$$rJ(x,\tilde{y})=f(x,\tilde{y})-c_y(\tilde{y})-w(x,\tilde{y})-\gamma J(x,\tilde{y}).$$

The firm pays a fixed wage $w(x,\tilde{y})$ to the worker, and when a shock arrives, either the pair is separated or the worker starts to search on the job, as the new match is not the best one with probability one under the continuous density assumption. Therefore, the firm always obtains zero value upon the arrival of the shock. We can immediately solve $w(x,\tilde{y})=f(x,\tilde{y})-c_y(\tilde{y})-(r+\gamma)k\frac{\tilde{\theta}}{\varphi_u\Lambda}m^{-1}\left(\frac{\tilde{\theta}}{\Lambda}\right)$. After the arrival of a shock, the firm's continuation value becomes $J(x',y')=0$ and hence $w(x',y')=f(x',y')-c_y(y')$.

■

B.2 Proof of Proposition 2

Proof. First of all, it is straightforward to see that there always exists a unique $\Delta(x,y)$ solving Equation (13). For any given $S^\star(x)$, the solution to Equation (14) is unique as well: $U(x)=\frac{b(x)}{r}$ if $S^\star(x)\leq\frac{b(x)}{r}$, and there is a unique $U(x)>\frac{b(x)}{r}$ solving Equation (14) when $S^\star(x)>\frac{b(x)}{r}$.

Second, Equation (12) implies that

$$S^\star(x)=\frac{f^\star(x)}{r+\gamma}+\frac{\gamma\int\max\{S(x',y'),U(x')\}dG(x',y')}{r+\gamma}. \tag{24}$$

Denote

$$A = \frac{\gamma \int \max\{S(x', y'), U(x')\} dG(x', y')}{r + \gamma}. \tag{25}$$

Then Equation (24) implies that $S^\star(x; A) = \frac{f^\star(x)}{r+\gamma} + A$. Since the solutions to Equations (13) and (14) are unique, we obtain uniquely $S(x', y'; A)$ and $U(x'; A)$ for a given $A \geq 0$. Plug these expressions back to Equation (25), and we get an equation about A:

$$A = \frac{\gamma \int \max\{S(x', y'; A), U(x'; A)\} dG(x', y')}{r + \gamma}. \tag{26}$$

Obviously, when $A = 0$, the RHS is larger than the LHS. Moreover, since $S^\star(x; A) = \frac{f^\star(x)}{r+\gamma} + A$ and $S(x, y; A) = S^\star(x; A) - \Delta(x, y)$, we obtain $\frac{\partial S}{\partial A} = 1$. Meanwhile, Equation (14) implies that $0 \leq \frac{\partial U}{\partial S^\star} < 1$ and hence $0 \leq \frac{\partial U}{\partial A} < 1$. Therefore, the derivative of the RHS with respect to A is larger than 0, but less than $\frac{\gamma}{r+\gamma} < 1$. As a result, there must exist a unique A solving the above equation. The uniqueness of A directly implies that $S(x, y)$ and $U(x)$ are unique as well. ■

B.3 Proof of Proposition 3

Proof. Since the optimal $\tilde{y} = \mu(x)$ is always chosen to maximize $f(x, y) - c_y(y)$, we obtain

$$f_x(x, \mu(x)) = c_y'(\mu(x)). \tag{27}$$

Total differentiation implies that

$$\mu'(x) = \frac{f_{xy}}{c_y'' - f_{yy}}. \tag{28}$$

By Assumption 1, $c_y'' - f_{yy} > 0$ and hence $\mu'(x) \geq 0$ if and only if $f(x, y)$ is supermodular: $f_{xy} = 0$. ■

B.4 Proof of Proposition 4

Proof. From Equation (13), $\Delta(x, y) = S^\star(x) - S(x, y)$ is increasing in $f^\star(x) - \left[f(x, y) - c_y(y)\right]$. $f^\star(x) - \left[f(x, y) - c_y(y)\right] = 0$ when $y = \mu(x)$, and Assumption 1 implies that $f^\star(x) - \left[f(x, y) - c_y(y)\right]$ is increasing in $|y - \mu(x)|$. Since the optimal $\tilde{y} = \mu(x)$ is always chosen to maximize $f(x, y) - c_y(y)$, we obtain

$$f_x(x, \mu(x)) = c_y'(\mu(x)). \tag{29}$$

And $y \in \mathcal{M}(x)$ if and only if $S(x, y) \geq U(x)$. Therefore, there exist $\overline{y}(x) \geq \underline{y}(x) \geq 0$ such that $y \in \mathcal{M}(x)$ if and only if $y \in [\underline{y}(x), \overline{y}(x)]$. In particular, the interior bounds $\underline{y}(x), \overline{y}(x)$ are determined by $S(x, \underline{y}) = U(x)$ and $S(x, \overline{y}) = U(x)$. ■

B.5 Proof of Proposition 5

Proof. The comparative static results with respect to $\Delta(x, y)$ all come from Equation (13). From that equation, we have: 1) When the degree of supermodularity ω increases, then $f^\star(x) - [f(x, y) - c_y(y)]$ goes up and hence $\Delta(x, y)$ goes up as well; 2) When the shock arrival rate γ increases, then $\Delta(x, y)$ should decrease to satisfy Equation (13); 3) When κ or σ changes, then $\Delta(x, y)$ should not change because Equation (13) is not affected. The change of Δ immediately implies the change in λ_e.

The comparative static results with respect to $S^\star(x)$ all come from Equation (12). When the degree of supermodularity ω increases, $\Delta(x, y)$ goes up and hence $S(x, y)$ goes down if $S^\star(x)$ stays the same. Since $f^\star$ is independent of ω, $S^\star(x)$ must decrease from Equation (12). Then $\lambda_u(x)$ should also go down from Equation (14). However, the impacts of other parameters on $S^\star$ are ambiguous. Take the comparative static of γ with respect to $S^\star(x)$ for example. There are two opposing effects from an increase in γ. On the one hance, if $S^\star(x)$ stays the same, $S(x, y)$ increases since Δ goes down while $U(x)$ does not change. This implies that $S^\star(x)$ should go up. On the other hand,

$$\gamma \left[\int \max\{S(x', y'), U(x')\} dG(x', y') - S^\star(x) \right]$$

becomes more negative as γ goes up when $\int \max\{S(x', y'), U(x')\} dG(x', y') - S^\star(x) < 0$. Numerical results suggest that the change of $S^\star$ is indeed ambiguous as γ increases. ■

Appendix C Numerical Algorithm

Step 1 Calculate the equilibrium value functions $S(x, y)$ and $U(x)$ for any (x, y). For any initial guess of $S(x, x)$, we can compute $S(x, x) - S(x, y)$ using m-files EW1 and EW2; and compute $S(x, x) - U(x)$ using m-files EW3 and EW4. For ng grids of x and a set of initial guesses $S_0(x, x)$, we can hence numerically evaluate

$$\int \max\{S(x', y'), U(x')\} dG(x', y'|x) = \sum_{i,j} \max\{S(x_i, y_j), U(x_i)\} \Delta x \Delta y \tag{30}$$

and get a new update $S_1(x, x)$. Keep this process until $S(x, x)$ converges.

Step 2 After solving $S(x, x)$, we can first characterize the boundaries. Then we also take ng grids of x uniformly distributed on $[0, 1]$, and $2 * ng + 1$ grids of y uniformly distributed on $[0, 2]$. We let $A(i, j) = 1$ if (x_i, y_j) is in the acceptance region, and $= 0$ otherwise. We can define vector xx corresponding to the i's with $A(i, j) = 1$, and vector yy corresponding to the j's with $A(i, j) = 1$. In other words, $A(xx(i), yy(i)) = 1$ for any i.

Step 3 From the equations of the stationary density, we get:

$$u(x) = \frac{\gamma+\delta}{\lambda_u m\left(\frac{\tilde{\theta}_u}{\lambda_u}\right)}\psi(x) - \frac{\int_{\underline{y}}^{\bar{y}} \phi(x,y)\lambda_y m\left(\frac{\tilde{\theta}_y}{\lambda_y}\right) dy}{\lambda_u m\left(\frac{\tilde{\theta}_u}{\lambda_u}\right)}, \tag{31}$$

and hence

$$\begin{aligned}
&\left(\lambda_u m\left(\frac{\tilde{\theta}_u}{\lambda_u}\right)+\delta\right)\left\{\frac{\gamma+\delta}{\lambda_u m\left(\frac{\tilde{\theta}_u}{\lambda_u}\right)}\psi(x) - \frac{\int_{\underline{y}}^{\bar{y}} \phi(x,y)\lambda_y m\left(\frac{\tilde{\theta}_y}{\lambda_y}\right) dy}{\lambda_u m\left(\frac{\tilde{\theta}_u}{\lambda_u}\right)}\right\} \\
= \quad &\gamma\int \Pr\Big(y \notin (\underline{y},\bar{y})|\tilde{x}\Big)\phi(\tilde{x},\tilde{y})d\tilde{x}d\tilde{y} + \gamma\int \Pr\Big(y \notin (\underline{y},\bar{y})|\tilde{x}\Big)\psi(\tilde{x})d\tilde{x} + \delta f_0(x).
\end{aligned}$$

We take $f_0 = 1$: the new entrant's x follows a uniform distribution. Discretizing implies that:

$$\begin{aligned}
&(MPu_i+\delta)\left\{\frac{\gamma+\delta}{MPu_i}\psi_i - \frac{\sum_j \phi_{i,j} MPe_{i,j}\Delta y}{MPu_i}\right\} \\
= \quad &\gamma\sum Prob_{i,s}\phi_{s,j}\Delta x\Delta y + \gamma\sum Prob_{i,s}\psi_s\Delta x + \delta,
\end{aligned}$$

where MPu and MPe denote the matching probability for the unemployed and employed workers respectively, and $Prob_{i,s}$ is the density that the new x is x_i, the old x is x_s, and the match is separated. Finally, we have

$$\left(\gamma+\delta+MPe_{i,j}\right)\phi_{i,j} = \gamma\sum g(x_i,y_j|x_t)\phi_{t,s}\Delta x\Delta y + \gamma\sum g(x_i,y_j|x_t)v_t\Delta x. \tag{32}$$

The above system of equations are N linear equations about N unknowns where N is the length of the vector xx. Solving this system of linear equations gives us the stationary densities.

Appendix D Further Results

D.1 More General Shocks

1. Shocks that depend on y only. This includes y fixed: $G(x', y'|y)$.

Now the value function has a term $H(y)$ that depends on y where $H(y) = \int \max\{U(x'), S(x', y)\}dG(x'|y)$.

The value function of the surplus now is:

$$(r+\gamma)S(x,y) = f(x,y) - c_y(y) + \gamma \int \max\{U(x'), S(x',y)\} dG(x'|y) + \max_{\lambda,\theta,\tilde{y}} \{\varphi_e \lambda m(\theta) [S(x,\tilde{y}) - S(x,y)] - c_\lambda(\lambda) - k\lambda \tag{33}$$

This affects the FOC of the choice of y:

$$f_y(x,y) - c_y'(y) + \gamma H'(y) = 0. \tag{34}$$

The SOC satisfies:

$$f_{yy}(x,y) - c_y''(y) + \gamma H''(y) < 0. \tag{35}$$

Evaluating the FOC at the equilibrium allocation $x = \mu(y)$ and taking the total derivative yields:

$$f_{yy}(\mu,y) + f_{xy}(\mu,y)\mu'(y) - c_y''(y) + \gamma H''(y) = 0. \tag{36}$$

Jointly with the SOC this implies there is PAM provided $f_{xy} > 0$.

2. Shocks that depend on y and x: $G(x'|x,y)$.

Now the term H depends on both x, y

$$(r+\gamma)S(x,y) = f(x,y) - c_y(y) + \gamma \int \max\{U(x'), S(x',y)\} dG(x'|x,y) + \max_{\lambda,\theta,\tilde{y}} \{\varphi_e \lambda m(\theta) [S(x,\tilde{y}) - S(x,y)] - c_\lambda(\lambda) - k\lambda\theta\} \tag{37}$$

and FOC condition is:

$$f_y(x,y) - c_y'(y) + \gamma H_y(x,y) = 0. \tag{38}$$

The SOC satisfies:

$$f_{yy}(x,y) - c_y''(y) + \gamma H_{yy}(x,y) < 0. \tag{39}$$

Evaluating the FOC at the equilibrium allocation $x = \mu(y)$ and taking the total derivative yields:

$$f_{yy}(\mu,y) + f_{xy}(\mu,y)\mu'(y) - c_y''(y) + \gamma \left(H_{yy}(\mu,y) + H_{xy}(\mu,y)\mu'\right) = 0. \tag{40}$$

Then there is PAM provided:

$$f_{xy}(\mu,y) + \gamma H_{xy}(\mu,y) > 0. \tag{41}$$

When is $H_{xy} > 0$? Let y be fixed and therefore G does not depend on y. Ignore the maximization over U, and define $\tilde{H}$ as:

$$\tilde{H}(x,y) = \int S(x,y) dG(x'|x) \tag{42}$$

then

$$\tilde{H}_y = \int S_y(x'|y)dG(x'|x) \tag{43}$$

and

$$\tilde{H}_{xy} = \int S_y g_x dx = -\int S_{xy} G_x(x'|x)dx' \tag{44}$$

as a result, under FOSD of G in x ($G_x < 0$) we obtain that $H_{xy} > 0$ whenever $X_{xy} > 0$.

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