# Moral Hazard: Dynamic Models

# Preliminary Lecture Notes

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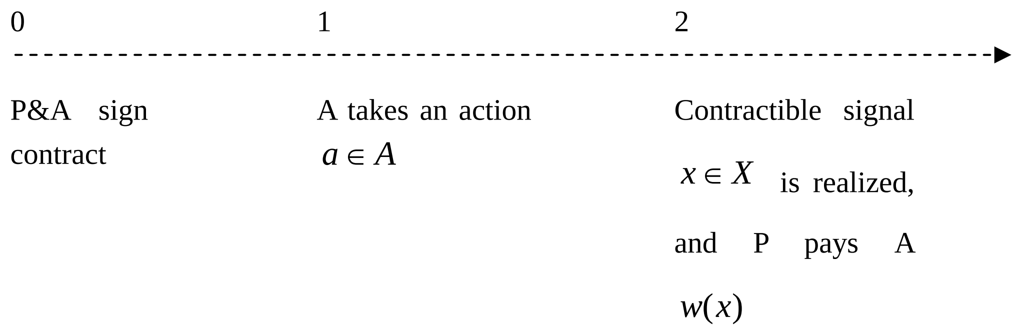
## Contents




# 1 Static Moral Hazard Model: An Overview

## 1.1 Model Setup

There are two players: the Principal (P) who owns the firm; and the Agent (A), who operates the firm.[1] Action $a \in A$ is NOT contractible and $A \subset \mathbb{R}$ is a compact set. $x \in X = [\underline{x}, \bar{x}]$ is a contractible signal. To be contractible, $x$ has to be observable, *ex ante describable* and *ex post verifiable*. A *contract* is denoted as $\{w(x)\}_{x \in X}$, where $w(x) \in \mathbb{R}$ is usually interpreted as the dollar payment from P to A. Each action $a$ yields a distribution $F(\cdot|a)$ of signals $x$ on $X$. This includes both continuous and discrete distributions. Let

$$X(a) = \text{Supp}F(\cdot|a) \subset X.$$

**Assumption 1** *(Common Support)* $X(a) = X, \forall a \in A.$

## 1.2 Utilities

The agent has a Bernoulli utility function $U(a, x, w) = u(w) - c(a)$, where $u' > 0$ and $u'' \leq 0$ guarantee risk aversion. As for $c(\cdot)$, we assume that effort is costly, $c' > 0$ and we usually

[1]The moral hazard models are widely applied to investigations of firms and organizations. However, this literature begins with the study of sharecropping (Cheung (1969) and Stiglitz (1974)).

assume that the marginal cost of effort increases with effort, $c'' > 0$.

An action-contract pair $(a, w(\cdot))$ generates expected payoffs:

$$\mathbb{E}_x[u(w(x))|a] - c(a).$$

**Remark 1** *A's utility is additively separable in payment $w$ and action $a$. This makes his risk preferences over money lotteries independent of his action. This simplifies by letting us rule out random contracts, i.e., contracts that specify for each $x$ a random payment.*

The principal has a Bernoulli utility function $V(a, x, w) = v(x - w)$, where $v' > 0$ and $v'' \leq 0$. An action-contract pair $(a, w(\cdot))$ generates expected payoffs:

$$\mathbb{E}_x[v(x - w(x))|a].$$

## 1.3 Main Results

The workhorse model sketched above makes two important assumptions: the agent is risk averse and the output is noisy. Therefore, rewarding the agent based on $x$ put randomness into his income. In particular, making the rewards vary more with $x$, which would normally occur if the incentives were made stronger, increases the amount of risk the agent bears. The optimal contract then trades off the cost of inefficient risk-bearing against the benefits of inducing the desired behavior less its cost to the agent. The result is typically that the agent bears excessive risk and delivers too little effort relative to the first best case. However, obtaining clear predictions about the shape of the performance contract beyond this basic one proved very difficult without putting stronger assumptions on the preferences and the information structures.

Given the difficulties of working with a general model, the highly tractable linear-exponential-normal specification (Linear Contracts, Normally Distributed Performance, and Exponential Utility) has become widely used. It can be shown that incentives are smaller if the marginal productivity of effort is lower, the agent is more risk averse, there is more noise in the performance measure, or the agent's effort choice is less responsive to increased incentives.

The empirical tests of the workhorse model find very mixed results. Prendergast (1999) summarized the empirical research on the risk/incentive tradeoff by stating, "there is some evidence that contracts are designed to optimally trade off risk against incentives" and "it would not appear that on the margin, the risk measures that have been considered are the true constraining factors on the provision of incentives." Meanwhile, research on executive compensations finds that CEO compensation is not tied to performance: a typical S&P 500 CEO receives between $3.25 (Jensen and Murphy, 1990) and $5.29 (Hall and Liebman, 1998) for every thousand-dollar increase in shareholder wealth.

In this course, we are going to discuss dynamic moral hazard models. Due to some dynamic benefits, the agent is willing to exert effort even though there is no static incentive. The models to be discussed include career concern models, relational contract models, and reputation models.

# 2 Career Concern Models

Career concerns were first discussed by Fama (1980), who argued that incentive contracts are not necessary because managers are disciplined through the managerial labor market:

superior performances will generate high wage offers; poor performances, low offers. Holmström (1999) showed that although such labor-market discipline can have substantial effects, it is not a perfect substitute for contracts: in the absence of contracts, managers typically work too hard in early years(while the market is still assessing the manager's ability) and not hard enough in later years. Gibbons and Murphy (1992) conclude from Fama's and Holmström's work that contracts are necessary to provide managers with optimal incentives; and the explicit incentives from the compensation contract should be strongest for workers close to retirement, because career concerns are weakest for these workers. Gibbons and Murphy (1992) provide with empirical support for this prediction in the relation between chief-executive compensation and stock-market performance.

## 2.1 A Two-Period Example

### 2.1.1 Model Settings

1. Manager, M, is risk neutral and $U = w_1 - C(e_1) + w_2 - C(e_2)$.

2. Technology: output at any of many identical, risk neutral firms is

$$x_t = e_t + a + u_t,$$

   where $a$ is a time-invariant characteristic (for example, individual ability), $e_t$ is the effort put M at time $t$, and $u_t$ is a transient shock.

3. Information structure:

   - $e_t$ privately chosen by M at time $t$;

- $x_t$ publicly observed at the end of period $t$;
- $a$, $u_1$ and $u_2$ initially unknown to *everyone*; independently and normally distributed with mean 0;
- $\tau = \frac{var(a)}{var(a)+var(u)}$.

4. No explicit contracts: $w_t$ fixed and paid at start of period $t$. Wage is determined in a competitive market: $w_t = Ex_t$.

### 2.1.2 Equilibrium Analysis

The equilibrium can be analyzed by backward induction. In period 2, manager sets $e_2 = 0$, since $w_2$ is fixed. Expecting this, firms will offer wage $w_2 = E(x_2|x_1) = E(a|x_1)$. Although the market cannot observe $e_1$, it can make inference about $e_1$ based on solving M's maximization problem. Denote $\hat{e}_1$ to be market's conjecture about $e_1$. This implies that the market believes that $a + u_1$ is $x_1 - \hat{e}_1$.

Notice that if $a \sim N(a_0, var(a))$ and we observe a signal $s_1 = a + u_1$, where $u_1 \sim N(0, var(u))$, then Bayesian updating implies that

$$E(a|s_1) = \frac{var(u)}{var(a) + var(u)} a_0 + \frac{var(a)}{var(a) + var(u)} s_1.$$

Therefore, we have $w_2 = E(a|x_1) = \tau(x_1 - \hat{e}_1)$. For manager, $e_1$ is chosen to maximize $\tau(Ex_1 - \hat{e}_1) - C(e_1)$. M's expected marginal benefit of $e_1$ is $\tau$ for all $\hat{e}_1$. Hence, equilibrium $e_1 > 0$ solves $C'(e_1) = \tau \leq 1$. The equilibrium is generally inefficient, because if effort is observable, it is optimal to choose $C'(e_1) = C'(e_2) = 1$.

## 2.2 Infinite Horizon Model

The two-period example can be extended to an infinite horizon model, where M's utility function is:

$$U = \sum_{t=1}^{\infty} \beta^{t-1}[w_t - C(e_t)].$$

### 2.2.1 Constant $a$ Case

If $a$ is a constant over time, then we can similarly define $\hat{e}_t$ to be market's conjecture about $e_t$, and denote $z_t = x_t - \hat{e}_t$. The posterior distributions of $a$ will stay normal with means and precisions given by:

$$m_{t+1} = \frac{h_t m_t + h_u z_t}{h_t + h_u} = \frac{h_u \sum_{s=1}^{t} z_s}{h_a + t h_u},$$

and

$$h_{t+1} = h_a + t h_u.$$

The precision $h$ is the inverse of the variance. Obviously, an increase in $e_t$ will affect $z_t$ and $m_s$ for all $s \geq t+1$. Denote

$$\gamma_t = \sum_{s=t+1}^{\infty} \beta^{s-t} \frac{h_u}{h_s},$$

and the FOC is such that $\gamma_t = C'(e_t)$. Obviously, $h_s$ increases as $s$ goes up and hence, $\gamma_t$ decreases in $t$. if ability is constant over time, equilibrium effort decreases over time: As firms learn more about manager's ability, they place less weight on new observations in updating their beliefs, so marginal return to effort goes down and converges to zero as $t \to \infty$. However, it is possible that $\gamma_t > 1$, so the agent may work inefficiently hard!

### 2.2.2 The Stationary Case

Holmström (1999) considered another case where $a_t$ follows a random walk: $a_{t+1} = a_t + \eta_t$. Then, The posterior distributions of $a_{t+1}$ will stay normal with the same means and precisions given by:

$$h_{t+1} = \frac{(h_t + h_u)h_\eta}{h_t + h_u + h_\eta}.$$

Now $h_t$ will not go to infinity with $t$ since the $\eta$-shocks keep adding uncertainty.

Denote $\mu_t = \frac{h_t}{h_t + h_u}$, and then we have:

$$\mu_{t+1} = \frac{1}{2 - \mu_t + r},$$

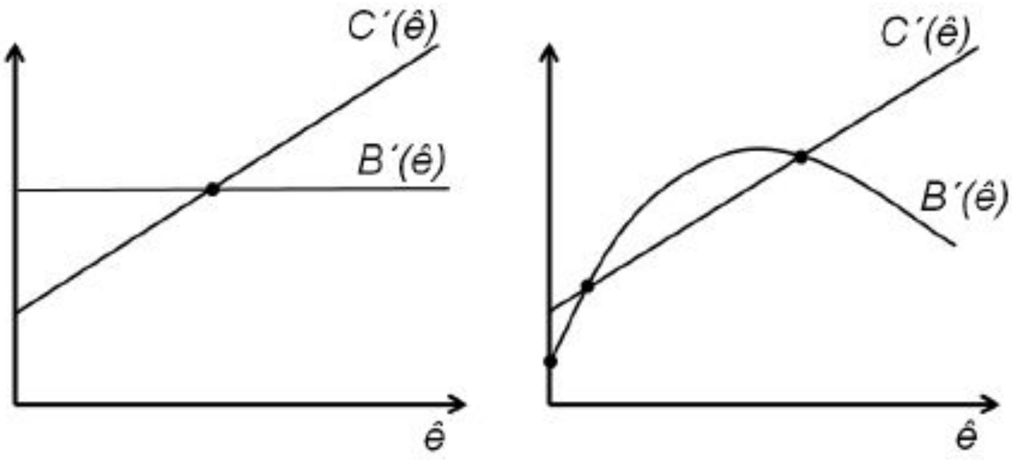
where $r = \frac{h_u}{h_\eta} = \frac{var(\eta)}{var(u)}$. At steady state, $\mu_{t+1} = \mu_t = \mu^\star$, where

$$\mu^\star = 1 + \frac{r}{2} - \sqrt{\frac{1}{4}r^2 + r} \in (0, 1).$$

The FOC is:

$$\frac{\beta(1 - \mu^\star)}{1 - \mu^\star\beta} = C'(e^\star).$$

**Proposition 1** *The stationary level of effort $e^\star$ is never greater than the efficient level $e^{FB}$, where $e^{FB}$ satisfies $C'(e) = 1$. It is equal to $e^{FB}$ if $\beta = 1$. It is closer to $e^{FB}$ the bigger is $\beta$, the higher is $var(\eta)$ and the lower is $var(u)$.*




$x = e + a + u$ (Holmström) $\qquad$ $x = ea + u$ (DJT)

## 2.3 Discussions

1. Additive technology vs. Dewatripont, Jewitt, and Tirole (1999)'s analysis of more general technology

   With $x = e + a + u$, manager's marginal benefit of effort is independent of market's conjecture $\hat{e}$; hence, equilibrium is unique. However, With $x = ea + u$, Dewatripont, Jewitt, and Tirole (1999) observe that marginal benefit of effort varies with $\hat{e}$, so multiple equlibria are possible.

2. There are several crucial simplifying assumptions in the model. First of all, from a technical point of view it is very convenient to assume that all parties are symmetrically informed. Otherwise, we get a reputation model discussed in the future. Second, the manager is assumed to be risk-neutral. And the market incentives discussed above do not protect the manager at all against risk and as such they are clearly suboptimal.

# 3 Extensions

## 3.1 Optimal Incentive Contracts in the Presence of Career Concerns

Gibbons and Murphy (1992) considered the design of optimal explicit contracts in the presence of career concerns. The setting is similar to the one in the previous section. Firms are risk neutral. But the manager is assumed to be risk averse with the following exponential utility function:

$$U = -exp\left\{-r\left(w_1 - C(e_1) + \delta(w_2 - C(e_2))\right)\right\},$$

where $C(e) = \frac{1}{2}ce^2$. There are two crucial assumptions about contracting possibilities: (1) short-term (i.e., one-period) contracts are linear in output, and (2) long-term (i.e., multi-period) contracts are not feasible. Therefore, $w_t = \alpha_t + \beta_t x_t$. In the second period, FOC implies that $ce_2 = \beta_2$ and the optimal $\beta_2$ should be:

$$\beta_2^\star = \frac{1}{1 + rc\sigma_2^2},$$

where

$$\sigma_2^2 = \frac{var(u)var(a)}{var(u) + var(a)} + var(u).$$

In equilibrium, the expected profits of the firms are zero, and hence $\alpha_2 = (1 - \beta_2^\star)E[x_2|x_1]$.

In the first period, the FOC is:

$$ce_1 = \beta_1 + \delta\tau(1 - \beta_2^\star),$$

and hence, the optimal $\beta_1$ should be:

$$\beta_1^\star = \frac{1}{1 + rc\sigma_1^2} - \delta\tau(1 - \beta_2^\star) - \frac{rc\delta\beta_2^\star var(a)}{1 + rc\sigma_1^2},$$

where $\sigma_1^2 = var(a) + var(u)$.

The main conclusion from this two-period model is that $\beta_1^\star < \beta_2^\star$. Three effects contribute to the result. The first term in the expression of $\beta_1^\star$ reflects a noise reduction effect: learning about the manager's ability causes the conditional variance of output to decline over time ($\sigma_2^2 < \sigma_1^2$), so the optimal trade-off between insurance and incentives shifts toward the latter over time. The second term is the career concerns effect; it implies that optimal explicit incentives are adjusted to account for career concerns incentives by imposing a lower pay-performance relation when career concerns are high. The third term reflects a human capital insurance effect: risk-averse mangers with uncertain ability want insurance against low realizations of ability; in our model this insurance must take the form of a reduction in the slope of the first-period contract.

Gibbons and Murphy (1992) also found empirical evidence supporting $\beta_1^\star < \beta_2^\star$. For example, it is estimated that a 10 percent change in shareholder wealth corresponds to 1.7 percent changes in cash compensation for CEOs less than three years from retirement, but only 1.3 percent pay changes for CEOs more than three years from retirement. Thus for a CEO earning \$562,000 (the sample average), a 10 percent change in shareholder wealth corresponds to a \$9,500 change in cash compensation for a CEO close to retirement, but only a \$7,300 change for a CEO far from retirement.

### 3.2 Comparative Performance Information (CPI) and Implicit Incentives

Meyer and Vickers (1997) considered a 2-manager&2-period model with comparative performance evaluation:

$$x_{ti} = e_{ti} + a_i + u_{ti},$$

$$x_{tj} = e_{tj} + a_j + u_{tj}.$$

$a_i$, $a_j$ identically distributed; all $u_{tk}$ identically distributed. Denote $\eta = corr(a_i, a_j)$ and $\rho = corr(u_{ti}, u_{tj})$. It remains true that $e_{2i} = 0$ and that $w_{2i}$ equals the conditional expectation of $a_i$, but that expectation is now conditional on $x_{1j}$ as well as $x_{1i}$. The variables $a_i$, $x_{1i}$, and $x_{1j}$ have a multivariate normal distribution with covariance matrix proportional to

$$\begin{pmatrix} \tau & \tau & \eta\tau \\ \tau & 1 & \kappa \\ \eta\tau & \kappa & 1 \end{pmatrix},$$

where $\kappa = (1-\tau)\rho + \tau\eta$ is the correlation between $x_{1i}$ and $x_{1j}$. Bayesian updating yields

$$w_{2i} = E[a_i | x_{1i}, x_{1j}] = \frac{\tau}{1-\kappa^2}\left[(1-\eta\kappa)(x_{1i} - \hat{e}_{1i}) + (\eta - \kappa)(x_{2i} - \hat{e}_{2i})\right].$$

FOC implies that $i$'s first period effort is given by:

$$C'(e_1) = \tau \left( \frac{1 - \eta\kappa}{1 - \kappa^2} \right) = \Psi \leq 1.$$

Notice that without CPI, the FOC is $C'(e_1) = \tau$. Therefore, we have:

**Proposition 2** *In the managerial career concerns model, effort incentives and efficiency are greater with performance comparisons than without if and only if* $\kappa(\rho - \eta) > 0$

Consider the special case $\eta = 0$ and $\rho \neq 0$. Then $\kappa(\rho - \eta) = (1 - \tau)\rho^2$ is always larger than 0. Comparative performance information improves effort incentives because the observation of $x_{1j}$ effectively reduces the variance of the "noise" $u_{1i}$, and so increases the weight on $x_{1i}$ in estimating $a_i$.

The above result also sheds a light on why comparative performance information (CPI) is not as widely used as theory would lead us to expect (Murphy, 1999). Traditionally, it is believed that comparative performance information (CPI), when used optimally, can improve incentives and efficiency in principal-agent relationships governed by *explicit contracts*. The above example shows that in the presence of career concerns, the overall effect of CPI on welfare is ambiguous and depends upon the sources of correlation in agents' performances.

## References

CHEUNG, S. (1969): "Transactions Costs, Risk Aversion and the Choice of Contractual Arrangements," *Journal of Law and Economics*, 19.

DEWATRIPONT, M., I. JEWITT, AND J. TIROLE (1999): "The Economics of Career Concerns," *Review of Economic Studies*, 66.

FAMA, E. (1980): "Agency Problems and the Theory of the Firm," *Journal of Political Economy*, 88, 288–307.

GIBBONS, R., AND K. MURPHY (1992): "Optimal Incentive Contracts in the Presence of Career Concerns," *Journal of Political Economy*, 100, 468–506.

HOLMSTRÖM, B. (1999): "Managerial Incentive Problems: A Dynamic Perspective," *Review of Economic Studie*, 66(1), 169–182.

MEYER, M., AND J. VICKERS (1997): "Performance Comparisons and Dynamic Incentives," *Journal of Political Economy*, 105, 547–81.

PRENDERGAST, C. (1999): "The Provision of Incentives in Firms," *Journal of Economic Literature*, 37, 7–63.

STIGLITZ, J. (1974): "Incentives and Risk Sharing in Sharecropping," *Review of Economic Studies*, 41, 219–255.