Random Authority*

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Abstract

This paper rationalizes matrix management in a multi-project organization in which decisions must be adapted to local conditions but also coordinated with each other. Project managers are privately informed about local conditions and communicate strategically via cheap talk. Matrix management is modeled as a randomization over deterministic authority allocations. We show that random authority is strictly optimal when the conflict between adaptation and coordination is very severe or the coordination need is very small. Moreover, the optimal degree of delegation changes non-monotonically in the coordination need when the incentives of the project managers are sufficiently aligned.

Keywords. Adaptation. Cheap Talk. Coordination. Matrix Organization. Random Authority.

JEL. D23. D83. L23. M11.

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1 Introduction

A central question for the internal organization of a firm is how to allocate *authority* within its members.[1] The traditional unity-of-command principle of management suggests a clear and single flow of authority from the top of an organization to its bottom. In practice, however, ambiguity or randomness arises about who will be making certain decisions in matrix organizations, due to their multiple manager structure.[2] For example, by investigating project management in R&D organizations, [21] argued that "in the industry as a whole, the structuring of authority apparently has been random." In this paper, we develop a model to study how random authority allocation affects an organization's overall performance, and investigate when random authority allocation outperforms deterministic authority allocations.

We consider an organization with two project managers ('he') and one functional manager ('she').[3] The interests of the functional manager and the two project managers are misaligned: the functional manager's objective is to maximize total profits, while each project manager is biased toward maximizing the profits of his own project rather than those of the overall organization. To maximize profits, decisions must be adapted to local conditions but also coordinated with each other. However, information about local conditions is held by project managers, and this makes the strategic communication between these managers very important.[4]

Following the literature, we assume that the organization lacks commitment such that incentive contracts cannot work. Therefore, the only way to provide communication incentives is through authority allocation. We consider both deterministic and random authority allocations in this paper. Specifically, we consider the following deterministic authority allocations: functional authority, where decision rights are centralized to the functional manager; project authority, where decision rights are delegated to project managers; and mixed authority, where the functional manager controls one project while the other one is delegated to another project manager. Random authority involves a randomization of these deterministic authority allocations.[5] The game proceeds as fol-

[1] See [6] and [19] for excellent surveys of the literature on authority in organizations.

[2] Discussions on matrix organizations can be found in [11], [15], [22], and [27].

[3] A matrix organization typically has more than one functional manager. Our model can be viewed as a simplification by assuming that the functional managers do not possess any private information, and there exists no coordination need between the two functional managers. See related discussions in Section 5.

[4] There is a growing literature investigating how the tradeoff between adaptation and coordination affects the delegation of authority in a strategic information transmission environment (see, e.g., [2, 3], [13], [16], [33, 34]).

[5] In matrix organizations, random authority can be implemented through the intervention of upper-level authority. As argued by [27], the aim of the intervention is to "strike a balance between functional authority and project authority". Another way of implementing random authority is from the result of joint bargaining between project managers and functional managers. We can model this bargaining by letting the functional manager and one project

lows. First, the organization chooses authority allocation to maximize expected profits. Second, after observing the authority allocation and information about local conditions, each project manager sends a cheap talk message to influence the posterior beliefs of the potential decision-maker(s) (henceforth DM). The message is observable by both the functional manager and the other project manager. Finally, the DMs are chosen according to the committed authority allocation, and the decisions are made based on the cheap talk messages.

We characterize the communication equilibria given an authority allocation in symmetric organizations, where the two projects are identical. Organizational performance is measured by the total expected profits in the most efficient communication equilibrium. Allocating authority to project managers has a natural advantage in terms of adapting decisions to local conditions, since decisions are made by the managers with the best information about those conditions. However, it also has a natural disadvantage in coordinating decisions. Therefore, the comparison of organizational performance under different authority allocations crucially depends on two parameters: *incentive misalignment*, which measures the biasedness of each project manager's incentives; and *coordination need*, which measures the relative importance of coordination in the profit function.

Our main theoretical result suggests that randomization between functional authority and project authority outperforms deterministic authority allocations under some parameter values. The optimal authority design depends on two factors: one is the relative importance of adaptation compared with coordination, while the other is the communication quality. Communication quality is the key reason for the optimality of random authority. If communication quality does not change with randomization probability, then there is no need to randomize: the optimal authority allocation is either functional authority or project authority, depending on the relative importance of adaptation compared with coordination. Random authority can be optimal because increasing the probability assigned to functional authority improves communication quality due to the fact that the project managers are more willing to share information with the functional manager as the interests of the functional manager are better aligned with those of the project manager. Moreover, the gain in communication quality is a concave function of the probability assigned to functional authority: as the probability goes up, the marginal improvement in communication quality diminishes. This creates the possibility that at an interior probability, the marginal loss in adaptation is equal to the marginal gain in communication quality, and hence the corresponding

manager simultaneously send another message chosen from a suitable set of messages. These additional messages determine the allocation of decision rights through a jointly controlled lottery following [5] and [30]. As a result, authority allocation is *de facto* random with pre-determined randomization probabilities.

random authority maximizes the total expected profits.

Random authority is optimal in two scenarios. In one scenario whenever functional authority and project authority perform equally well, random authority is strictly optimal. Intuitively, this is when the conflict between adaptation and coordination is most severe, and random authority provides a better way to handle the conflict. By continuity, there then exists a range of incentive misalignment around this indifference point for which random authority is strictly optimal. In the second scenario given any incentive misalignment, random authority is strictly optimal when the coordination need is sufficiently small. As shown by [2] (henceforth ADM), when the coordination need is sufficiently small, project authority always outperforms functional authority, and there is a huge difference in the quality of communication between functional authority and project authority. By adding a small probability of centralization to project authority, the organization can enjoy a big improvement in communication quality, and hence random authority is very likely to improve organizational performance. On the contrary, as the coordination need approaches infinity, the difference in the quality of communication between functional authority and project authority vanishes, and hence random authority cannot outperform the deterministic authority allocations in the limit.

We conduct a comparative statics analysis to examine how an increase in the coordination need affects optimal authority allocation. Given an optimal authority allocation, we measure the *degree of delegation* by the probability assigned to project authority. Interestingly, it is found that the optimal degree of delegation changes non-monotonically in the coordination need when the incentive misalignment is sufficiently small. An increase in the coordination need has two opposing effects on the optimal authority allocation. First, it leads to a higher probability assigned to functional authority as coordination becomes relatively more important. Second, it leads to a lower probability assigned to functional authority as the difference in the quality of communication diminishes. When the incentives are sufficiently aligned, the gain in coordination is limited, and the optimal degree of delegation converges to one, even when the coordination need is arbitrarily large. Meanwhile, when the coordination need is arbitrarily small, the optimal degree of delegation converges to one as well, as adaptation becomes much more important than coordination. This implies that when random authority is optimal, the optimal degree of delegation has to change non-monotonically in the coordination need. We show by example that the optimal degree of delegation first decreases and then increases in the coordination need. When the incentive misalignment is sufficiently large, the first effect can dominate the second one, and thus the optimal degree of delegation always decreases in the coordination need.

Our paper has three interesting implications for the management of firms in the real world. First, our paper provides a new rationale for ambiguous authority in matrix organizations. It is widely believed that ambiguous authority is a significant weakness of matrix organizations ([35]). Our theoretical finding shows the opposite: random authority is sometimes beneficial and can improve organizational performance. This in turn suggests that managers and employees must be trained to be tolerant of the ambiguity inherent in matrix organizations. Second, one determinant of coordination need is the uniqueness of local conditions: the relative importance of coordination declines as the local conditions become increasingly unique and require the expertise of the project managers. Hence, our model implies that matrix management is likely to be adopted when local conditions are very unique. This implication is consistent with some informal evidence. For example, [7] finds that matrix management is often adopted by hospitals facing great complexity. Historically, the two-boss matrix system worked very effectively for the aerospace industry: one boss focused on technical excellence and owned the specialists, while the other acted like a general contractor ([22]). This is another situation where the local conditions are very unique.

Finally, our paper provides a theoretical foundation for the organizational configurations of multinational firms. As argued by [23] and [32], a multinational firm will adopt different strategies depending on the degrees of geographical dispersion and organizational coordination. If the firm faces a relatively low degree of dispersion and a high degree of coordination in marketing and sales activities, the optimal strategy is to be a global exporter for whom production is largely concentrated in the home country. If, on the contrary, the firm faces a relatively low degree of coordination and a high degree of dispersion, a multidomestic strategy enables the firm to focus on local adaptation while cross-border organizational coordination is kept to a minimum. If the firm faces both a high degree of coordination and a high degree of dispersion, the conflict between adaptation and coordination is most severe. Consistent with our theoretical prediction, a transnational strategy based on a matrix structure is adopted in this situation to allow the firm to simultaneously pursue local adaptation and global coordination.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 describes the model and characterizes the communication equilibria under random authority. Section 4 analytically investigates when random authority is optimal. Section 5 concludes the paper.

2 Related Literature

From a modeling perspective, our paper is closely related to ADM. They only compare centralization (corresponding to functional authority) and decentralization (corresponding to project authority) in symmetric organizations, and show that the optimal degree of delegation is non-increasing in the coordination need. Our paper provides a more complete characterization of the optimal allocation of authority in symmetric organizations, and shows that random authority allocation can outperform deterministic authority allocations. Moreover, by considering random authority, we find that the optimal degree of delegation can be non-monotonic in the coordination need.

In the literature, [24] provide the pioneering theoretical study that investigates when matrix organization can be optimal. However, in [24], the main difference between the matrix organization and functional/project authority stems from the number of middle managers: matrix organization has the highest number of middle managers, and hence has the highest probability of finding coordination opportunities as well. Naturally, this implies that matrix organization is optimal when the cost of employing middle managers is low (compared to the opportunity cost of the CEO). Although the insight is important, we feel that [24] ignored many other important aspects of matrix organizations such as communication incentives and ambiguity of authority. This motivated us to create a new model that incorporated these issues.

Our paper is also related to [14]. Both papers aim to explain the existence of "strategic ambiguity" in organizations.[6] They show that for a multi-task principal-agent environment, randomization between two compensation schedules can effectively prevent gaming and induce more balanced efforts. Therefore, it is optimal to introduce a random contract if the degree of complementarity is sufficiently high to overcome the risk imposed by randomness. However, in our paper, random authority is likely to be suboptimal if the degree of complementarity is high (large coordination need), because the difference in communication quality diminishes

The idea of random authority is different from contingent allocation of authority, which is viewed as another reason for the existence of strategic ambiguity on authority allocation. In this branch of literature (see e.g., [1], [18] and [29]), there is a deterministic rule that decides how authority is allocated when certain contingencies occur, to better handle major changes in the environment, unexpected events, or the advent of special problems. In our paper, however, authority is randomly allocated to better handle the tradeoff between adaptation and coordination.

[6]In organizations, the term "strategic ambiguity" means intentionally being non-transparent about certain practices.

One may argue that it is not surprising that random authority can be optimal. In the standard Crawford-Sobel cheap talk game, introducing uncertainty indeed enables improved information transmission (see, e.g., [30]).[7] Although the stochastic mechanism considered by [30] can always outperform all cheap talk equilibria, random authority cannot always outperform deterministic authority allocations. In particular, in the standard Crawford-Sobel cheap talk game, random authority is always suboptimal.[8] The key intuition is that the set of corresponding cheap talk equilibria does not change as we change the probability of centralization. As a result, total expected profits are simply a convex combination of expected profits under functional authority and those under project authority. Hence, random authority can never improve organizational performance. With two projects, as we change the probability of centralization, the communication incentives and the set of possible cheap talk equilibria will also change. As a result, random authority may outperform deterministic authority allocations.

3 The Model

3.1 Setup

Consider an organization consisting of two operating projects ($j = 1, 2$), and three players: two project managers in charge of the projects and one functional manager. Project j generates profits that depend on its local conditions, described by $\theta_j \in \mathbb{R}$, and on the decisions taken in both projects, $d_1 \in \mathbb{R}$ and $d_2 \in \mathbb{R}$. In particular, the profits of project j are given by

$$\pi_j = -(d_j - \theta_j)^2 - \delta(d_j - d_i)^2.$$

The first squared term captures the *adaptation loss* that project j incurs if decision d_j is not perfectly adapted to its local condition θ_j, and the other squared term captures the *coordination loss* that project j incurs if the two decisions are not perfectly coordinated. Parameter $\delta \in \mathbb{R}_+$ measures the relative importance of coordination in the profit function.

[7]However, as shown by [20] and [28], the optimal communication mechanism is deterministic in this setting.

[8]The comparison of centralized and decentralized organizations in the standard Crawford-Sobel cheap talk game has been investigated by [12].

3.1.1 Information

Each project manager j privately observes his local information θ_j , but does not know the realization $\theta_{i\neq j}$. The functional manager does not observe any private information. It is common knowledge, however, that θ_j is uniformly distributed on $[-s_j, s_j]$, with $s_j \in \mathbb{R}_+$. The draws of θ_1 and θ_2 are independent. Throughout this paper, we focus on symmetric organizations (i.e., $s_1 = s_2 = s$).[9]

3.1.2 Preferences

The functional manager's objective is to simply maximize the total profits, and thus $U_{FM} = \sum_j \pi_j$. Manager j has a preference such that: $U_j = \lambda\pi_j + (1-\lambda)\pi_{i\neq j}$, where $\lambda_j \in [\frac{1}{2}, 1]$. Obviously, when $\lambda = \frac{1}{2}$, the incentives of the functional manager and the project managers are perfectly aligned; when $\lambda > \frac{1}{2}$, manager j is biased toward his own project's profits. Parameter λ thus captures the incentive misalignment in the organization.

3.1.3 Potential Authority Allocations

So far, our model setup is exactly the same as the one in ADM. The main difference lies in the possible authority allocations. In this paper, we consider the following deterministic authority allocations:

1. Functional Authority: Both of the decision rights are held by the functional manager (F);

2. Project Authority: Both of the decision rights are held separately by project managers (P);

3. Mixed Authority 1: The functional manager controls project 1 and delegates project 2 to manager 2 (M1);

4. Mixed Authority 2: The functional manager controls project 2 and delegates project 1 to manager 1 (M2).

Obviously, functional authority and project authority correspond to centralization and decentralization, which has been analyzed by ADM, respectively. Mixed authority corresponds to partial centralization, which has been analyzed by [33].[10] On top of these four deterministic authority

[9] The analysis of asymmetric organizations (i.e., $s_1 \neq s_2$) can be found in the Online Appendix.

[10] [33] also considers directional authority, where both of the decision rights are held by one project manager. We do not allow this possibility because it is obviously suboptimal in our symmetric setting.

allocations, we allow a general random authority (RA) allocation where the organization commits to deterministic authority allocations with a probability vector $\rho = (\rho_1, \rho_2, \rho_3, \rho_4)$, where ρ_i is the probability of using deterministic authority allocation i. Obviously, our setting includes the deterministic authority allocations if $\rho_i = 1$. Although the organization can commit to a random authority allocation rule, it lacks commitment such that incentive contracts cannot work in our model. Therefore, the only way to provide communication incentives is through authority allocation, and for simplicity, we assume that communication occurs only in one round.

Random authority captures the key features of matrix organizations quite well. In matrix organizations, functional authority is overlayed by project authority. To solve this dual authority problem, "the use of an ambiguous authority definition was the most common adaptation" ([21]), and this creates situations that call for the intervention of an upper-level authority. As argued by [27], the aim of the intervention is to "strike a balance between functional authority and project authority". In other words, the upper-level authority will swing between functional authority and project authority to seek a balance of power. This can be viewed as a way to implement random authority.[11]

As shown in Figure 1, the game proceeds as follows. First, an (possibly random) authority allocation $\rho = (\rho_1, \rho_2, \rho_3, \rho_4)$ is chosen to maximize the total expected profits $\mathrm{E}[\pi_1 + \pi_2]$. Second, the local conditions θ_j are realized and observed by project manager j. And then, prior to the realization of *ex post* authority allocation, manager $j \in \{1, 2\}$ simultaneously sends cheap talk messages $m_j \in \mathcal{M} = [-s, s]$, knowing the committed authority allocation $\rho = (\rho_1, \rho_2, \rho_3, \rho_4)$. The messages are observable to all players. Finally, the authority allocation is realized, and decisions d_1 and d_2 are made. Each DM makes the decision(s) given the cheap talk messages and realized authority allocation.

3.2 Equilibrium Analysis

In this section, we provide a complete characterization of the communication equilibria under the authority allocation $\rho = (\rho_1, \rho_2, \rho_3, \rho_4)$. Following the literature, we consider perfect Bayesian equilibria of the game, which are characterized by three features: (a) communication rules $p_j(m_j|\theta_j)$ for the project manager $j \in \{1, 2\}$, which specify the probability of sending message $m_j \in \mathcal{M}_j$ conditional on the observed state θ_j; (b) decision rules for the DMs, and (c) belief functions $q_j(\theta_j|m_j)$ for

[11] In reality, the intervention can be affected by factors such as corporate culture. It fits our model as long as these factors are payoff-irrelevant.

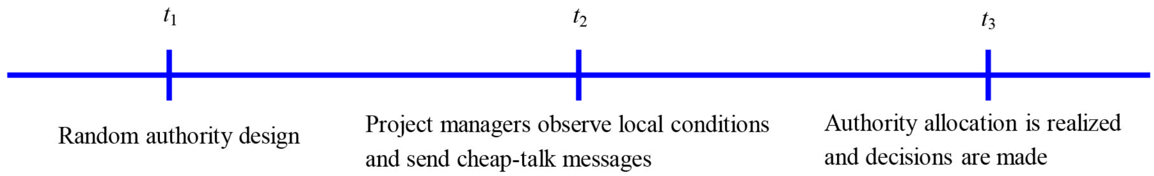

Figure 1: Timeline

the receivers of message m_j.[12] In Section 3.2.1, we first characterize decision making under different deterministic authority allocations, taking as given the DMs' posterior beliefs over θ_j. Section 3.2.2 and 3.2.3 characterize the communication equilibria, and Section 3.2.4 derives organizational performance under the authority allocation $\rho = (\rho_1, \rho_2, \rho_3, \rho_4)$.

3.2.1 Decision Making

Under functional authority, the functional manager chooses $\{d_1, d_2\}$ to maximize $\mathrm{E}[\pi_1 + \pi_2|m]$, where $m = (m_1, m_2)$. ADM have shown that decisions are convex combinations of the functional manager's posterior expectations of θ_j given m:

$$\begin{aligned} d_1^F &= \gamma_F \mathrm{E}[\theta_1|m] + \gamma_F^{'} \mathrm{E}[\theta_2|m], \\ d_2^F &= \gamma_F^{'} \mathrm{E}[\theta_1|m] + \gamma_F \mathrm{E}[\theta_2|m], \end{aligned} \tag{1}$$

where $\gamma_F = (1+2\delta)/(1+4\delta), \gamma_F^{'} = 2\delta/(1+4\delta)$.

Under project authority, manager $j \in \{1,2\}$ chooses d_j to maximize $\mathrm{E}[U_j|m]$ given the other manager's strategy. From ADM, decisions are given as below:

$$\begin{aligned} d_1^P &= (1-k)\theta_1 + \gamma_P \mathrm{E}[\theta_1|m] + \gamma_P^{'} \mathrm{E}[\theta_2|m], \\ d_2^P &= (1-k)\theta_2 + \gamma_P^{'} \mathrm{E}[\theta_1|m] + \gamma_P \mathrm{E}[\theta_2|m], \end{aligned} \tag{2}$$

where $k = \delta/(\lambda+\delta), \gamma_P = k\delta/(\lambda+2\delta), \gamma_P^{'} = k(\lambda+\delta)/(\lambda+2\delta)$.

Under mixed authority 1 (M1), the functional manager chooses d_1 and manager 2 chooses d_2. Similarly, decisions are convex combinations of the local conditions and the posterior expectations

[12] Since the local conditions are independently distributed, the receivers can only update their beliefs from the messages received. In other words, belief q_j only depends on m_j.

about local conditions $E[\theta_j|m]$:

$$\begin{aligned} d_1^{M1} &= \frac{1}{1+2\delta}\mathrm{E}[\theta_1|m] + \frac{2\delta}{1+2\delta}\mathrm{E}[d_2|m], \\ d_2^{M1} &= \frac{\lambda}{\lambda+\delta}\theta_2 + \frac{\delta}{\lambda+\delta}\mathrm{E}[d_1|\theta_2, m], \end{aligned} \tag{3}$$

Notice that $\mathrm{E}[d_2|m]$ and $\mathrm{E}[d_1|\theta_2, m]$ can be obtained by taking the expectations over Eq. (3). Substitute them back into Eq. (3) and we can obtain the equilibrium decisions rules

$$\begin{aligned} d_1^{M1} &= \gamma_M \mathrm{E}[\theta_1|m] + \gamma_M^{'} \mathrm{E}[\theta_2|m], \\ d_2^{M1} &= (1-k)\theta_2 + k\gamma_M \mathrm{E}[\theta_1|m] + k\gamma_M^{'} \mathrm{E}[\theta_2|m], \end{aligned} \tag{4}$$

where $k = \delta/(\lambda+\delta), \gamma_M = (\lambda+\delta)/(\lambda+\delta+2\lambda\delta), \gamma_M^{\prime} = 2\lambda\delta/(\lambda+\delta+2\lambda\delta)$.

The analysis for mixed authority 2 (M2) is similar, and the equilibrium decision rules are given as below:

$$\begin{aligned} d_1^{M2} &= (1-k)\theta_1 + k\gamma_M^{'} \mathrm{E}[\theta_1|m] + k\gamma_M \mathrm{E}[\theta_2|m], \\ d_2^{M2} &= \gamma_M^{'} \mathrm{E}[\theta_1|m] + \gamma_M \mathrm{E}[\theta_2|m], \end{aligned} \tag{5}$$

It can be seen that as δ increases, the project manager(s) with decision rights puts less weight on her private information and more weight on a weighted average of posterior beliefs. When $\delta \to \infty$, the equilibrium decisions d_1 and d_2 will both converge to $\frac{1}{2}(\mathrm{E}[\theta_1|m] + \mathrm{E}[\theta_2|m])$ under functional authority and project authority, and to $\frac{1}{1+2\lambda}(\mathrm{E}[\theta_i|m] + 2\lambda \mathrm{E}[\theta_{j\neq i}|m])$ under mixed authority i.

3.2.2 Strategic Communication and Information Distortion

In order to analyze communication equilibria under random authority, it is useful to first calculate project managers' incentives to misrepresent information. As in ADM, we need to determine the optimal expectation that the sender expects others to hold. Denote by $\nu_j = \mathrm{E}[\theta_j|m]$ the receivers' expectation of θ_j given that the received message is m, and suppose manager j can simply choose any ν_j. Then the optimal expectation $\nu_j^\star$ manager j would like others to hold should satisfy

$$\nu_j^\star = \mathrm{argmax}_{\nu_j} \mathrm{E}[U_j|d^{RA}, \theta_j]. \tag{6}$$

Then, we can define the degree of information distortion b_{RA}^j for manager $j \in \{1, 2\}$ under a fixed authority allocation $\rho = (\rho_1, \rho_2, \rho_3, \rho_4)$ as:

$$b_{RA}^j(\rho) = \frac{\nu_j^* - \theta_j}{\theta_j}.$$

Employing the law of iterated expectations, we can solve the explicit expressions of b_{RA}^j under any authority allocation ρ, which turns out to be independent of θ_j

$$b_{RA}^1(\rho) = -1 + \frac{\chi_1(\rho_1, \rho_2, \rho_3, \rho_4)}{\chi_2(\rho_1, \rho_2, \rho_3, \rho_4)}, \quad b_{RA}^2(\rho) = -1 + \frac{\chi_1(\rho_1, \rho_2, \rho_4, \rho_3)}{\chi_2(\rho_1, \rho_2, \rho_4, \rho_3)}, \tag{7}$$

where

$$\begin{aligned}\chi_1(\rho_1, \rho_2, \rho_3, \rho_4) = \rho_1 \lambda \gamma_F + \rho_2 \big\{\lambda \gamma_P - (1-k)[\lambda \gamma_P + \delta(\gamma_P - \gamma_P')]\big\} + \rho_3 \lambda \gamma_M \\ + \rho_4 \big\{\lambda(k\gamma_M') - (1-k)[\lambda(k\gamma_M') + \delta(k\gamma_M' - \gamma_M')]\big\},\end{aligned}$$

and

$$\begin{aligned}\chi_2(\rho_1, \rho_2, \rho_3, \rho_4) = \rho_1 \big\{\lambda \gamma_F^2 + (1-\lambda)(\gamma_F')^2 + \delta(\gamma_F - \gamma_F')^2\big\} + \rho_2 \big\{\lambda \gamma_P^2 + (1-\lambda)(\gamma_P')^2 + \delta(\gamma_P - \gamma_P')^2\big\} \\ + \rho_3 \big\{\lambda \gamma_M^2 + (1-\lambda)(k\gamma_M)^2 + \delta(\gamma_M - k\gamma_M)^2\big\} + \rho_4 \big\{\lambda(k\gamma_M')^2 + (1-\lambda)(\gamma_M')^2 + \delta(k\gamma_M' - \gamma_M')^2\big\}.\end{aligned}$$

Obviously, when $\rho_1 = 1$, $b_{RA}^1 = b_{RA}^2 = b_F$, which measures the incentives to misrepresent information under centralization in ADM; when $\rho_2 = 1$, $b_{RA}^1 = b_{RA}^2 = b_P$, which measures the incentives to misrepresent information under decentralization in ADM; when $\rho_3 = 1$, $b_{RA}^1 = b_{M1}^1$ and $b_{RA}^2 = b_{M1}^2$. It is straightforward to show that

$$b_{M1}^1 > b_F, \quad b_P > b_F, \quad \text{and} \quad b_P > b_{M1}^2.$$

Manager i is more willing to convey information if the decision $d_{j \neq i}$ is taken by the functional manager, because the preferences of a project manager are more closely aligned with those of the functional manager than with those of the other project manager. However, b_{M1}^1 can be either higher or lower than b_{M1}^2.

3.2.3 Equilibrium Characterization

Under authority allocation $\rho = (\rho_1, \rho_2, \rho_3, \rho_4)$, we say a combination of communication rules, decision rules and belief functions $\{p_1, p_2, d_1, d_2, q_1, q_2\}$ constitutes a perfect Bayesian equilibrium if: (i) Whenever $p_j(m_j|\theta_j) > 0$, then $m_j \in \text{argmax}_{m \in \mathcal{M}_j} \text{E}[U_j|d, \theta_j, \rho]$; (ii) The decision rules $d_j(.)$ satisfy Eq. (1) (2, 3 and 4, respectively) once functional authority (project authority, mixed authority 1 and mixed authority 2, respectively) is realized; and (iii) the belief functions satisfy the Bayes rule whenever possible, that is , $q_j(\theta_j|m) = p_j(m_j|\theta_j)/\int_P p_j(m_j|\theta_j)d\theta_j$, where $P = \{\theta_j : p_j(m_j|\theta_j) > 0\}$.

As in [10] and ADM, all equilibria are interval equilibira. The state space $[-s, s]$ is partitioned into intervals and project manager j only reveals some information indicating which interval the local condition θ_j belongs to. Denote by $t_{j,i}, i \in \{0, 1, ..., N\}$ the dividing points of the partition with $t_{j,0} = -s$ and $t_{j,N} = s$. Then $t_{j,i}$ satisfies the difference equation stated in the following lemma. The proofs of the following and all subsequent results can be found in the Appendix.

Lemma 1 (Communication Equilibria) : *Given any $\delta \in (0, \infty)$ and any probability vector ρ, then for every positive integer $N_j, j \in \{1, 2\}$, there exists at least one equilibrium $(p_j(.), d_j(.), q_j(.))$ satisfying:*

(i) $p_j(m_j|\theta_j)$ is uniform, supported on $[t_{j,i-1}, t_{j,i}]$ if $\theta_j \in (t_{j,i-1}, t_{j,i})$;

(ii) $q_j(\theta_j|m_j)$ is uniform, supported on $[t_{j,i-1}, t_{j,i}]$ if $m_j \in (t_{j,i-1}, t_{j,i})$;

(iii) $t_{j,i+1} - t_{j,i} = t_{j,i} - t_{j,i-1} + 4b^j_{RA} t_{j,i}$ for $i = 1, 2, ..., N_j - 1$, with b^j_{RA} defined by Eq. (7);

(iv) $d^l_j(m, \theta_j) = d^l_j$ for $j \in \{1, 2\}$ and $l \in \{F, P, M1, M2\}$ which are defined by Eq. (1), (2), (4) and (5), respectively.

Lemma 1 illustrates all the outcome-equivalent interval equilibria under the random authority allocation $\rho = (\rho_1, \rho_2, \rho_3, \rho_4)$. On the one hand, communication is noisy, as long as the managers' incentives are not perfectly aligned; on the other hand, for any $\delta > 0$, some information will be communicated to reduce the losses from miscoordination. Moreover, Lemma 1 also shows that the number of equilibrium intervals can be constructed to approach infinity because at state $\theta = 0$, the incentives of the sender and the receivers are perfectly aligned. Following the literature, we will focus on the limiting equilibrium as $N_j \to \infty$, which is the most efficient equilibrium that maximizes the total expected profits.[13]

Lemma 2 (Efficiency) : *For $j \in \{1, 2\}$, the limit of strategy profiles and beliefs $(p_j(.), d_j(.), q_j(.))$*

[13] See e.g., [8], [9], and [31].

as $N_j \to \infty$ is a perfect Bayesian equilibrium of the communication game. In this equilibrium the total expected profits $\mathrm{E}[\sum_{j=1}^{2} \pi_j]$ *are higher than in any other equilibrium.*

3.2.4 Communication Quality and Organizational Performance

This subsection computes communication quality and organizational performance under random authority. Following ADM, communication quality is defined as the residual variance of the information communication: $\mathrm{E}\big[\,(\theta_j - \mathrm{E}(\theta_j|m_j))^2\,\big]$. The explicit expression of communication quality is shown in the following lemma.

Lemma 3 (Communication Quality) : *In the most efficient equilibrium, the communication residual variance is given by*

$$\mathrm{E}[(\theta_j - \mathrm{E}[\theta_j|m_j])^2] = S^j_{RA}(\rho)\sigma^2 \;\; for \; j \in \{1,2\} \tag{8}$$

where $S^j_{RA}(\rho) = \frac{b^j_{RA}(\rho)}{3+4b^j_{RA}(\rho)}$ *and* $\sigma^2 = \frac{s^2}{3}$.

Lemma 3 is analogous to Lemma 1 in ADM, which shows that $S_l = \frac{b_l}{3+4b_l}$ for $l = \{F, P\}$. The proofs are thus omitted. $1 - S^j_{RA}$ can be viewed as a measure of communication quality. As b^j_{RA} increases, the senders have a stronger incentive to misrepresent information, and hence communication quality will decline. Based on this lemma, we can further calculate the organization's expected profits in the most efficient equilibrium under random authority, and these profits are used to measure organizational performance. Obviously, an increase in S_{RA} reduces communication quality, and leads to a decrease in the expected profits.

Proposition 1 (Organizational Performance) : *Under the random authority structure ρ, the organization's expected profits in the most efficient equilibrium are given by:*[14]

$$\begin{aligned}
\pi_{RA}(\rho) = \;\; & - \;\; \rho_1 \left\{2A_F + (1-A_F)\left(S^1_{RA}(\rho) + S^2_{RA}(\rho)\right)\right\}\sigma^2 \\
& - \;\; \rho_2 \left\{2A_P + B_P\left(S^1_{RA}(\rho) + S^2_{RA}(\rho)\right)\right\}\sigma^2 \\
& - \;\; \rho_3 \left\{2A_M + \left(B_M S^1_{RA}(\rho) + C_M S^2_{RA}(\rho)\right)\right\}\sigma^2 \\
& - \;\; \rho_4 \left\{2A_M + \left(C_M S^1_{RA}(\rho) + B_M S^2_{RA}(\rho)\right)\right\}\sigma^2
\end{aligned} \tag{9}$$

[14] It is straightforward to check that when $\rho_1 = 1$, $\pi_{RA} = \pi_F = -2\big(A_F + (1-A_F)S_F\big)\sigma^2$, and when $\rho_2 = 1$, $\pi_{RA} = \pi_P = -2\big(A_P + B_P S_P\big)\sigma^2$. Thus, the expected profits calculated in ADM are special cases of our results.

where

$$A_F = \frac{2\delta}{1+4\delta},\ A_P = \frac{2(\lambda^2+\delta)\delta}{(\lambda+2\delta)^2}, B_P = \frac{(4\lambda^3+6\lambda^2\delta+2\delta^2-\lambda^2)\delta^2}{(\lambda+\delta)^2(\lambda+2\delta)^2}, A_M = \frac{\delta(\delta+2\lambda^2+4\delta\lambda^2)}{(\delta+\lambda+2\delta\lambda)^2},$$
$$B_M = \frac{(1+2\delta)\lambda(2\delta+\lambda)}{(\delta+\lambda+2\delta\lambda)^2},\ \textit{and}\ C_M = \frac{4\delta^2\lambda(\delta^2+\lambda^3+\delta\lambda+2\delta\lambda^3)}{(\delta+\lambda)^2(\delta+\lambda+2\delta\lambda)^2}. \tag{10}$$

Eq. (9) indicates an important feature of our model. In the standard Crawford-Sobel cheap talk game, the set of cheap talk equilibria does not change as we change the probability of centralization. As a result, the total expected profits are just a convex combination of expected profits under functional authority and those under project authority. However, in our model, the set of cheap talk equilibria changes as we change probability vector ρ. This is reflected by the change in communication quality S_{RA}. Therefore, the total expected profits are non-linear in the probability ρ_i. This implies that there may exist an interior probability vector ρ, which maximizes the total expected profits.

In our model, it is crucial to assume that the messages are observable to all players. If communication is private and the project managers can send different messages to different players, then the above feature may disappear. For example, let $\rho_3 = \rho_4 = 0$, and then the change of ρ_1 and ρ_2 has no impact on the set of cheap talk equilibria at all. No matter what ρ is, it is always an equilibrium such that the project managers privately communicate with the functional manager following an arbitrary cheap talk equilibrium under functional authority; and meanwhile they privately communicate with each other following another arbitrary cheap talk equilibrium under project authority. In this situation, random authority is always suboptimal.

4 Optimal Authority Allocation

This section investigates optimal authority allocation on the basis of the previous section. It seems very intuitive that random authority can be optimal. In principle, when the organization is restricted to choosing deterministic authority allocations, the authority allocation chosen may not fully adjust to the change of underlying environments, and this undermines the organization's ability to handle the tradeoff between adaptation and coordination. Introducing random authority thus helps the organization to change the authority allocation smoothly to better balance adaptation and coordination. The main question is under what conditions random authority outperforms deterministic authority allocations.

This section is organized as follows. First, section 4.1 shows the sub-optimality of mixed authority in symmetric organizations, and paves the way for characterizing the optimal (random) authority allocation. Section 4.2 fist numerically depicts the optimal authority allocation, and then theoretically derives different scenarios under which random authority outperforms deterministic authority allocations . Section 4.3 investigates how the optimal degree of delegation changes with the underlying parameters.

4.1 Sub-optimality of Mixed Authority

From Proposition 1, the optimal authority allocation solves:

$$\begin{gathered}\text{Max } \pi_{RA}(\rho_1,\rho_2,\rho_3,\rho_4)\\ s.t.\ \rho_j \in [0,1],\ \text{and } \sum_{j=1}^{4}\rho_j = 1.\end{gathered} \tag{11}$$

[33] shows numerically that mixed authority is always dominated by either project authority or functional authority in symmetric organizations since it is suboptimal to treat the two identical projects differently. But this intuition cannot apply if random authority is allowed. Naturally, one may wonder whether $\rho_3 = \rho_4 > 0$ can be optimal. The next observation compares two extreme cases: $\tilde{\rho}_1 = (0.5, 0.5, 0, 0)$ and $\tilde{\rho}_2 = (0, 0, 0.5, 0.5)$, and finds that it is never optimal to set $\rho_3 = \rho_4 = \frac{1}{2}$.

Observation 1 *For any pair of* (λ, δ)*, the organizational performance satisfies* $\Pi(\tilde{\rho}_1) \geq \Pi(\tilde{\rho}_2)$.

Under both $\tilde{\rho}_1$ and $\tilde{\rho}_2$, $S^1_{RA} = S^2_{RA} \triangleq S_{RA}$. Then, the comparison of $\Pi(\tilde{\rho}_1)$ and $\Pi(\tilde{\rho}_2)$ consists of two parts: (i) the comparison of adaptation loss, which is $A_F + A_P - 2A_M$; and (ii) the comparison of coordination loss, which is $(1 - A_F + B_P)S_{RA}(\tilde{\rho}_1) - (B_M + C_M)S_{RA}(\tilde{\rho}_2)$. It can be verified that the adaptation loss is larger under $\tilde{\rho}_2$: $A_F + A_P - 2A_M < 0$, and the coordination loss is larger under $\tilde{\rho}_1$: $1 - A_F + B_P - (B_M + C_M) > 0$. Moreover, the difference in communication quality is small since under both $\tilde{\rho}_1$ and $\tilde{\rho}_2$, the decision rights of each project are held by the functional manager with probability 0.5 and held by the project manager with probability 0.5. As $S_{RA} < \frac{1}{4}$ and $1 - A_F + B_P - (B_M + C_M) = 2A_M - A_F - A_P$, the adaptation loss dominates the coordination loss, and this implies that $\Pi(\tilde{\rho}_1) \geq \Pi(\tilde{\rho}_2)$.

Following the idea of Observation 1, we can intuitively conjecture that any authority allocation $(\rho_1, \rho_2, \rho_3, \rho_4)$ with $\rho_3 = \rho_4 > 0$ is suboptimal, because the organizational performance can be improved by switching to authority allocation $(\rho_1+\epsilon, \rho_2+\epsilon, \rho_3-\epsilon, \rho_4-\epsilon)$ ($\epsilon > 0$ sufficiently small). The proof, however, is very tedious, and can be found in the Online Appendix. Furthermore, we prove in the Online Appendix that it is always suboptimal to set $\rho_3 > 0$ or $\rho_4 > 0$.

Proposition 2 *In symmetric organizations, it is always suboptimal to include mixed authority in the authority allocation (i.e., any $\rho = (\rho_1, \rho_2, \rho_3, \rho_4)$ with $\rho_3 > 0$ or $\rho_4 > 0$ is suboptimal).*[15]

4.2 When is Random Authority Optimal?

In symmetric organizations, Proposition 2 enables us to simply focus on randomization over functional authority (the functional manager makes both decisions) and project authority (project managers independently make their decisions). Since $\rho_3 = \rho_4 = 0$, both project managers have identical degrees of information distortion b_{RA} from Eq. (7) and thus we have $S^1_{RA} = S^2_{RA} \triangleq S_{RA}$. The organization's expected profits in the most efficient equilibrium then can be expressed as:

$$\pi_{RA} = -2\left\{[\rho_1 A_F + \rho_2 A_P] + [\rho_1(1-A_F) + \rho_2 B_P]\, S_{RA}\right\}\sigma^2, \tag{12}$$

where the coefficients are given in Eq. (10).

We first numerically depict the optimal authority allocation in the (λ, δ)-space. The top-left triangle of Figure 2 is the region where project authority is optimal, the top-right triangle is the region where functional authority is optimal, and the bottom triangle is the region where random authority is optimal. If we only consider deterministic authority allocations, the dashed line in Figure 2 is the boundary of the functional authority region and the project authority region as shown in ADM. The findings in Figures 2 can be summarized as follows.

Small δ ($\delta < 0.25890$): As shown by ADM, when coordination need δ is less than 0.19257, project authority always outperforms functional authority. We obtain two more insights by considering random authority. First, no matter how small the coordination need is, adding some minor probability of centralization into project authority can always improve organizational performance. Second, for any coordination need $\delta \in (0, 0.25890)$, random authority is strictly optimal when the incentives are sufficiently misaligned and vice versa.

[15] In the working paper version, we also show that the optimal authority allocation can include mixed authority as long as the two projects are not identical (i.e., $s_1 \neq s_2$).

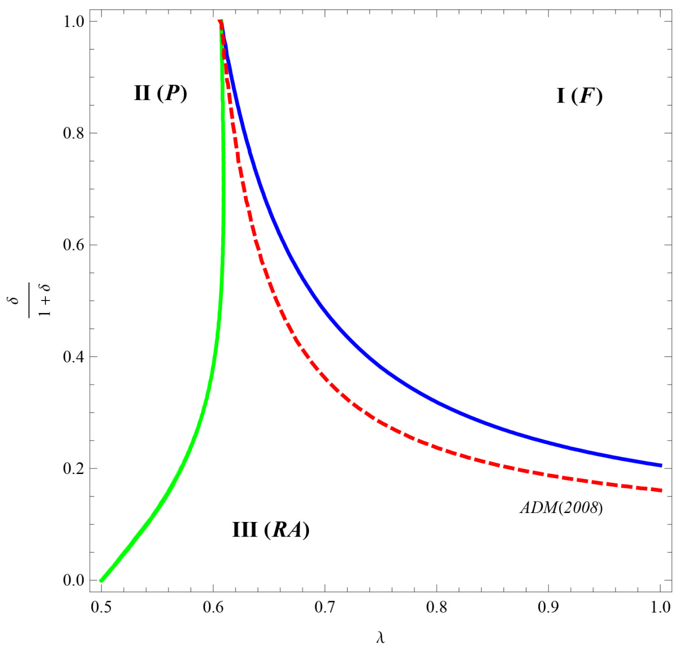

Figure 2: Optimal Authority Allocation in Two Dimensions

Large δ ($\delta > 0.25890$): If δ is large, there exists a unique λ (shown by the dashed line) at which functional authority and project authority perform equally well. Random authority is strictly optimal in an interval of λ around this indifference point. This interval shrinks as δ increases, and eventually degenerates into a single point as δ approaches infinity.

Based on the above numerical findings, the next theorem fully characterizes under what conditions employing random authority can better handle the tradeoff between adaptation and coordination.

Theorem 1 *Random authority outperforms deterministic authority allocations in the following circumstances:*

1. *Whenever functional authority and project authority have the same organizational performance, random authority is strictly optimal.*

2. *Fix any $\lambda > 0.5$. There exists $\bar{\delta}(\lambda) > 0$ such that random authority is strictly optimal whenever $0 < \delta < \bar{\delta}(\lambda)$.*

3. *Fix any $\delta > 0$. There exist $\underline{\lambda}$ and $\bar{\lambda}$ such that $0.5 < \underline{\lambda} < \bar{\lambda} \leq 1$ and random authority outperforms deterministic authority allocations whenever $\lambda \in (\underline{\lambda}, \bar{\lambda})$. Moreover, as δ approaches*

infinity, the set of λ under which random authority is strictly optimal converges to a measure zero set.

Authority allocation affects the quality of public communication in the organization, which is the driving force of Theorem 1. As the interests of the functional manager are better aligned with those of the project manager, the project managers have a stronger incentive to communicate with the functional manager (vertical communication) than to communicate with the other project manager (horizontal communication), and hence $1 - S_F > 1 - S_P$. Randomization between functional authority and project authority achieves communication quality $1 - S_{RA}(\rho)$, which lies between $1-S_F$ and $1-S_P$. Moreover, as we increase the probability of centralization, communication quality $1 - S_{RA}(\rho)$ will increase in a concave way: the marginal increase becomes higher as the probability ρ_1 decreases, and vice versa; as we increase the probability of decentralization, communication quality $1 - S_{RA}(\rho)$ will decrease in a convex way: the marginal decrease becomes lower as the probability ρ_2 decreases, and vice versa.

The change of communication quality makes it possible that random authority better handles the tradeoff between adaptation and coordination. Under functional authority the functional manager lacks the right information to adapt to local conditions; while under project authority local managers lack the right incentives to ensure effective coordination. Suppose first that project authority outperforms functional authority. Compared to project authority, random authority improves communication quality by occasionally centralizing decision-making, but the possibility of centralization also leads to an adaptation loss. Since the loss is linear in the probability of centralization, by adding a small probability of centralization to project authority, the gain in communication quality may outweigh the loss. In particular, this could occur when (i) the difference between the adaptation advantage of project authority and the coordination advantage of functional authority is small; or (ii) the difference in the quality of horizontal and vertical communication is large, leading to a large gain in communication quality. If, on the contrary, functional authority outperforms project authority, adding a small probability of decentralization to functional authority improves adaptation, but reduces communication quality. In this case, random authority may outperform functional authority when (i) the difference between the the coordination advantage of functional authority and adaptation advantage of project authority is small; or (ii) the difference in the quality of horizontal and vertical communication is small.

In summary, whether random authority is optimal crucially depends on the difference in comparative advantage and the difference in communication quality. Random authority is more likely to be optimal when the difference in comparative advantage is small. When project authority outperforms functional authority, random authority is more likely to be optimal when the difference in communication quality is large, and vice versa. An increase in the coordination need δ or the incentive misalignment λ will increase the relative comparative advantage of functional authority over project authority: as coordination becomes more important or incentives become more misaligned, it is more likely that functional authority dominates project authority. Meanwhile, as shown by ADM, an increase in δ will decrease the difference in communication quality while an increase in λ will increase the difference in communication quality.

Based on the above discussions, we can provide very intuitive explanations for each assertion in Theorem 1. First, when functional authority and project authority perform equally well, the conflict between adaptation and coordination is most severe, and hence the difference in comparative advantage is very small. Then random authority can improve organizational performance by better handling the tradoff between adaptation and coordination. By continuity, there exists a range of incentive misalignment around the indifference point for which random authority is strictly optimal. From Figure 2, the range is not symmetric around the indifference point. When project authority dominates functional authority, random authority is more likely to be optimal. This is because the dashed line in Figure 2 is downward sloping, implying that at this line, either the difference in communication quality is very large (large λ) or coordination is relatively very important (large δ). Both make random authority more likely to dominate project authority, and less likely to dominate functional authority.

Second, when $\delta > 0$ is very small, the difference in communication quality is very large, but still project authority outperforms functional authority because adaptation is much more important. Adding a small probability of centralization to project authority thus can induce a huge gain in communication quality. If $\delta > 0$ is sufficiently small, the gain outweighs the adaptation loss and makes random authority optimal. Moreover, we see from Figure 2 that for any fixed small coordination need, random authority is strictly optimal when the incentives are sufficiently misaligned and vice versa. This is because the difference in communication quality increases in the incentive misalignment λ. As λ becomes larger, random authority is more likely to be optimal since the difference in communication quality increases.

Finally, the limiting result of $\delta \to \infty$ can be understood by considering two cases. First, when λ is small such that project authority outperforms functional authority, it becomes less profitable to employ random authority as $\delta \to \infty$, since the difference in communication quality vanishes. Second, when λ is large such that functional authority outperforms project authority, random authority is suboptimal too, since the difference in comparative advantage is very large. As δ goes to infinity, the set of λ under which random authority is strictly optimal converges to $\{\frac{17}{28}\}$. At this point, functional authority, project authority and all randomization between these two deterministic authority allocations generate the same expected profits.

In practice, there are both successful and unsuccessful stories of matrix management. On the one hand, we have seen a proliferation of matrix organizations in a variety of industries such as aerospace, automotive, banking, chemical, communications, computer, defense, electronics, financial, and energy ([11]; [17]). On the other hand, matrix management has also caused many failures. For example, GE had experienced success using a matrix form in their aircraft business, but its application in the appliance business failed and was abandoned ([4]). People at various levels at Xerox referred to their matrix structure as the culprit behind their company's decline in the 1980s ([22]).

Our theoretical predictions provide explanations for the success or failure of matrix management in different firms, and shed light on why organizations adopt or abandon matrix structures. These predictions can be tested using cross-sectional data. For example, [7] surveyed 1,375 hospitals, and found that 346 hospitals or 28 percent of those responding had adopted matrix management between 1961 and 1978. Of these, 96 hospitals abandoned matrix management during the same period. This dataset can be used to test our theory if we could obtain more information on coordination need and incentive misalignment.

4.3 Comparative Statics

Given an optimal authority allocation, we can define the optimal degree of delegation by $\rho_2^\star$. This subsection investigates how $\rho_2^\star$ changes with the underlying parameter. Our next theorem shows that the optimal degree of delegation changes non-monotonically in coordination need δ when incentives are sufficiently aligned.

Theorem 2 *For any $\lambda \in (\frac{1}{2}, \frac{17}{28})$, the optimal degree of delegation changes non-monotonically in coordination need δ.*

This result seems very surprising because if we only consider deterministic authority allocations, the optimal degree of delegation should be non-increasing in δ as shown by ADM. Intuitively,

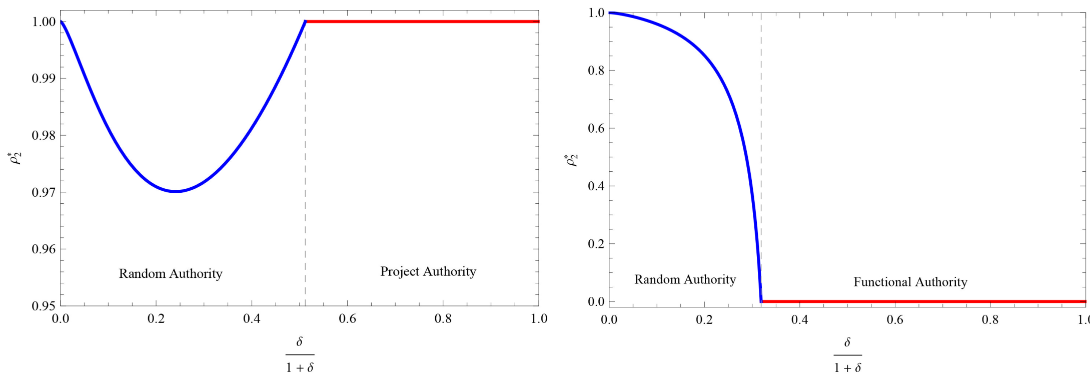

Figure 3: Optimal Degree of Delegation as a Function of δ. A. $\lambda = 0.6$. B. $\lambda = 0.8$.

as coordination becomes relatively more important, there should be a natural tendency toward centralization to improve coordination. This intuition turns out to be incomplete because it neglects the effect on communication quality. As coordination becomes relatively more important, the difference in communication quality diminishes as well.

Therefore, an increase in the coordination need has two opposing effects on optimal authority allocation. On the one hand, to pursue better coordination, it is optimal to assign a higher probability to functional authority. On the other hand, as the difference in communication quality diminishes, it is optimal to assign a lower probability to functional authority. When the incentives are sufficiently aligned, the gain in coordination is limited so that the adaptation advantage of project authority always outweighs the coordination advantage of functional authority. Hence, as the difference in communication quality vanishes (when the coordination need is arbitrarily large), the optimal degree of delegation must converge to one to pursue the adaptation advantage. Meanwhile, when the coordination need is arbitrarily small, the optimal degree of delegation converges to one as well, as adaptation becomes much more important than coordination. From Theorem 1, for any $\lambda \in (\frac{1}{2}, \frac{17}{28})$, $\rho_2^\star < 1$ for $\delta \in (0, \bar{\delta})$. Then, the optimal degree of delegation must satisfy $\rho_2^\star \to 1$ as $\delta \to 0$ or $\delta \to \bar{\delta}$. This in turn implies the property of non-monotonicity as $\rho_2^\star$ is continuous in δ. Panel A of Figure 3 shows a numerical example where $\lambda = 0.6$. In this case, the optimal degree of delegation, $\rho_2^\star$, first decreases and then increases in δ.

If $\lambda > \frac{17}{28}$, it is difficult to obtain an analytical result on comparative statics. Intuitively, when the incentives are sufficiently misaligned, the coordination advantage of functional authority outweighs the adaptation advantage of project authority when coordination is sufficiently important.. Although an increase in δ has two opposing effects, we may thus conjecture that the coordination

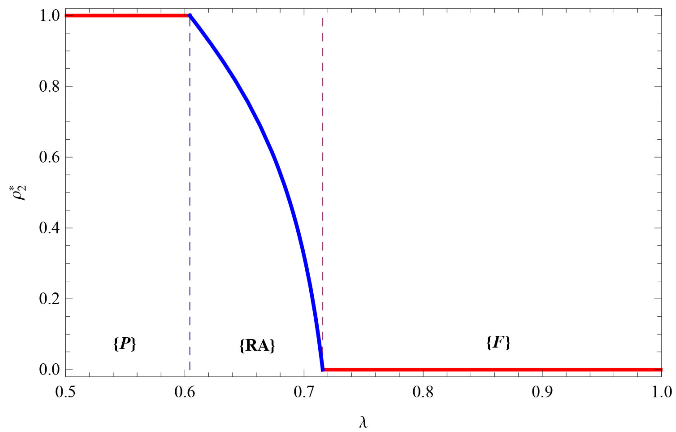

Figure 4: Optimal Degree of Delegation as a Function of λ

effect may always dominate the communication quality effect. As a result, the optimal degree of delegation, $\rho_2^\star$, is monotonically decreasing in δ. Panel B of Figure 3 confirms this when $\lambda = 0.8$.

Figure 4 numerically describes how the optimal degree of delegation changes with λ for a given δ. An increase in λ also has two effects on optimal authority allocation. First, the increase of incentive misalignment reduces the project managers' incentives to coordinate with each other, which requires that a higher probability be assigned to functional authority to promote coordination. Second, the increase of incentive misalignment increases the difference in communication quality, which also makes it more profitable to assign a higher probability to functional authority. Since these two effects work in the same direction, $\rho_2^\star$ should decrease in λ, as shown in Figure 4.

The predictions on comparative statics can potentially be tested using panel data. In particular, if there is an exogenous variation in the coordination need in a group of firms, we may exploit this variation to investigate the effect on degree of decentralization. The model suggests a non-monotonic response when managers put a heavy weight on firm-wide profits, and a monotonic response when managers are very biased toward their own projects. It is also possible to develop a continuous measure of the degree of decentralization by asking a number of senior managers which of a list of critical project decisions are or are not within the authority of the functional manager.[16] The higher the agreement that functional manager has authority over the project, the lower the degree of decentralization.

[16] In fact, [21] employed this method to measure of degree of authority ambiguity in firms.

5 Concluding Remarks

Optimal authority allocations within an organization require that there is a fit between the organizational design and its underlying environments. While economists often focus on the simple dichotomy between centralization and decentralization, authority allocation is in reality more complicated. This paper develops a model to rationalize the use of matrix management, one which contradicts the traditional view that authority should be clearly defined.

The key ingredient of our model is a multi-project organization in which decisions must be adapted to local conditions but also coordinated with each other. Project managers are privately informed about local conditions and communicate strategically via cheap talk. Matrix management is modeled as a randomization over deterministic authority allocations, and this randomization may help the organization to better handle the tradeoff between adaptation and coordination. In symmetric organizations, random authority between functional authority and project authority is strictly optimal when the conflict between adaptation and coordination is very severe or the coordination need is very small. However, there also exist situations where random authority is strictly dominated by deterministic authority allocations. Our findings can potentially shed light on the adoption and abandonment of matrix management in the real world.

Another interesting finding is the non-monotonic relationship between the underlying environment and the optimal degree of delegation in organizations. As far as we know, there is no such finding when focusing on the simple dichotomy between centralization and decentralization. By allowing random authority, the optimal degree of delegation can change continuously with the underlying environment. As a result, non-monotonicity may occur, especially when the change of the underlying environment induces opposing effects on the optimal degree of delegation.

One of the most crucial simplifying assumption in our model is that there is simply one functional manager. Having just one functional manager makes functional authority equivalent to centralization. Ideally, in a complete version of our model, there would be two functional managers and two project managers. Each manager would have private information and communicate with other managers. Similar to our model, matrix organizations can still be captured by random authority where the managers simultaneously send a cheap-talk message and the decision rights are allocated randomly. Analyzing this complete model is in general complicated, and we believe that our main results still hold in the more complicated model, and our model can also be viewed as a first step to analyze the role of random authority in that more complicated model.

As discussed earlier, many predictions of our model can be tested once cross-sectional or panel data is available. While there are many case studies on matrix organizations, a rigorous econometric study is still a fruitful direction for future research. Moreover, our model is applicable to many other situations beyond business organizations. As argued by [26], the concept of a matrix is anything but new. For example, a family can be described as a matrix because there are potentially two bosses, the mother and the father. Meanwhile, government departments have been operating dual authority structures for more than a hundred years ([25]). It would be interesting to extend our framework to these situations.

Appendix: Omitted Proofs

Define the random variable $\overline{m}_i$ as the posterior expectation of the state θ_i by the receiver after observing message m_i. We first state two lemmas from ADM.

Lemma 4 *For any communication equilibria, we have for any $j \in \{1,2\}$ and $i \neq j$, $E_{\theta_j}[\overline{m}_i \overline{m}_j] = E_{\theta_j}[\theta_i \overline{m}_j] = E_{\theta_j}[\theta_j] = E_{\theta_j}[\overline{m}_j] = E_{\theta_j}[\theta_i\theta_j] = 0$;(ii) $E[\overline{m}_j\theta_j] = E[\overline{m}_j^2]$ for $j \in \{1,2\}$.*

Lemma 5 *$E[\overline{m}_j^2]$ is strictly increasing in the number of N_j for $j = 1,2$; and given the authority allocation $\rho = (\rho_1, \rho_2, \rho_3, \rho_4)$, we have*

$$E(\overline{m}_j^2) = \frac{s^2}{3}\left\{1 - \frac{1}{4}\left[\frac{(x^{3N_j}-1)(x-1)^2}{(x^{N_j}-1)^3(x^2+x+1)} + \frac{3x^{N_j}(x-1)^2}{(x^{N_j}-1)^2 x}\right]\right\} \tag{13}$$

where $x = x_{RA}^j(\rho)$ is defined in Eq. (16).

A.1 Proof of Lemma 1

Proof. Obviously, in any perfect Bayesian equilibrium, the optimal decision rules satisfy Eq. (1) (2, 4, 5, resp.) once functional authority F ($P, M1, M2$, resp.) is realized. The proof consists of two parts. The first part is to prove that for any communication equilibria, the communication rules in equilibrium are interval equilibria. This part is the same as that in ADM, and thus omitted.

The second part is to derive the difference equations for all the outcome-equivalent communication equilibria. Let t_j be a partition of $[-s, s]$, and any message $m_j \in (t_{j,i-1}, t_{j,i})$ be denoted by $m_{j,i}$, and $\overline{m}_{j,i}$ be the receivers' expected value of θ_j after receiving a message $m_{j,i}$. For any critical types $t_{j,i}$ where $i \in \{1, 2, ..., N_j - 1\}$, manager $j \in \{1,2\}$ must be indifferent between sending messages $m_{j,i}$ and $m_{j,i+1}$. In other words, given the state $t_{1,i}$,

$$E[U_1|t_{1,i}, \overline{m}_{1,i}] - E[U_1|t_{1,i}, \overline{m}_{1,i+1}] = \rho_1 W_1 + \rho_2 W_2 + \rho_3 W_3 + \rho_4 W_4 = 0,$$

where

$$\begin{aligned}
W_1 &= \left(\lambda(\gamma_F \overline{m}_{1,i+1} - t_{1,i})^2 + (1-\lambda)(1-\gamma_F)^2 \overline{m}_{1,i+1}^2 + \delta(2\gamma_F - 1)^2 \overline{m}_{1,i+1}^2\right) \\
&- \left(\lambda(\gamma_F \overline{m}_{1,i} - t_{1,i})^2 + (1-\lambda)(1-\gamma_F)^2 \overline{m}_{1,i}^2 + \delta(2\gamma_F - 1)^2 \overline{m}_{1,i}^2\right); \\
W_2 &= \left(\lambda\big(\gamma_P \overline{m}_{1,i+1} - k t_{1,i}\big)^2 + (1-\lambda)(\gamma_P')^2 \overline{m}_{1,i+1}^2 + \delta\big((\gamma_P - \gamma_P')\overline{m}_{1,i+1} + (1-k)t_{1,i}\big)^2\right) \\
&- \left(\lambda\big(\gamma_P \overline{m}_{1,i} - k t_{1,i}\big)^2 + (1-\lambda)(\gamma_P')^2 \overline{m}_{1,i}^2 + \delta\big((\gamma_P - \gamma_P')\overline{m}_{1,i} + (1-k)t_{1,i}\big)^2\right); \\
W_3 &= \left(\lambda\big(\gamma_M \overline{m}_{1,i+1} - t_{1,i}\big)^2 + (1-\lambda)k^2\gamma_M^2 \overline{m}_{1,i+1}^2 + \delta(1-k)^2\gamma_M^2 \overline{m}_{1,i+1}^2\right) \\
&- \left(\lambda(\gamma_M \overline{m}_{1,i} - t_{1,i})^2 + (1-\lambda)k^2\gamma_M^2 \overline{m}_{1,i}^2 + \delta(1-k)^2\gamma_M^2 \overline{m}_{1,i}^2\right); \\
W_4 &= \left(\lambda k^2\big(\gamma_M' \overline{m}_{1,i+1} - t_{1,i}\big)^2 + (1-\lambda)(\gamma_M' \overline{m}_{1,i+1})^2 + \delta(1-k)^2(\gamma_M' \overline{m}_{1,i+1} - t_{1,i})^2\right) \\
&- \left(\lambda k^2\big(\gamma_M' \overline{m}_{1,i} - t_{1,i}\big)^2 + (1-\lambda)(\gamma_M' \overline{m}_{1,i})^2 + \delta(1-k)^2(\gamma_M' \overline{m}_{1,i} - t_{1,i})^2\right).
\end{aligned}$$

Substituting $\overline{m}_{1,i} = (t_{1,i-1} + t_{1,i})/2$, we have that $E[U_1|t_{1,i}, \overline{m}_{1,i}] - E[U_1|t_{1,i}, \overline{m}_{1,i+1}] = 0$ iff $t_{1,i} = (t_{1,i+1} + t_{1,i-1})/(2 + 4b_{RA}^1)$, where b_{RA}^1 is defined in Eq. (7). Rearranging it yields:

$$t_{1,i+1} - t_{1,i} = t_{1,i} - t_{1,i-1} + 4b_{RA}^1 \times t_{1,i}.$$

The same reasoning can be applied to manager 2, and thus we have

$$t_{j,i+1} - t_{j,i} = t_{j,i} - t_{j,i-1} + 4b^j_{RA} t_{j,i}, \quad \text{for } j \in \{1,2\}, \tag{14}$$

Solving the difference equation (14) yields

$$t_{j,i} = a_0 (x^j_{RA})^i + a_1 (y^j_{RA})^i \quad \text{for } j \in \{1,2\}. \tag{15}$$

where a_0, a_1 are constants.

The expressions of x^j_{RA} and y^j_{RA} are:

$$x^j_{RA} = \left(1 + 2b^j_{RA}\right) + \sqrt{\left(1 + 2b^j_{RA}\right)^2 - 1} \quad \text{and} \quad y^j_{RA} = \left(1 + 2b^j_{RA}\right) - \sqrt{\left(1 + 2b^j_{RA}\right)^2 - 1}, \tag{16}$$

which satisfy $x^j_{RA} y^j_{RA} = 1$, with $x^j_{RA} > 1$.

Using the boundary conditions $t_{j,0} = -s, t_{j,N_j} = s$, we get the expressions of $t_{j,i}$ for $j \in \{1,2\}$:

$$t_{j,i} = \frac{s}{(x^j_{RA})^{N_j} - (y^j_{RA})^{N_j}} \left\{ (x^j_{RA})^i (1 + (y^j_{RA})^{N_j}) - (y^j_{RA})^i (1 + (x^j_{RA})^{N_j}) \right\} \quad \text{for } 0 \le i \le N_j. \tag{17}$$

■

A.2 Proof of Lemma 2

Proof. It is straightforward to verify that the limit of strategy profiles and beliefs $(p_j(.), d_j(.), q_j(.))$ as $N_j \to \infty$ constitutes a perfect Bayesian equilibrium of the communication game. We will only prove that the total expected profits are increasing in N_j. Under random authority structure ρ, the total expected profits consist of four parts:

$$\pi_{RA} = \rho_1 \pi_F + \rho_2 \pi_P + \rho_3 \pi_{M1} + \rho_4 \pi_{M2}, \tag{18}$$

where

$$\begin{aligned}
\pi_F &= -E\big[(d^F_1 - \theta_1)^2 + (d^F_2 - \theta_2)^2 + 2\delta(d^F_1 - d^F_2)^2\big], \\
\pi_P &= -E\big[(d^P_1 - \theta_1)^2 + (d^P_2 - \theta_2)^2 + 2\delta(d^P_1 - d^P_2)^2\big], \\
\pi_{M1} &= -E\big[(d^{M1}_1 - \theta_1)^2 + (d^{M1}_2 - \theta_2)^2 + 2\delta(d^{M1}_1 - d^{M1}_2)^2\big], \\
\pi_{M2} &= -E\big[(d^{M2}_1 - \theta_1)^2 + (d^{M2}_2 - \theta_2)^2 + 2\delta(d^{M2}_1 - d^{M2}_2)^2\big].
\end{aligned} \tag{19}$$

Using Eq. (1) and (2) and Lemma 4, we have

$$\begin{aligned}
E[(d^F_1 - \theta_1)^2] &= \sigma^2 - (2 - \gamma_F)\gamma_F E(\overline{m}_1^2) + (1 - \gamma_F)^2 E(\overline{m}_2^2), \\
E[(d^F_2 - \theta_2)^2] &= \sigma^2 - (2 - \gamma_F)\gamma_F E(\overline{m}_2^2) + (1 - \gamma_F)^2 E(\overline{m}_1^2), \\
E[(d^F_1 - d^F_2)^2] &= (2\gamma_F - 1)^2 (E(\overline{m}_1^2) + E(\overline{m}_2^2).
\end{aligned} \tag{20}$$

$$\begin{aligned}
E[(d^P_1 - \theta_1)^2] &= k^2\sigma^2 - (2k\gamma_P - \gamma_P^2) E(\overline{m}_1^2) + (\gamma'_P)^2 E(\overline{m}_2^2), \\
E[(d^P_2 - \theta_2)^2] &= k^2\sigma^2 - (2k\gamma_P - \gamma_P^2) E(\overline{m}_2^2) + (\gamma'_P)^2 E(\overline{m}_1^2), \\
E[(d^P_1 - d^P_2)^2] &= \big[(\gamma_P - \gamma'_P)^2 + 2(1-k)(\gamma_P - \gamma'_P)\big](E(\overline{m}_1^2) + E(\overline{m}_2^2) \\
&+ 2(1-k)^2\sigma^2.
\end{aligned} \tag{21}$$

Moreover, from Eq. (4) and Lemma 4, we obtain:

$$
\begin{aligned}
E[(d_1^{M1}-\theta_1)^2] &= \sigma^2 - 2\gamma_M E(\overline{m}_1^2) + \gamma_M^2 E(\overline{m}_1^2) + (\gamma_M')^2 E(\overline{m}_2^2), \\
E[(d_2^{M1}-\theta_2)^2] &= k^2 \left\{\sigma^2 + \gamma_M^2 E(\overline{m}_1^2) + (\gamma_M')^2 E(\overline{m}_2^2) - 2\gamma_M' E(\overline{m}_2^2)\right\}, \\
E[(d_1^{M1}-d_2^{M1})^2] &= (1-k)^2 \left\{\sigma^2 + \gamma_M^2 E(\overline{m}_1^2) + (\gamma_M')^2 E(\overline{m}_2^2) - 2\gamma_M' E(\overline{m}_2^2)\right\}, \\
E[(d_1^{M2}-\theta_1)^2] &= k^2 \left\{\sigma^2 + \gamma_M^2 E(\overline{m}_2^2) + (\gamma_M')^2 E(\overline{m}_1^2) - 2\gamma_M' E(\overline{m}_1^2)\right\}, \\
E[(d_2^{M2}-\theta_2)^2] &= \sigma^2 - 2\gamma_M E(\overline{m}_2^2) + \gamma_M^2 E(\overline{m}_2^2) + (\gamma_M')^2 E(\overline{m}_1^2), \\
E[(d_1^{M2}-d_2^{M2})^2] &= (1-k)^2 \left\{\sigma^2 + \gamma_M^2 E(\overline{m}_2^2) + (\gamma_M')^2 E(\overline{m}_1^2) - 2\gamma_M' E(\overline{m}_1^2)\right\}.
\end{aligned} \tag{22}
$$

Hence, we can calculate the first derivatives as below:

$$
\begin{aligned}
\frac{\partial \pi_F}{\partial E[\overline{m}_j^2]} &= \gamma_F = \frac{1+2\delta}{1+4\delta} > 0, \quad \frac{\partial \pi_P}{\partial E[\overline{m}_j^2]} = \frac{\delta^2\left(6\delta\lambda^2 - \lambda^2 + 4\lambda^3 + 2\delta^2\right)}{(\lambda+\delta)^2(\lambda+2\delta)^2} > 0, \\
\frac{\partial \pi_{M1}}{\partial E[\overline{m}_1^2]} &= \frac{\partial \pi_{M2}}{\partial E[\overline{m}_2^2]} = 2\gamma_M - \left[1+k^2+2\delta(1-k)^2\right]\gamma_M^2 = \frac{(1+2\delta)\lambda(\lambda+2\delta)}{(\lambda+\delta+2\lambda\delta)^2} > 0, \\
\frac{\partial \pi_{M1}}{\partial E[\overline{m}_2^2]} &= \frac{\partial \pi_{M2}}{\partial E[\overline{m}_1^2]} = [2k^2 + 4\delta(1-k)^2]\gamma_M' + \left[1+k^2+2\delta(1-k)^2\right] \times (\gamma_M')^2 \\
&= \frac{4\lambda\delta^2(\lambda^3+\delta^2+\lambda\delta+2\lambda^3\delta)}{(\lambda+\delta)^2(\lambda+\delta+2\lambda\delta)^2} > 0.
\end{aligned}
$$

Since $\frac{\partial \pi_{RA}}{\partial E[\overline{m}_1^2]} = \rho_1 \frac{\partial \pi_F}{\partial E[\overline{m}_1^2]} + \rho_2 \frac{\partial \pi_P}{\partial E[\overline{m}_1^2]} + \rho_3 \frac{\partial \pi_{M1}}{\partial E[\overline{m}_1^2]} + \rho_4 \frac{\partial \pi_{M2}}{\partial E[\overline{m}_1^2]}$ and $\frac{\partial \pi_{RA}}{\partial E[\overline{m}_2^2]} = \rho_1 \frac{\partial \pi_F}{\partial E[\overline{m}_2^2]} + \rho_2 \frac{\partial \pi_P}{\partial E[\overline{m}_2^2]} + \rho_3 \frac{\partial \pi_{M1}}{\partial E[\overline{m}_2^2]} + \rho_4 \frac{\partial \pi_{M2}}{\partial E[\overline{m}_2^2]}$, we have $\frac{\partial \pi_{RA}}{\partial E[\overline{m}_j^2]} > 0$. Finally, from Lemma 5, we have that $E[\overline{m}_j^2]$ increases with N_j and therefore the total expected payoffs π_{RA} increases in the number of intervals N_j. ■

A.3 Proof of Proposition 1

Proof. When functional authority and project authority are realized, the expected profits are identical to that in ADM, and henceforth we directly have:

$$
\pi_F = -\left(2A_F + (1-A_F)\left(S_{RA}^1 + S_{RA}^2\right)\right)\sigma^2, \tag{23}
$$
$$
\pi_P = -\left(2A_P + B_P\left(S_{RA}^1 + S_{RA}^2\right)\right)\sigma^2, \tag{24}
$$

Moreover, from Eq. (19), (22) and lemma 3, we obtain:

$$
\pi_{M1} = -\left(2A_M + \left(B_M S_{RA}^1 + C_M S_{RA}^2\right)\right)\sigma^2, \tag{25}
$$
$$
\pi_{M2} = -\left(2A_M + \left(C_M S_{RA}^1 + B_M S_{RA}^2\right)\right)\sigma^2. \tag{26}
$$

Finally, combining Eq. (18), (23), (24), (25), and (26) will yield the results. ■

A.4 Proof of Observation 1

Proof. From Proposition 1 (Organizational Performance), we obtain:

$$
\Pi(\tilde{\rho}_1) - \Pi(\tilde{\rho}_2) = -\frac{3\lambda\delta^2(2\lambda-1)^2\mathcal{W}_1}{(4\delta+1)(\delta+\lambda)(2\delta+\lambda)(2\delta\lambda+\delta+\lambda)\mathcal{W}_2},
$$

where

$$\begin{aligned}
\mathcal{W}_1 &= -\lambda^8+2\delta\lambda^6 z_1+\delta^2\lambda^5 z_2+\delta^3\lambda^4 z_3+\delta^4\lambda^3 z_4+4\delta^5\lambda^2 z_5+2\delta^6\lambda z_6+2\delta^7 z_7+32\delta^8 z_8+128\delta^9 z_9,\\
\mathcal{W}_2 &= \left(8\delta^4(8\lambda-1)+\delta^3\left(84\lambda^2+8\lambda-1\right)+\delta^2\lambda(8\lambda(5\lambda+3)-1)+2\delta\lambda^3(4\lambda+7)+3\lambda^4\right)\\
&\times\ \left(\delta^3(4\lambda(\lambda(4\lambda+3)+1)-1)+\delta^2(\lambda(2\lambda+1)(2\lambda+9)-1)\lambda+8\delta(\lambda+1)\lambda^3+3\lambda^4\right)>0
\end{aligned}$$

and

$$\begin{aligned}
z_1 &= \left(3\lambda^2-11\lambda+1\right), z_2=\left(44\lambda^3-48\lambda^2-117\lambda+16\right),\\
z_3 &= \left(56\lambda^4+152\lambda^3-502\lambda^2-276\lambda+55\right),\\
z_4 &= \left(352\lambda^4-192\lambda^3-1676\lambda^2-280\lambda+103\right),\\
z_5 &= \left(16\lambda^5+204\lambda^4-512\lambda^3-691\lambda^2+8\lambda+28\right),\\
z_6 &= \left(144\lambda^5+288\lambda^4-2408\lambda^3-1128\lambda^2+177\lambda+34\right),\\
z_7 &= \left(320\lambda^5-528\lambda^4-2608\lambda^3-288\lambda^2+160\lambda+9\right),\\
z_8 &= \left(16\lambda^4-68\lambda^3-76\lambda^2+9\lambda+3\right), z_9=\left(-8\lambda^2-2\lambda+1\right).
\end{aligned}$$

It is straightforward to verify that given any $\lambda \in [\frac{1}{2}, 1]$, $z_j < 0$ for $1 \le j \le 9$. Therefore, $\mathcal{W}_1 < 0$, which implies that $\Pi(\tilde{\rho}_1) - \Pi(\tilde{\rho}_2) \ge 0$. ■

A.5 Proof of Theorem 1

Proof. To prove Theorem 1, we will first derive the expression of $\frac{d\pi_{RA}(\rho_2)}{d\rho_2}$. By plugging $\rho_1 = 1-\rho_2$ into Eq. (12) and with a little abuse of notation, let $\pi_{RA}(\rho_2)$ be the resulting function. Take the first derivative with respect to ρ_2, and we obtain:[17]

$$\frac{d\pi_{RA}(\rho_2)}{d\rho_2}=\frac{(1-\mu)\mu\lambda(2\lambda-1)\left(a(\lambda,\mu)\rho_2^2+b(\lambda,\mu)\rho_2+c(\lambda,\mu)\right)}{(1+3\mu)\left[(3-2\lambda)\mu+\lambda^2+\mu^2(2-\lambda)(1-\mu)\right](\psi_2+\psi_3\rho_2)^2}, \tag{27}$$

where $\mu = \frac{\delta}{1+\delta}$ and

$$\begin{aligned}
a(\lambda,\mu) &= -(1-\mu)^2\Big\{-3\lambda^8+\mu\lambda^6\left(3-56\lambda+43\lambda^2\right)+\mu^2\lambda^5\left(25-346\lambda+372\lambda^2-19\lambda^3\right)\\
&+\ \mu^3\left(\lambda^3+67\lambda^4-911\lambda^5+893\lambda^6+388\lambda^7-189\lambda^8\right)\\
&+\ \mu^4\lambda^2\left(5+52\lambda-956\lambda^2-190\lambda^3+3204\lambda^4-1720\lambda^5+191\lambda^6\right)\\
&+\ \mu^5\lambda\left(8-29\lambda-134\lambda^2-2710\lambda^3+669\lambda^4-3299\lambda^5-48\lambda^6+169\lambda^7\right)\\
&+\ \mu^6\left(4-34\lambda+183\lambda^2-1900\lambda^3+2596\lambda^4+2469\lambda^5-4874\lambda^6+2180\lambda^7-297\lambda^8\right)\\
&+\ 3\mu^7\left(2-26\lambda+139\lambda^2-865\lambda^3+2329\lambda^4-2697\lambda^5+1473\lambda^6-372\lambda^7+35\lambda^8\right)\Big\},
\end{aligned}$$

[17] The expressions of ψ_2 and ψ_3 are shown in the proof of Lemma 6.

$$
\begin{aligned}
b(\lambda,\mu) &= -2(1-\mu)\left(\lambda+(2-\lambda)\mu\right)^2\Big\{3\lambda^6+\mu\lambda^4\left(-3+44\lambda-40\lambda^2\right)\\
&- \mu^2\lambda^3\left(14-160\lambda+168\lambda^2+21\lambda^3\right)+\mu^3\lambda^2\left(-19+170\lambda+32\lambda^2-472\lambda^3+168\lambda^4\right)\\
&- \mu^4\lambda\left(4+23\lambda-606\lambda^2+1242\lambda^3-576\lambda^4+23\lambda^5\right)\\
&+ \mu^5\left(4-72\lambda+383\lambda^2-386\lambda^3-477\lambda^4+716\lambda^5-192\lambda^6\right)\\
&+ 3\mu^6\left(2-36\lambda+209\lambda^2-488\lambda^3+510\lambda^4-232\lambda^5+35\lambda^6\right)\Big\},\\
c(\lambda,\mu) &= (\lambda+(2-\lambda)\mu)^3\Big\{3\lambda^5+\mu\lambda^3\left(-3+38\lambda-40\lambda^2\right)\\
&- \mu^2\lambda^2\left(9-89\lambda+94\lambda^2+21\lambda^3\right)+\mu^3\lambda\left(-9+54\lambda+94\lambda^2-356\lambda^3+168\lambda^4\right)\\
&- \mu^4\left(3+26\lambda-320\lambda^2+658\lambda^3-356\lambda^4+23\lambda^5\right)\\
&- \mu^5\left(26-137\lambda+78\lambda^2+395\lambda^3-542\lambda^4+192\lambda^5\right)\\
&+ 3\mu^6\left(-13+94\lambda-245\lambda^2+291\lambda^3-162\lambda^4+35\lambda^5\right)\Big\}.
\end{aligned}
$$

Moreover, the sign of $\frac{d\pi_{RA}(\rho_2)}{d\rho_2}$ at $\rho_2=1$ is the same as the sign of $s(\lambda,\mu)$, where

$$
\begin{aligned}
s(\lambda,\mu) &\triangleq a(\lambda,\mu)+b(\lambda,\mu)+c(\lambda,\mu)\\
&= -\mu^3(1+3\mu)^2\Big\{\lambda^3\left(1-8\lambda+12\lambda^2\right)+\mu\lambda^2\left(5-39\lambda+73\lambda^2-36\lambda^3\right)\\
&+ 3\mu^4\left(10-70\lambda+149\lambda^2-118\lambda^3+31\lambda^4\right)+\mu^2\lambda\left(8-47\lambda+57\lambda^2-29\lambda^3+36\lambda^4\right)\\
&+ \mu^3\left(4+10\lambda-181\lambda^2+335\lambda^3-129\lambda^4-12\lambda^5\right)\Big\}.
\end{aligned}
$$

The next lemma shows that π_{RA} is a concave function of ρ_2. Therefore, there is an interior maximizer of π_{RA} when $\frac{d\pi_{RA}(\rho_2)}{d\rho_2}=0$ holds for some $\rho_2\in(0,1)$.

Lemma 6 *Given any pair of* $(\lambda,\mu)\in\left\{(\frac{1}{2},1)\times(0,1)\right\}$, *the second derivative* $\frac{d^2\pi_{RA}(\rho_2)}{d\rho_2^2}<0$ *for any* $\rho_2\in[0,1]$.

Proof. From Eq. (27), we obtain

$$
\operatorname{sign}\left\{\frac{d^2\pi_{RA}(\rho_2)}{d\rho_2^2}\right\}=\operatorname{sign}\left\{\frac{6(1-\mu)^2\mu^4(1+3\mu)(2\lambda-1)^2\lambda\big((1-\mu)\lambda+2\mu\big)^2\psi_1}{(\psi_2+\psi_3\rho_2)^3}\right\}, \tag{28}
$$

where

$$
\begin{aligned}
\psi_1 &= \lambda^3-6\lambda^4+\mu\lambda^2(3-14\lambda-\lambda^2)+\mu^2\lambda(3-6\lambda-21\lambda^2+10\lambda^3)\\
&+ \mu^3(1+2\lambda-17\lambda^2+10\lambda^3),\\
\psi_2 &= (2\mu+\lambda(1-\mu))^2\big(1-(1-\lambda)(1-\mu)\big)\big(3\lambda+(5\lambda-1)\mu\big),\\
\psi_3 &= -(1-\mu)\Big\{3\lambda^4+\mu\lambda^3(14-\lambda)+\mu^2\left(-7\lambda^4+12\lambda^3+14\lambda^2+\lambda+1\right)\\
&+ \mu^3\lambda\left(5\lambda^3-26\lambda^2+30\lambda-1\right)\Big\}.
\end{aligned}
$$

Rearranging yields:

$$
\begin{aligned}
\psi_1 &= -\lambda^3(6\lambda-1)-\mu\lambda^2(-3+14\lambda+\lambda^2)-\mu^2\times[3\lambda(2\lambda-1)+10\lambda(1-\lambda)]\\
&- \mu^3\times\left[(2\lambda-1)(2\lambda+1)+2\lambda(2\lambda-1)+9\lambda^2\right]-10\mu^2\lambda^3(1-\mu)-\mu^2\lambda^3.
\end{aligned}
$$

Henceforth, for any $\lambda > \frac{1}{2}$ and $\mu > 0$ we have $\psi_1 < 0$. It is straightforward to verify that

$$\psi_2 + \psi_3 = \mu^2(1+3\mu)\Big((2\lambda-1)\lambda + 3\lambda^2 + 5\lambda\mu(1-\mu) + (2\lambda-1)\mu + 2\lambda\mu\Big) > 0,$$

which implies that $\psi_2 + \rho_2\psi_3 > 0$ for any ρ_2. Therefore, we obtain $\frac{d^2\pi_{RA}(\rho_2)}{d\rho_2^2} < 0$ for any $\rho_2 \in [0,1]$. ■

Theorem 1 then follows directly from lemma 7 to lemma 9 below.

Lemma 7 *Whenever functional authority and project authority have the same organizational performance, random authority is strictly optimal.*

Proof. If functional authority and project authority have the same organizational performance, $\pi_{RA}(0) = \pi_{RA}(1)$. Since the expected profits are concave in the allocation of authority as shown by lemma 6, we must have $\pi_{RA}(\rho) > \pi_{RA}(0) = \pi_{RA}(1)$ for any $\rho \in (0,1)$. Hence, random authority is strictly optimal.

Consider any pair of parameters $(\lambda^\star, \delta^\star)$ under which functional authority and project authority have the same organizational performance. Since random authority is strictly optimal, we must have $c(\lambda^\star, \mu^\star) > 0$ and $s(\lambda^\star, \mu^\star) < 0$. By continuity, there must exist a neighborhood of $(\lambda^\star, \delta^\star)$ such that $c(\lambda, \mu) > 0$ and $s(\lambda, \mu) < 0$. In this neighborhood, random authority is strictly optimal as well. ■

Lemma 8 *For any $\lambda \in (\frac{1}{2}, 1]$, random authority is always strictly optimal when $\delta > 0$ is sufficiently small.*

Proof. Recall that the sign of $\frac{d\pi_{RA}}{d\rho_2}$ is the same as that of $g(\rho_2) = a(\lambda,\mu)\rho_2^2 + b(\lambda,\mu)\rho_2 + c(\lambda,\mu)$. Obviously, $g(0) = c(\lambda,\mu)$ and $g(1) = s(\lambda,\mu)$.

From the expressions of $c(\cdot)$ and $s(\cdot)$, we have for any $\lambda \in (\frac{1}{2}, 1]$:

$$\lim_{\mu\to 0} c(\lambda,\mu) = 3\lambda^8 > 0, \quad \text{and} \quad \lim_{\mu\to 0} \frac{s(\lambda,\mu)}{\mu^3} = -3\lambda^8(2\lambda-1)(6\lambda-1) < 0.$$

Hence, for any $\lambda \in (1/2, 1)$ and sufficiently small $\epsilon > 0$, $g(0) > 0$ and $g(1) < 0$ for any $\delta \in (0,\epsilon)$. Then $g(\rho_2) = 0$ at some interior probability, and the corresponding random authority will dominate deterministic authority allocations. ■

Lemma 9 *Fix any $\delta > 0$. There exist $\underline{\lambda}$ and $\bar{\lambda}$ such that $\frac{1}{2} < \underline{\lambda} < \bar{\lambda} \leq 1$ and random authority dominates deterministic authority whenever $\lambda \in (\underline{\lambda}, \bar{\lambda})$. Moreover, as $\delta \to \infty$, the set of λ under which random authority is strictly optimal converges to a measure zero set.*

Proof. Note that $g(0) = c(\lambda,\delta) > 0$ and $g(1) = s(\lambda,\delta) < 0$ together constitutes a sufficient condition for random authority being strictly optimal. Let $\lambda^\star(\delta) = \frac{17}{28} + \kappa \times \frac{1}{1+\delta}$, and then we have $c(\lambda,\delta)|_{\lambda=\lambda^\star(\delta)} > 0$ and $s(\lambda,\delta)|_{\lambda=\lambda^\star(\delta)} < 0$ for any $\kappa \in [\frac{1}{90}, \frac{1}{15}]$.

Denote $\bar{\lambda}(\delta) = \frac{17}{28} + \frac{1}{15(1+\delta)}$ and $\underline{\lambda}(\delta) = \frac{17}{28} + \frac{1}{90(1+\delta)}$. For any δ, $c(\lambda,\delta) > 0$ and $s(\lambda,\delta) < 0$ if $\underline{\lambda} < \lambda < \bar{\lambda}$.

Finally, it is easy to verify that $\delta \to \infty$, $\mu = \frac{\delta}{1+\delta} \to 1$, and hence both $a(\lambda,\mu)$ and $b(\lambda,\mu)$ converge to 0. Therefore, in the limit where $\delta \to \infty$, random authority cannot be strictly optimal. ■

A.6 Proof of Theorem 2

Proof. The non-monotonicity result follows directly from the followings observations. (i) When $\delta \to 0$, the optimal delegation probability $\rho_2^\star \to 1$. To see it, note that

$$\lim_{\mu \to 0} a(\lambda, \mu) = 3\lambda^8, \lim_{\mu \to 0} b(\lambda, \mu) = -6\lambda^8, \text{and} \lim_{\mu \to 0} c(\lambda, \mu) = 3\lambda^8.$$

Obviously, when $\mu = 0$, $g(1) = 0$ for any λ. By continuity, ρ_2 solving equation $g(\rho_2) = 0$ will converge to 1 as $\delta \to 0$. Hence, the optimal delegation probability $\rho_2^\star \to 1$ as $\delta \to 0$.

(ii) For any $\lambda \in (\frac{1}{2}, \frac{17}{28})$, the optimal delegation probability is given by $\rho_2^\star = 1$ when $\delta \to \infty$. This is because

$$\lim_{\delta \to \infty} g(\rho_2) = \lim_{\mu \to 1} g(\rho_2) = -32(4\lambda - 1)(28\lambda - 17).$$

Thus, for any $\lambda \in (\frac{1}{2}, \frac{17}{28})$, $\frac{d\pi_{RA}(\rho_2)}{d\rho_2} > 0$ will always hold, implying that $\rho_2^\star = 1$.

(iii) As shown by lemma 8, random authority is strictly optimal when $\delta > 0$ is sufficiently small. In other words, $\rho_2^\star < 1$ when $\delta > 0$ is sufficiently small. Since $\rho_2^\star$ is continuous in δ, it must be non-monotonic in δ. ■

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